Information Sharing in Financial Markets

Itay Goldstein, Yan Xiong and Liyan Yang*

02/04/22

Abstract

This paper studies information sharing between strategic investors who are privately informed about asset fundamental with different precision levels. We find that a coarsely informed investor would always share her information "as is" if her counterparty investor is well informed about the fundamental. By doing so, the coarsely informed investor invites the well informed investor to trade against her information, thereby offsetting her informed order flow and reducing the price impact. In equilibrium, the coarsely informed investor gains from the information sharing and the well informed investor loses from it. Our model sheds new light on phenomena such as communication on social media, investors' trading strategies based on sentiment, and information networks in financial markets.

Keywords: Information sharing, communication, sentiment, asset pricing

JEL: D82, G14, G18

*Goldstein: University of Pennsylvania, itayg@wharton.upenn.edu. Xiong: Hong Kong University of Science and Technology, yanxiong@ust.hk. Yang: Rotman School of Management, University of Toronto, liyan.yang@rotman.utoronto.ca. An earlier version of this paper circulated under the title "Whoever Has Will Be Given More: Information Sharing in Financial Markets." We thank Swaminathan Balasubramaniam, Snehal Banerjee, Ing-Haw Cheng, Jean Edouard Colliard, Jason Donaldson, Phil Dybvig, Hanming Fang, Thierry Foucault, Chong Huang, Naveen Khanna, Richard E. Kihlstrom, Pete Kyle, Mina Lee, Jiasun Li, Michael Sockin, Günter Strobl, Laura Veldkamp, Xavier Vives, Vladimir Vladimirov, Chaojun Wang, Bart Zhou Yueshen, Yao Zeng, and the conference/seminar participants at the 2021 Northern Finance Association, the 2022 AFA Annual Meeting, the 2022 Econometric Society Conference, Future of Financial Information Conference, Accounting and Economics Society Webinars, Mid-Atlantic Research Conference in Finance (MARC), the 2020 Greater China Area Finance Conference, Virtual Finance Theory Seminar, Renmin University of China, ShanghaiTech University, Tsinghua University, University of Chinese Academy of Sciences, University of Nevada, Las Vegas, University of Technology Sydney, University of Utah, University of Warwick, and Zhejiang University.

1 Introduction

In financial markets, some well-known proverbs such as "a barking dog never bites" and "a loaded wagon makes no noise" vividly describe the situation in which market participants who own superior information carefully hide and trade on it. In other words, a dog that makes a good bite is the one that is the most silent. Meanwhile, these proverbs suggest that the market participants who make the most noise (a barking dog) rarely express true insights. While it is relatively rare to observe the investment genius share their insights in the public, there has been a great number of investment opinions and analysis shared on social media such as Seeking Alpha, StockTwits, and Reddit. Then, why does a barking dog bark? Is that just noise?

Information sharing also appears to be commonplace among professional investors. For example, Shiller and Pound (1986) provide survey evidence that a majority of institutional investors in the NYSE attribute their recent trades to discussions with peers. Hong, Kubik, and Stein (2005) find that a mutual fund manager's trading is similar to other fund managers located in the same city and interpret this finding in terms of information spread by word of mouth. More recently, investment conferences as a new type of industry event have become popular. In these conferences, professional investors pitch their investment ideas to the wide audiences including activists, fundamental equity funds, investment advisors, and sell-side analysts; these presented investment ideas are closely followed in the financial media and on investment blogs (Luo, 2018). Why do these professional investors share their investment ideas? In this investor community, who shares information with whom?

Some common intuition for an investor's information-sharing behavior holds that by sharing her privileged information, the investor can manipulate markets (Benabou and Laroque, 1992), or accelerate price discovery towards the direction that is in favor of her existing positions (Ljungqvist and Qian, 2016). However, such intuition implies that only investors with known superior information have incentives to share their information, which cannot explain why information sharing is such a widespread phenomenon in financial markets (e.g., social media).

In this paper, we propose that even an investor with coarse information has a strategic motive to genuinely share her information. This novel theory complements the existing explanations for investors' information-sharing behavior. It can be further applied to understand the role of social media in financial markets, the nascent sentiment trading strategy implemented by institutional investors, and the formation of and information transmission within an information network.

To study a strategic investor's information-sharing incentives in financial markets, we adopt a standard Kyle (1985) framework and extend it by considering two investors endowed with private information of different precision and allowing for information sharing between them. The market consists of noise traders, competitive market makers, and two risk-neutral rational investors. A single risky asset is traded in the financial market. One investor perfectly learns about the fundamental of the asset and we refer to him as H. The other investor only observes a noisy signal of the fundamental and we refer to her as L. Investors can share their own private information with each other; for example, L can choose whether to share her information, and if so, how much to share with H. The rest of the model is standard: the two investors trade on their respective endowed information and the shared information, if any; they then submit market orders to maximize their expected profits; and the orders are executed by market makers at the conditional expected value of the asset given the total order flows.

The central finding of the paper is that in financial markets information can transmit from the less informed investor to the more informed one; that is, H never shares his information whereas L always genuinely shares her information. Such information-sharing behavior has further consequences for the involved investors' profits and market quality. Specifically, after the information sharing, L makes higher profits whereas H becomes worse off, and market liquidity worsens but both market efficiency and total trading volume increase.

Why is L willing to share her information "as is"? The key driving force for her information-sharing behavior is the novel *trading-against-error effect*. This effect crucially hinges on the fact that H is able to identify the error in L's endowed information. After observing the shared information, H tends to trade against the error in the shared information, thereby offsetting L's informed order flow and reducing its price impact. Specifically, H well understands L's trading strategy and can calculate her trading demand that is not justified on the basis of the asset fundamental. For example, if the error component in the shared information is positive, L tends to overly buy or inadequately short the risky asset. From the perspective of H, however, this is pure noise.

If, say, L overly buys the asset, after observing the shared information, H knows that the asset price is pushed too high so that he optimally refrains from buying too many units of the asset. Alternatively, if L inadequately shorts the risky asset because of the noise component in her endowed information, H knows that the asset price is not low enough and he will short more of the asset. In either case, H trades against the shared information, corrects the mispricing, and makes profits accordingly. In some sense, H provides liquidity to L. Overall, the trading-against-error effect encourages L to share her information.

Further, the more genuine the shared information, the more accurately H can calculate L's demand that is driven by the error in her endowed information, the more aggressively H trades against the shared information, and the less price impact triggered by L's order flow. Consequently, the trading-against-error effect makes L not only share her information, but do so truthfully.

Next, why is H never sharing his information? Different from that H trades against the information shared by L, L always trades alongside H's shared information. This is because for L, any piece of information shared by the more informed investor is instrumental for her to make better forecast of the asset fundamental. Therefore, for H any information sharing can only dissipate his informational advantage and erode his profits, which prevents him from sharing his information. In this way, information can only flow from L to H.

Such information sharing has further consequences for the profits of the involved investors and market quality. We find that relative to the economy without information sharing, when to share information is permitted, L makes higher profits whereas H becomes worse off. As analyzed above, L benefits from the trading-against-error effect in information sharing. Nonetheless, why is H worse off? While H gains by detecting the error component in the shared information, trading against it, and correcting the mispricing accordingly, he loses because of the more competitive pressure from L and the more aggressive pricing by market makers. First, with her order flow partially offset, L is less concerned about the price impact and engages in more aggressive trading accordingly. As such, H is forced to trade less aggressively on his fundamental information. Second, as H trades against the shared information (and thus the error in L's endowed information), the two investors' aggregate order flow becomes more correlated with the fundamental. Faced with an effectively more informed investor side, market makers raise the price impact to manage the increased adverse-selection risk, thereby decreasing market liquidity. Taken together, both effects hurt H, ironically leaving him worse off despite the additional piece of free information.

As for market quality, we find that relative to the economy without information sharing, when to share information is permitted, market liquidity is lower whereas market efficiency and total trading volume are higher. Again, the key lies in the fact that H tends to trade against the shared information. Because this trading-against-error effect can reduce the noise in the two investors' aggregate order flow, market makers raise the price impact to manage the increasing adverse-selection risk, resulting in a lower market liquidity. Meanwhile, less noise in the total order flow suggests that it is more correlated with the asset fundamental. Therefore, market efficiency improves, i.e., asset prices contains more fundamental information. Information sharing is also associated with more total trading volume; specifically, while H's trading volume decreases after information sharing, both L's and market makers' trading volume increases, and overall the total trading volume increases.

Finally, we consider several extensions of the baseline model and show that the novel trading-against-error effect robustly exists and information sharing remains a prevalent phenomenon. First, even if H is not perfectly informed about the asset fundamental, as long as he can relatively accurately sift the error component in the shared information, L will genuinely share her information so as to benefit from the counterparty offsetting her informed order flow. In other words, information can be transmitted from a coarsely informed investor to a relatively well (though not perfectly) informed investor. Second, our baseline model suggests that if possible, H would like to commit not to reading the shared information. However, we find that in the presence of multiple Hs, even if they are able to make such a commitment, in equilibrium all Hs may choose to read and trade against the shared information. This constitutes a prisoner's dilemma for Hs because they would have been better off if they together commit to not using L's shared information. Third, when peer Ls are present, despite the loss of informational advantage, each L may still share her information if Ls' private information contains common noise (e.g., sentiment). Fourth, even though the shared information may be leaked to the public during the communication process, we find that as long as other investors (e.g., market makers) have low capabilities to interpret the information whereas H has superior ability to do so, L still has incentives to share her private information.

Overall, our theory provides a novel perspective of information-sharing behavior in financial markets and sheds new light on the related phenomena. First, as for the barking-dog questions raised in the beginning, we argue that the investment opinions expressed on social media such as StockTwits may not be mere noise, but instead can represent the true information owned by the posters. By making their information observable to the well informed investors such as hedge funds, these social media investors can have their order flow partially offset. Indeed, as the machine learning technology advances, analyzing the sentiment on the social media becomes feasible and gains popularity among hedge funds. Our theory further suggests that such a sentiment-based trading strategy may not be good for well informed hedge funds as they can become worse off after trading against the sifted sentiment (error). Second, the two investors in our model can represent the very basic component of any information network in financial markets. In this sense, our theory provides an answer to such fundamental questions as how information network is formed and who shares information with whom.

Related Literature Previous research has identified other possible reasons for why investors share their information. For example, insiders can use privileged information to manipulate markets (Benabou and Laroque, 1992). Ljungqvist and Qian (2016) suggest that in the face of noise trader risk, arbitrageurs with short positions may reveal their information to accelerate price correction, thereby circumventing limits to arbitrage. The idea that information revelation can be used to accelerate price correction is particularly relevant for investors with short-term incentives (Kovbasyuk and Pagano, 2015; Liu, 2017; Schmidt, 2019). In addition, by injecting noise into the spread information, an investor gains advantage over uninformed followers (Van Bommel, 2003) and commits to aggressive trading to other informed investors (Indjejikian, Lu, and Yang, 2014); by disclosing a mixture of fundamental information and her position, an investor induces market makers to move the asset price in a manner favorable to her (Pasquariello and Wang, 2016). Foucault and Lescourret (2003) show that information sharing is possible between traders with different types of information (fundamental vs. non-fundamental information). In a contemporaneous paper, Balasubramaniam (2020) shows that competing traders share information when they disagree enough with each other.

We contribute to this literature by offering a complementary explanation for information sharing/revelation in financial markets. Our explanation is unique in the following aspects. First, in our model information transmits from the *less* informed investor to the *more* informed one, whereas in the other explanations the direction of information flow is the opposite. The key underlying this insight is that the information receiver tends to trade against the shared information, whereas in the existing explanations the receiver's trading is aligned with what the shared information suggests. This unique direction of information flow demonstrates that information sharing can be a widespread phenomenon in financial markets; that is, not only well informed investors, but also the ones with (very) coarse information, would be willing to share their information. Second, in our model the information sender is better off whereas the receiver becomes worse off after the information sharing. However, in the existing explanations, both should make profits from the information sharing at the expense of third parties (e.g., Indjejikian, Lu, and Yang, 2014; Foucault and Lescourret, 2003). Third, our explanation does not require that the information sender owns initial positions or has short-term incentives. Unlike other explanations in which the investor "talks for her book" (e.g., Pasquariello and Wang, 2016; Schmidt, 2019), in our model the investor does not have any book yet and she instead reveals information to help build it.

Our paper is also related to the large literature on information transmission in financial markets. Starting from Admati and Pfleiderer (1986, 1988, 1990), there have been studies on how the informed agent monetizes her private information by selling it (e.g., Allen, 1990; Naik, 1997; Cespa, 2008; García and Sangiorgi, 2011). Fishman and Hagerty (1995) rationalize the sales of information by arguing that via it informed traders can commit to aggressive trading, thereby forcing other informed traders to trade less aggressively. Biais and Germain (2002) study how to structure a combination of proprietary trading with indirect information sales (setting up a fund) to increase the overall profits from proprietary trades and fund trades. In addition to the transmission of information for a fee, the informed agents in financial markets may disclose their private information for various reasons such as fear of negative inferences (Grossman and Hart, 1980; Grossman, 1981; Milgrom, 1981) and elimination of screening (Glode, Opp, and Zhang, 2018). Again, one commonality of this literature is that information transmits from the more informed agents to the less informed ones, and our model uniquely predicts that information can flow in the opposite direction.

Another strand of related literature studies noise/supply information in financial markets. Ganguli and Yang (2009) examine investors' incentives of acquiring information about the fundamental and the noise/supply in a static model, whereas Farboodi and Veldkamp (2020) consider a dynamic setting and study how financial data technology affects agents' information choices, trading strategies, and market outcomes. Other related works include Madrigal (1996), Cao, Lyons, and Evans (2003), Paul and Rytchkov (2018), among others. We contribute to this literature by focusing on the noise component in the investor's private information, rather than that in the asset supply, and we highlight its effect on investors' information-sharing behavior. Further, in our setting, the well informed investors do not actively search for the noise/supply information (e.g., Farboodi and Veldkamp, 2020), instead such information is voluntarily shared by the coarsely informed investors.

Our paper is also broadly related to the literature on communication and information network in financial markets. DeMarzo, Vayanos, and Zwiebel (2003) propose a model in which individuals with bounded rationality are subject to persuasion bias and fail to account for repetition in the information they receive; as such, the influence of an individual on group opinions depends not only on accuracy but also on her connectedness. Han and Hirshleifer (2016) study how the process by which ideas are transmitted affects active versus passive investment behavior. We contribute to this literature by identifying a novel information-sharing incentive and exploring its implications for market quality.

2 Model Setup

We consider a Kyle-type model (Kyle, 1985) and extend its analysis to allow for information sharing between investors. The economy has three dates, $t = 0, 1, 2$. Figure 1 describes the timeline of the economy. There is a single risky asset with a date-2 liquidation value $\tilde{v}$, where $\tilde{v} \sim N(0, 1)$.[1] The risky asset can be interpreted as a listed firm's stock. The financial market operates on date 1, and it is populated by three groups of agents: competitive market makers, noise traders, and two heterogeneously informed rational investors. As standard in the literature, market makers set the price based on the weak market-efficiency rule and noise traders submit exogenous random market orders. There are two rational investors that own private information about the fundamental of the risky asset and their information is of different precision. On $t = 0$, information can be shared between the two rational investors.

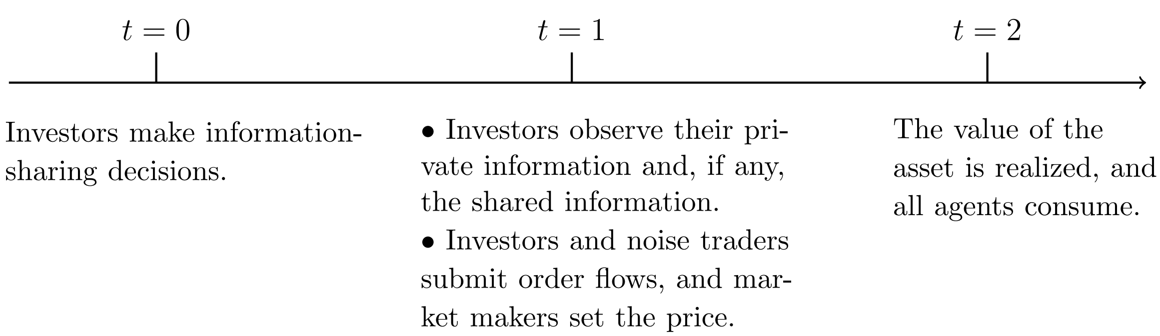


Figure 1: Timeline

We denote the two rational investors H and L. H owns more precise information about the fundamental and he can be a corporate executive or a sophisticated hedge fund manager that possesses high-quality information about the firm's fundamental. Suppose that H perfectly observes $\tilde{v}$. L is coarsely informed about the fundamental, and she can only observe a private noisy signal as follows:

$$\tilde{y} = \tilde{v} + \tilde{e}, \text{ where } \tilde{e} \sim N(0, \rho^{-1}). \tag{1}$$

[1] The normalization that $\tilde{v}$ has a zero mean and a unit standard deviation is without loss of generality. Instead, if we assume $\tilde{v} \sim N(\bar{v}, \sigma_v^2)$, then all our results would hold as long as we reinterpret the information precisions as signal-to-noise ratios.

The parameter $\rho \in (0, +\infty)$ governs the quality of L's private information. If $\rho \to 0$, L is almost uninformed about the asset fundamental, whereas if $\rho \to \infty$, L knows the fundamental as precisely as H. L can represent investors that actively collect information but are still coarsely informed.

On $t = 0$, the two rational investors simultaneously make information-sharing decisions to maximize their respective expected trading profits. Specifically, H can share a garbled signal with L as follows:

$$\tilde{s}_H = \tilde{v} + \tilde{\varepsilon}_H, \text{ where } \tilde{\varepsilon}_H \sim N\left(0, \tau_H^{-1}\right),$$

whereas L's shared information is as follows:

$$\tilde{s}_L = \tilde{y} + \tilde{\varepsilon}_L, \text{ where } \tilde{\varepsilon}_L \sim N\left(0, \tau_L^{-1}\right).$$

The precisions of the shared information τ_H and τ_L are controlled by H and L, respectively, and can range between 0 and $+\infty$; that is, $\tau_i \in [0, +\infty]$, where $i \in \{H, L\}$. If $\tau_i = 0$, investor i's shared information is not informative at all, or equivalently investor i is not sharing any private information. If $\tau_i = +\infty$, then investor i shares the private information "as is."[2]

Trading occurs on $t = 1$. Let $\tilde{p}$ denote the date-1 price of the risky asset in the financial market. Conditional on the endowed private information, as well as the shared information (if any), investor $i \in \{H, L\}$ places market order $\tilde{x}_i$ to maximize the expected trading profits as follows:

$$E[\tilde{x}_i(\tilde{v} - \tilde{p})|\mathcal{F}_i], \tag{2}$$

where $\mathcal{F}_i$ indicates investor i's information set: $\mathcal{F}_H = \{\tilde{v}, \tilde{s}_H, \tilde{s}_L\}$ and $\mathcal{F}_L = \{\tilde{y}, \tilde{s}_H, \tilde{s}_L\}$. Noise traders place market order $\tilde{u}$, where $\tilde{u} \sim N(0, \sigma_u^2)$ (with $\sigma_u > 0$) and $\tilde{u}$ is independent

[2]We assume that investors can commit themselves to a information-sharing policy before receiving private information. The otherwise involved strategic communication issue (e.g., Sobel, 1985) is beyond the scope of the current paper. Moreover, even if we restrict that investors can share either all or none of their information, our results remain robust.

of all other random shocks. Therefore, the total order flow faced by market makers are

$$\tilde{\omega} = \tilde{x}_H + \tilde{x}_L + \tilde{u}. \tag{3}$$

Then competitive market makers set price $\tilde{p}$ according to the weak-efficiency rule,

$$\tilde{p} = E[\tilde{v}|\tilde{\omega}]. \tag{4}$$

3 Information Sharing in Financial Markets

In this section, we first characterize the two investors' optimal trading strategies on $t = 1$ given their information-sharing decisions, and then move backward to solve for their optimal information-sharing strategies on $t = 0$.

3.1 Trading on $t = 1$

Given any pair of the two investors' information-sharing strategy (τ_H, τ_L) on $t = 0$, we solve for their optimal trading strategies and market makers' equilibrium pricing rule on $t = 1$.

We consider a linear pricing rule for market makers $\tilde{p} = \lambda\tilde{w}$, where the total order flow $\tilde{\omega}$ is specified by equation (3), and linear trading rules for the two investors: $\tilde{x}_H = \alpha_v\tilde{v} + \alpha_H\tilde{s}_H + \alpha_L\tilde{s}_L$ and $\tilde{x}_L = \beta_y\tilde{y} + \beta_H\tilde{s}_H + \beta_L\tilde{s}_L$, where coefficients $\{\alpha_v, \alpha_H, \alpha_L, \beta_y, \beta_H, \beta_L\}$ are endogenously determined. The coefficients α_v and β_y respectively represent the trading aggressiveness of H and L when they make decisions based on their endowed information. The coefficients $\alpha_H, \alpha_L, \beta_H,$ and β_L capture the strategic interaction between the two investors when trading on the shared information.

With the information set $\mathcal{F}_H = \{\tilde{v}, \tilde{s}_H, \tilde{s}_L\}$, H's posterior beliefs about the value of the risky asset, L's endowed information, and noise trading are respectively as follows:

$$E[\tilde{v}|\mathcal{F}_H] = \tilde{v},\ E[\tilde{y}|\mathcal{F}_H] = \tilde{v} + \frac{\tau_L}{\rho + \tau_L}(\tilde{s}_L - \tilde{v}),\ \text{and}\ E[\tilde{u}|\mathcal{F}_H] = 0.$$

Then, H's conditional expected trading profits in (2) can be expressed as follows:

$$E[\tilde{x}_H(\tilde{v}-\tilde{p})|\mathcal{F}_H] = \tilde{x}_H\left(\tilde{v}-\lambda\left(\tilde{x}_H+\beta_y\left(\tilde{v}+\frac{\tau_L}{\rho+\tau_L}(\tilde{s}_L-\tilde{v})\right)+\beta_H\tilde{s}_H+\beta_L\tilde{s}_L\right)\right). \quad (5)$$

Maximizing H's profits yields his optimal trading rule, $\tilde{x}_H = \alpha_v\tilde{v}+\alpha_H\tilde{s}_H+\alpha_L\tilde{s}_L$, with

$$\alpha_v = \frac{\rho+\tau_L-\lambda\rho\beta_y}{2\lambda(\rho+\tau_L)}, \ \alpha_H = -\frac{\beta_H}{2}, \text{ and } \alpha_L = -\frac{\beta_L}{2}-\frac{\beta_y\tau_L}{2(\rho+\tau_L)}. \quad (6)$$

For L, given her information set $\mathcal{F}_L = \{\tilde{y}, \tilde{s}_H, \tilde{s}_L\}$, we can express her conditional expected trading profits as follows:

$$E[\tilde{x}_L(\tilde{v}-\tilde{p})|\mathcal{F}_L] = \tilde{x}_L\left(\frac{\rho\tilde{y}+\tau_H\tilde{s}_H}{1+\rho+\tau_H}-\lambda\left(\tilde{x}_L+\alpha_v\frac{\rho\tilde{y}+\tau_H\tilde{s}_H}{1+\rho+\tau_H}+\alpha_H\tilde{s}_H+\alpha_L\tilde{s}_L\right)\right). \quad (7)$$

Again, maximizing the profits yields L's optimal trading rule, $\tilde{x}_L = \beta_y\tilde{y}+\beta_H\tilde{s}_H+\beta_L\tilde{s}_L$, with

$$\beta_y = \frac{\rho(1-\lambda\alpha_v)}{2\lambda(1+\rho+\tau_H)}, \ \beta_H = -\frac{\alpha_H}{2}+\frac{(1-\lambda\alpha_v)\tau_H}{2\lambda(1+\rho+\tau_H)}, \text{ and } \beta_L = -\frac{\alpha_L}{2}. \quad (8)$$

Equations (6) and (8) are the reaction functions, which jointly determine the equilibrium values of $(\alpha_v, \alpha_H, \alpha_L, \beta_y, \beta_H, \beta_L)$ as shown in the following lemma.

Lemma 1. *Given the two investors' information-sharing strategies (τ_H, τ_L) and market makers' pricing rule λ, H's and L's equilibrium trading strategies on $t = 1$ are characterized by $\tilde{x}_H = \alpha_v\tilde{v}+\alpha_H\tilde{s}_H+\alpha_L\tilde{s}_L$ and $\tilde{x}_L = \beta_y\tilde{y}+\beta_H\tilde{s}_H+\beta_L\tilde{s}_L$ with*

$$\begin{aligned}
\alpha_v &= \big(2\tau_H(\tau_L+\rho)+2(\rho+1)\tau_L+\rho(\rho+2)\big)\Omega^{-1}, \\
\alpha_H &= -2\tau_H(\tau_L+\rho)(3\Omega)^{-1}, \\
\alpha_L &= -2\rho\tau_L(3\Omega)^{-1}, \\
\beta_y &= \rho(\tau_L+\rho)\Omega^{-1}, \\
\beta_H &= 4\tau_H(\tau_L+\rho)(3\Omega)^{-1}, \\
\beta_L &= \rho\tau_L(3\Omega)^{-1},
\end{aligned}$$

where $\Omega = \lambda(4\tau_H(\tau_L+\rho)+4(\rho+1)\tau_L+\rho(3\rho+4))$.

Lemma 1 states that while H trades in the same direction as suggested by his endowed information ($\alpha_v > 0$), he trades against not only his own shared information ($\alpha_H < 0$) but also the one shared by L ($\alpha_L < 0$). By contrast, L trades alongside all pieces of information she has access to; that is, $\beta_y, \beta_H, \beta_L > 0$. What determines the two investors' different trading rules?

First, it is intuitive that each investor's trading should be in the same direction as suggested by their own endowed information; that is, $\alpha_v > 0$ and $\beta_y > 0$. The endowed information is informative about the asset fundamental. So when the signal indicates a positive (negative) return, the investor tends to buy (sell) the risky asset.

Second, while L's trading is aligned with the information shared by H, H trades against the information shared by L, namely, $\beta_H > 0$ and $\alpha_L < 0$. As the less informed side, when L receives the information shared by H, though the signal might be noisy, L can still employ the information to make better forecasts of the asset fundamental. So her trading direction is aligned with what H's shared information suggests. However, since H has already been well informed of the asset fundamental, the information shared by L is of no additional use to him in forecasting the fundamental. Still, H uses this information. Why? Note that L's trading follows her endowed information $\tilde{y}$. Since $\tilde{y} = \tilde{v} + \tilde{e}$ is only a noisy signal of the asset fundamental, by trading on it, L trades not only "correctly" on the fundamental $\tilde{v}$, but also "incorrectly" on the error $\tilde{e}$. The latter noise trading can move asset price away from the fundamental value and if H could detect it, he would always have incentives to trade against it and make profits accordingly. For H, L's shared information exactly serves this purpose. Specifically, with the fundamental information $\tilde{v}$ and L's shared information, H can infer the error in L's endowed information as follows:

$$E[\tilde{e}|\mathcal{F}_H] = \frac{\tau_L}{\rho + \tau_L} (\tilde{s}_L - \tilde{v}) . \tag{9}$$

Since the inference $E[\tilde{e}|\mathcal{F}_H]$ always shares the same sign as that of $\tilde{s}_L$, it appears that H trades against $\tilde{s}_L$, namely, $\alpha_L < 0$. We refer to this novel effect as the *trading-against-error effect*. To further see the intuition, let's consider an illustrative example in which L buys the risky asset. When $\tilde{e} > 0$, L tends to buy an additional amount of the asset than is justified on the basis of the fundamental value. Understanding that this trading is merely driven by error, H will sell an additional amount $\alpha_L E[\tilde{e}|\mathcal{F}_H]$ of the risky asset, which can partly offset L's trading demand based on the error $\tilde{e}$. As such, L's informed order flows are partially canceled, which enables her to execute her trade at a better price and lose less from trading on the error $\tilde{e}$. In some sense, H "provides" liquidity to L. As will be shown later, this effect proves central to our information-sharing results.

Third, while H trades against the information shared by himself ($\alpha_H < 0$), L's trading is aligned with her own shared information ($\beta_L > 0$). As shown by the best response of H in equations (6) and that of L in equations (8), the coefficient of one investor's trading demand on the own shared information crucially depends how the counterparty investor trades on it, namely, $\alpha_H = -\frac{\beta_H}{2}$ and $\beta_L = -\frac{\alpha_L}{2}$. Let's examine H's trading strategy first. By sharing the garbled information $\tilde{s}_H$ with L, H understands that L trades in the same direction as suggested by $\tilde{s}_H$ as it helps her better predict the asset fundamental. Meanwhile, trading on $\tilde{s}_H$ suggests that L's trading injects the added noise $\tilde{\varepsilon}_H$ into the price, which induces H to trade against $\tilde{s}_H$ to correct the overshot price. Therefore, as $\beta_H > 0$, it must follow that $\alpha_H < 0$. Similarly, L well understands that H's using her shared information $\tilde{s}_L$ also incorporates her added noise $\tilde{\varepsilon}_L$ into the price. Since H trades against $\tilde{s}_L$, L ends up trading in the same direction as suggested by her own shared information; that is, as $\alpha_L < 0$, we have $\beta_L > 0$.

Next, after receiving the total order flow from the two investors and noise traders, market makers set the price for the risky asset: $\tilde{p} = \lambda\tilde{\omega}$. Based on the weak-efficiency rule (4) and investors' optimal trading rules as specified by Lemma 1, the equilibrium pricing rule can be expressed as a function of τ_H and τ_L as follows:

$$\lambda(\tau_H, \tau_L) = \frac{\sqrt{\begin{array}{r}\rho\tau_L\left(8\tau_H\left(8\tau_H + 14\rho + 17\right) + \rho(48\rho + 113) + 72\right) + 4\tau_L^2\left(\tau_H + \rho + 1\right)\left(8\tau_H + 8\rho + 9\right) \\ +4\rho^2\tau_H\left(8\tau_H + 12\rho + 17\right) + 9(\rho(2\rho + 5) + 4)\rho^2\end{array}}}{3\sigma_u\left(4\tau_L\left(\tau_H + \rho + 1\right) + \rho\left(4\tau_H + 3\rho + 4\right)\right)}. \tag{10}$$

The following proposition summarizes the subgame equilibrium on $t = 1$.

Proposition 1 (Trading)**.** *Given investors' information-sharing strategy* (τ_H, τ_L)*, on* $t = 1$ *the asset price is* $\tilde{p} = \lambda\tilde{\omega}$*, where* λ *is specified by equation (10), and investors' equilibrium trading rules are specified by Lemma 1.*

3.2 Information Sharing on $t = 0$

Now we study the optimal information sharing between the two rational investors on $t = 0$. The following proposition characterizes their optimal information-sharing strategies.

Proposition 2 (Information sharing)**.** *In equilibrium, H does not share any of his information whereas L fully shares her information; that is,* $\tau_H^* = 0$ *and* $\tau_L^* = +\infty$*.*

Proposition 2 states that when to share information is permitted, information won't flow from the more informed investor (investor H) to the less informed one (investor L); quite surprisingly, it transmits in the opposite direction from L to H.

We first explain why L would like to share her information genuinely. We ask how L's information-sharing decision τ_L affects her expected profits given τ_H. Inserting the two investors' optimal trading rules as specified by Lemma 1 into L's conditional expected profits (7) and taking expectation yields her unconditional expected trading profits on $t = 0$ as follows:

$$\pi_L = \lambda E[\tilde{x}_L^2] = \frac{1}{4\lambda} E\Big[E\big[\tilde{v} - \lambda(\alpha_v\tilde{v} + \alpha_H\tilde{s}_H + \alpha_L\tilde{s}_L)|\tilde{y}, \tilde{s}_H, \tilde{s}_L\big]\Big]^2.$$

We then use the chain rule to decompose how τ_L affects L's information-sharing incentive, which is summarized by the following equation:

$$\begin{aligned} \frac{\mathrm{d}\pi_L}{\mathrm{d}\tau_L} \quad = \quad & \underbrace{\frac{\partial\pi_L}{\partial\alpha_L}\frac{\partial\alpha_L}{\partial\tau_L}}_{\text{trading-against-error effect} >0} + \underbrace{\frac{\partial\pi_L}{\partial\alpha_v}\frac{\partial\alpha_v}{\partial\tau_L}}_{\text{competition} <0} + \underbrace{\frac{\partial\pi_L}{\partial\alpha_H}\frac{\partial\alpha_H}{\partial\tau_L}}_{<0} \\ & + \underbrace{\frac{\partial\pi_L}{\partial\lambda}\frac{\partial\lambda}{\partial\tau_L}}_{\text{liquidity} <0} + \underbrace{\frac{\partial\pi_L}{\partial\tau_L}}_{<0} \; > \; 0, \end{aligned} \tag{11}$$

where

$$
\begin{cases}
\frac{\partial \pi_L}{\partial \alpha_L} &= -\frac{1}{3} < 0, \\
\frac{\partial \alpha_L}{\partial \tau_L} &= -\frac{2\rho^2(4\tau_H+3\rho+4)}{3\lambda(4\tau_H(\tau_L+\rho)+4(\rho+1)\tau_L+\rho(3\rho+4))^2} < 0,
\end{cases}
$$

$$
\begin{cases}
\frac{\partial \pi_L}{\partial \alpha_v} &= -\frac{4\tau_H(\tau_L+\rho)+\rho(4\tau_L+3\rho)}{3(4\tau_H(\tau_L+\rho)+4(\rho+1)\tau_L+\rho(3\rho+4))} < 0, \\
\frac{\partial \alpha_v}{\partial \tau_L} &= \frac{2\rho^2(\tau_H+\rho+1)}{\lambda(4\tau_H(\tau_L+\rho)+4(\rho+1)\tau_L+\rho(3\rho+4))^2} > 0,
\end{cases}
$$

$$
\begin{cases}
\frac{\partial \pi_L}{\partial \alpha_H} &= -\frac{1}{3} < 0, \\
\frac{\partial \alpha_H}{\partial \tau_L} &= \frac{2\rho^2\tau_H}{3\lambda(4\tau_H(\tau_L+\rho)+4(\rho+1)\tau_L+\rho(3\rho+4))^2} > 0,
\end{cases}
$$

$$
\begin{cases}
\frac{\partial \pi_L}{\partial \lambda} &= -\frac{\begin{pmatrix} 16\tau_H\left((7\rho+4)\rho\tau_L+(4\rho+2)\tau_L^2+(3\rho+2)\rho^2\right)+32\tau_H^2(\tau_L+\rho)^2 \\ +\rho\left((48\rho+59)\rho\tau_L+32(\rho+1)\tau_L^2+9(2\rho+3)\rho^2\right) \end{pmatrix}}{9\lambda^2(4\tau_H(\tau_L+\rho)+4(\rho+1)\tau_L+\rho(3\rho+4))^2} < 0, \\
\frac{\partial \lambda}{\partial \tau_L} &= \frac{\rho^2(20\tau_H(\tau_L+\rho)+20(\rho+1)\tau_L+\rho(27\rho+20))}{18\lambda\sigma_u^2(4\tau_H(\tau_L+\rho)+4(\rho+1)\tau_L+\rho(3\rho+4))^3} > 0,
\end{cases}
$$

$$
\begin{cases}
\frac{\partial \pi_L}{\partial \tau_L} &= -\frac{\rho^2}{9\lambda(4\tau_H(\tau_L+\rho)+4(\rho+1)\tau_L+\rho(3\rho+4))^2} < 0.
\end{cases}
$$

As shown in equation (11), the effects of τ_L on L's profits (and thus incentives of information sharing) can be decomposed into three groups: (i) the one through its effect on the rival H's trading rules (α_L, α_v, and α_H), (ii) the one through its effect on market makers' pricing rule (λ), and (iii) the direct effect. The overall effect is positive so that L would like to share all her information. We then discuss these effects one by one.

First and most importantly, the key that underlies L's information-sharing behavior is the trading-against-error effect. Specifically, as L shares more precise information, H is able to infer the error in her endowed information more accurately (see equation (9)), thereby trading more aggressively against the shared information, i.e., $\frac{\partial \alpha_L}{\partial \tau_L} < 0$ (note that $\alpha_L < 0$). As L's informed order flows are more offset, she gains better execution price and higher profits accordingly, i.e., $\frac{\partial \pi_L}{\partial \alpha_L}\frac{\partial \alpha_L}{\partial \tau_L} > 0$. Interestingly, as clear in equation (11), this is the only positive force that induces L to share her information, which suggests that it must be strong enough to overturn all the other negative forces that discourage L's information sharing.

The information-sharing decision τ_L also affects H's trading on his endowed information $\tilde{v}$ and his own shared information $\tilde{s}_H$, and both effects discourage L from sharing information. Specifically, as L shares more precise information, H trades more aggressively on his endowed information, i.e., $\frac{\partial \alpha_v}{\partial \tau_L} > 0$. This is because by observing L's shared information, H not only knows the fundamental perfectly, but also his rival investor better, which gains him more competitive advantage. This effect reduces L's information-sharing incentives $\frac{\partial \pi_L}{\partial \alpha_v}\frac{\partial \alpha_v}{\partial \tau_L} < 0$. Moreover, as L shares more precise information, H uses less of his own shared information, i.e., $\frac{\partial \alpha_H}{\partial \tau_L} > 0$ (note that $\alpha_H < 0$). In the extreme case $\tau_H = 0$, this effect is shut down completely, i.e., $\alpha_H = 0$.

Next, as L shares more precise information and thus H trades against the shared information more aggressively, the aggregate order flow from the two investors becomes more correlated with the asset fundamental and market makers respond by raising the price impact, i.e., $\frac{\partial \lambda}{\partial \tau_L} > 0$. Ultimately, L's profits can be eroded, i.e., $\frac{\partial \pi_L}{\partial \lambda}\frac{\partial \lambda}{\partial \tau_L} < 0$. Finally, τ_L also affects L's profits directly, independent of the channels through the rival investor's trading strategies or market makers' pricing rule. This direct effect arises from L's added noise $\tilde{\varepsilon}_L$. Intuitively, as L shares more precise information, the term $\tilde{\varepsilon}_L$ becomes less volatile and its effect on L's profits diminished.

We then investigate H's information-sharing incentives. Similarly, we can derive H's unconditional trading profits as follows:

$$\pi_H = \lambda E[\tilde{x}_H^2] = \frac{1}{4\lambda} E\Big[E\big[\tilde{v} - \lambda(\beta_y \tilde{y} + \beta_H \tilde{s}_H + \beta_L \tilde{s}_L)|\tilde{v}, \tilde{s}_H, \tilde{s}_L\big]\Big]^2,$$

and use the chain rule to decompose the effect of τ_H on his profits. Having established that L would like to genuinely share her information, we now fix $\tau_L = \infty$ and explore H's information-sharing behavior. The following equation decomposes the effect τ_H on H's profits:

$$\begin{aligned} \frac{\mathrm{d}\pi_H}{\mathrm{d}\tau_H} = & \underbrace{\frac{\partial \pi_H}{\partial \beta_H}\frac{\partial \beta_H}{\partial \tau_H}}_{\text{information leakage} <0} + \underbrace{\frac{\partial \pi_H}{\partial \beta_y}\frac{\partial \beta_y}{\partial \tau_H}}_{>0} + \underbrace{\frac{\partial \pi_H}{\partial \beta_L}\frac{\partial \beta_L}{\partial \tau_H}}_{>0} \\ & + \underbrace{\frac{\partial \pi_H}{\partial \lambda}\frac{\partial \lambda}{\partial \tau_H}}_{\text{liquidity}>0} + \underbrace{\frac{\partial \pi_H}{\partial \tau_H}}_{<0} < 0, \end{aligned} \tag{12}$$

where

$$\begin{cases} \frac{\partial \pi_H}{\partial \beta_H} &= -\frac{1}{3} < 0, \\ \frac{\partial \beta_H}{\partial \tau_H} &= \frac{\rho+1}{3\lambda(\tau_H+\rho+1)^2} > 0, \end{cases} \quad \begin{cases} \frac{\partial \pi_H}{\partial \beta_y} &= -\frac{1}{3} < 0, \\ \frac{\partial \beta_y}{\partial \tau_H} &= -\frac{\rho}{4\lambda(\tau_H+\rho+1)^2} < 0, \end{cases}$$

$$\begin{cases} \frac{\partial \pi_H}{\partial \beta_L} &= -\frac{1}{3} < 0, \\ \frac{\partial \beta_L}{\partial \tau_H} &= -\frac{\rho}{12\lambda(\tau_H+\rho+1)^2} < 0, \end{cases} \quad \begin{cases} \frac{\partial \pi_H}{\partial \lambda} &= -\frac{8\tau_H+8\rho+9}{36\lambda^2(\tau_H+\rho+1)} < 0, \\ \frac{\partial \lambda}{\partial \tau_H} &= -\frac{1}{72\lambda\sigma_u^2(\tau_H+\rho+1)^2} < 0, \end{cases}$$

$$\begin{cases} \frac{\partial \pi_L}{\partial \tau_L} &= -\frac{1}{36\lambda(\tau_H+\rho+1)^2} < 0. \end{cases}$$

The key effect of H's information sharing on his own profits is the information-leakage effect, which strongly reduces his incentives to share information. With superior information about the asset fundamental, H owns an informational advantage over L. So, any piece of information sharing with L can only dissipate H's informational advantage. Recall that L uses H's shared information to better inform her trading decisions by trading alongside it, i.e., $\beta_H > 0$. With more precise information shared by H, more information is leaked to L and she trades more aggressively on it, i.e., $\frac{\partial \beta_H}{\partial \tau_H} > 0$. This greatly erodes H's competitive advantage and reduces his trading profits, i.e., $\frac{\partial \pi_H}{\partial \beta_H}\frac{\partial \beta_H}{\partial \tau_H} < 0$. Overall, the information-leakage effect prevails and H refrains from revealing any of his information, i.e., $\frac{d\pi_H}{d\tau_H} < 0$ so $\tau_H^* = 0$.

In addition, as in the analysis for L's information-sharing decisions, there are other forces at play in determining H's sharing decisions. First, as H shares more precise information, his shared information crowds out L's endowed information $\tilde{y}$ and her shared information $\tilde{s}_L$ in her trading rules, that is, $\frac{\partial \beta_y}{\partial \tau_H} < 0$ and $\frac{\partial \beta_L}{\partial \tau_H} < 0$. And both effects encourage H to share his information, i.e., $\frac{\partial \pi_H}{\partial \beta_H}\frac{\partial \beta_y}{\partial \tau_H} > 0$ and $\frac{\partial \pi_H}{\partial \beta_L}\frac{\partial \beta_L}{\partial \tau_H} > 0$. This is because if H's shared information becomes more precise, L trades more on this information and uses less her other information ($\tilde{y}$ and $\tilde{s}_L$) accordingly. Now H gains a strategic advantage by knowing his rival investor's trading rule better. Thus, H's profits improve through this channel.

Second, like the liquidity effect in L's information-sharing incentives, τ_H also affects H's profits through its effect on pricing rule. However, different from the effect of τ_L on π_L, as H shares more precise information, market makers reduces price impact, that is, $\frac{\partial \lambda}{\partial \tau_H} < 0$. Why do market makers respond differently to the two investors' information sharing behavior? As the more informed investor, when H shares more his information with L, the two investors' private information becomes more homogeneous, which induces the two investors to compete with each other more aggressively. This intense inter-investor competition reveals more information to market makers, reducing their informational disadvantage and adverse selection risk. As a response, market makers decreases price impact and the improved market liquidity benefits H. Finally, the analysis of the direct effect of τ_H on H's profits is the same as the direct effect in L's information-sharing incentives.

To sum, by sharing information, L invites H to trade against her shared information, offsetting her informed order flow and gaining her a better execution price. On the other hand, any piece of information sharing can greatly dissipate H's informational advantage. Taken together, the strong trading-against-error effect encourages L to not only share her information, but do so "as is," and the strong information-leakage effect discourages H from information sharing.

Having established the equilibrium information-sharing strategy, according to Proposition 1 we can characterize the asset price and investors' trading strategies along the equilibrium path. The following corollary summarizes the results.

Corollary 1. *In equilibrium, the asset price is* $\tilde{p} = \lambda^* \tilde{\omega}$, *where*

$$\lambda^* = \frac{\sqrt{9+8\rho}}{6\sigma_u\sqrt{1+\rho}}. \tag{13}$$

Investors H *and* L *submit market orders* $\tilde{x}_H = \alpha_v^* \tilde{v} + \alpha_L^* \tilde{y}$ *and* $\tilde{x}_L = (\beta_y^* + \beta_L^*)\tilde{y}$, *respectively, where*

$$\alpha_v^* = \frac{3\sigma_u\sqrt{1+\rho}}{\sqrt{9+8\rho}},\ \alpha_L^* = -\frac{\rho\sigma_u}{\sqrt{(1+\rho)(9+8\rho)}},\ \textit{and}\ \beta_y^* + \beta_L^* = \frac{2\rho\sigma_u}{\sqrt{(1+\rho)(9+8\rho)}}, \tag{14}$$

and their respective unconditional trading profits are as follows:

$$\pi_H^* = \frac{(9+4\rho)\sigma_u}{6\sqrt{(1+\rho)(9+8\rho)}}\ \textit{and}\ \pi_L^* = \frac{2\rho\sigma_u}{3\sqrt{(1+\rho)(9+8\rho)}}. \tag{15}$$

3.3 Implications

In this section, we examine the effect of information sharing on investors' profits and market quality by comparing the equilibrium outcomes with a benchmark economy without information sharing ($\tau_H = \tau_L = 0$). Based on Proposition 1, the following corollary immediately follows which summarizes the equilibrium in this benchmark economy (note the superscript 0 represents the benchmark).

Corollary 2. *Suppose there is no information sharing: $\tau_H = \tau_L = 0$. In equilibrium, the asset price is $\tilde{p} = \lambda^0 \tilde{\omega}$, where*

$$\lambda^0 = \frac{\sqrt{4 + 5\rho + 2\rho^2}}{(4 + 3\rho)\sigma_u}. \tag{16}$$

Investors H and L submit market orders $\tilde{x}_H = \alpha_v^0 \tilde{v}$ and $\tilde{x}_L = \beta_y^0 \tilde{y}$, respectively, where

$$\alpha_v^0 = \frac{(2 + \rho)\sigma_u}{\sqrt{4 + 5\rho + 2\rho^2}} \text{ and } \beta_y^0 = \frac{\rho\sigma_u}{\sqrt{4 + 5\rho + 2\rho^2}}, \tag{17}$$

and their respective unconditional expected profits are as follows:

$$\pi_H^0 = \frac{(2 + \rho)^2\sigma_u}{(4 + 3\rho)\sqrt{4 + 5\rho + 2\rho^2}} \text{ and } \pi_L^0 = \frac{\rho(1 + \rho)\sigma_u}{(4 + 3\rho)\sqrt{4 + 5\rho + 2\rho^2}}. \tag{18}$$

When there is no information transmission between the two strategic investors, they trade on their own private information to maximize their respective trading profits, taking into account the competition between them and the optimal response of market makers. Thanks to the more precise private information, H owns an informational advantage over L and trades more aggressively on his private information. That is, $\alpha_v^0 > \beta_y^0$. As such, H makes higher trading profits: $\pi_H^0 > \pi_L^0$.

With this benchmark, we now examine the implications of information sharing on the two investors' profits and market quality, as summarized in the following proposition.

Proposition 3. *Compared with the economy without information sharing, when information sharing is permitted,*

(i) *Investor L is better off whereas investor H is worse off and their combined profits are higher; that is,* $\pi_L^* > \pi_L^0$, $\pi_H^* < \pi_H^0$, *and* $\pi_H^* + \pi_L^* > \pi_H^0 + \pi_L^0$.

(ii) *Market liquidity is lower whereas market efficiency and total trading volume are higher; that is,* $\lambda^* > \lambda^0$, $m^* > m^0$, *and* $TV^* > TV^0$.

We first investigate how information sharing affects the two investors' trading profits. Part (i) of Proposition 3 summarizes the results. Why is L better off with information sharing? As analyzed above, this is due to the novel trading-against-error effect. In the benchmark without information sharing, L gains from trading on the fundamental component but loses from trading on the error component. Nonetheless, L cannot distinguish the two components and have to trade on them simultaneously. With information sharing permitted, H's trading against $\tilde{e}$ helps hide L's informed order flow to the benefit of L. For example, when $\tilde{e} > 0$, L tends to buy an additional amount of $(\beta_y^* + \beta_L^*)\tilde{e}$ than is justified on the basis of the fundamental value $\tilde{v}$. Meanwhile, since H trades against L's shared information ($\alpha_L^* < 0$), he tends to sell an additional amount $|\alpha_L^*|\tilde{e}$ of the risky asset, which partly offsets L's trading demand. As such, L can execute her order at a better price and lose less from the trading on the error in her private information.

Yet, how can H become worse off after receiving more information? With more order flow being offset, L becomes less concerned about the losses associated with the trading on the error and in turn trades more aggressively. Mathematically, recall that with information sharing permitted, L's trading strategy is $\tilde{x}_L = (\beta_y + \beta_L)\tilde{y}$ and the trading aggressiveness is captured by the coefficient $\beta_y + \beta_L$. A direct comparison of L's trading aggressiveness in equilibrium with information sharing permitted (see equations (14)) to that in the benchmark economy without information sharing (see equations (17)) yields that $\beta_y^* + \beta_L^* > \beta_y^0$; that is, L trades more aggressively after sharing her information. In turn, H is forced to trade less aggressively on his endowed information, that is, $\alpha_v^* + \alpha_L^* < \alpha_v^0$. At the same time, as will be shown in part (ii) of Proposition 3, market liquidity becomes lower after information sharing to the further detriment of H's profits.

The whole investor side makes higher profits after information sharing ($\pi_H^* + \pi_L^* > \pi_H^0 + \pi_L^0$) because with information sharing the two investors can better internalize the competition between them. One numerical example indicates that, given $\rho = 1$ and $\sigma_u = 1$, by sharing information, L's profits increases by 32.7% whereas H's profits drop by 4.1%; meanwhile, two investors' total profits increase by 2.6%.

Having established that H makes lower profits with the shared information, it is intuitive that H would be better off if he could commit not to using the received information; that is, when contemplating the optimal trading rule, H commits that $\alpha_L = 0$. But without additional assumptions, can he credibly make this commitment? No. This is evident from H's optimal trading strategy as specified by equations (14). Specifically, knowing that L trades on information $\tilde{y}$ and being able to filter out the error in this information, H always has the tendency to trade against it, correct the price, and make profits accordingly. In other words, after receiving the information from L, H cannot help using this piece of information despite its negative consequences for his profits. We discuss the extended economy in which H has such a commitment power to not listen to the shared information in Section 4.3.

Next, how is information sharing affecting market quality? We examine market liquidity, market efficiency (price discovery), and trading volume. Part (ii) of Proposition 3 summarizes the results. Market liquidity is measured by the Kyle's λ, which measures the effect of noise trading on prices, and so it is an inverse measure of market depth: more liquid markets have a smaller λ. A straightforward comparison of the equilibrium λ^* as specified by (13) and that in the benchmark λ^0 as specified by (16) reveals that when information sharing is permitted market liquidity decreases; that is, $\lambda^* > \lambda^0$. This is because with L's private information shared, H trades against the error in the shared information, thereby reducing the noise in the total order flow. As such, market makers increase the price impact to manage the increasing adverse-selection risk, which dampens market liquidity.

Following the literature (e.g. Kyle, 1985), We measure market efficiency (price discovery) by the precision of the asset payoff conditional on its price, i.e., $m \equiv Var\left(\tilde{v}|\tilde{p}\right)^{-1}$. Intuitively, when the price aggregates a great deal of information, the residual uncertainty of the fundamental $\tilde{v}$ conditional on the price $\tilde{p}$ is low, and thus market efficiency is high. According to Corollaries 1 and 2, it is easy to show that market efficiency improves when information sharing is permitted; that is, $m^* > m^0$. As argued above, after information sharing, the total order flow becomes more correlated with the fundamental. Accordingly, the price can aggregate more information about the fundamental.

Finally, following Vives (2010), we measure total volume traded, denoted by TV, by the sum of the expected absolute value of the demands coming from the different agents in the model divided by 2, as follows:[3]

$$TV = \frac{1}{2}\Big(E\big[|\tilde{x}_H| + |\tilde{x}_L| + |\tilde{\omega}| + |\tilde{u}|\big]\Big).$$

We find that information sharing is associated with higher total trading volume, that is, $TV^* > TV^0$. Specifically, after information sharing is permitted, since L trades more aggressively and H is forced to trade less aggressively, L's trading volume increases whereas H's decreases, that is, $E\left[|\tilde{x}_L|\right]$ increases but $E\left[|\tilde{x}_H|\right]$ decreases. Overall, market maker's trading volume increases, namely, $E\left[|\tilde{\omega}|\right]$ increases.

4 Extensions

In this section, we consider several extensions to demonstrate the robustness of our key result; that is, a coarsely informed investor has a strategic incentive to genuinely share her information with the well informed investor. Meanwhile, these extensions help our model better map to the reality.

4.1 Ex-post Information Sharing

The main purpose of this section is to better connect our model to the social media trading setting. Recall that in the baseline model, investors are assumed to make information-sharing decisions before observing their private information (see Figure 1). This assumption involves commitment issues. Now, we consider an alternative setting in which after observing the realization of private signals, each investor decides whether or not to share it.

Denote investor i's information-sharing set D_i, where $i \in \{H, L\}$. For instance, when the realization of L's private signal $\tilde{y} \in D_L$, L shares $\tilde{y}$ with H; otherwise L does not reveal any

[3] Our result remains robust under alternative measure of trading volume, e.g., $TV = \frac{1}{2}E[|\tilde{\omega}|]$ (Bernhardt and Miao, 2004).

of her information. In the general case, we should have $D_i \subseteq \mathbb{R}$. However, if an equilibrium involves "interior" information-sharing set ($D_i \neq \emptyset$ and $D_i \subset \mathbb{R}$) i.e., an investor shares upon some signal realizations but does not upon others, then the linearity breaks down and we are unable to analytically solve the equilibrium. Therefore, we only focus on the "corner" equilibrium in which $D_i = \emptyset$ or $D_i = \mathbb{R}$.

Furthermore, to deal with off-equilibrium beliefs, we assume that upon observing a deviation in an investor's equilibrium information sharing decision, other market participants do not update beliefs regarding the distribution of the deviant investor's private signal. One justification is that they interpret the deviation as a tremble and assume that trembles are uncorrelated with the investor's information. This also echoes the idea of passive beliefs that are commonly used in the signalling literature (e.g., McAfee and Schwartz, 1994).

The following proposition summarizes the equilibrium in this ex-post information sharing setting.

Proposition 4 (Ex-post information sharing)**.** *Suppose that investors make information-sharing decisions after observing the realization of their private signals.*

(1) *That neither L nor H shares information cannot be sustained in equilibrium.*

(2) *That H shares his information cannot be sustained in equilibrium.*

(3) *There exists an equilibrium in which L always fully shares her information, whereas H never shares his information.*

Part (1) of Proposition 4 states that in the ex-post information-sharing setting, there must be information transmitted between the two investors. That is, the "silence" cannot be sustained in equilibrium. Parts (2) and (3) further state that relative H, L has more incentive to share her information. Specifically, as any information sharing by the more informed investor only dissipates his information advantage, H never reveals his information, as shown in Part (2) of the proposition. By contrast, due to the trading-against-error effect, there always exists an equilibrium in which regardless of her signal realization, L shares information with H: by sharing noisy information with H, L has her order flow partially offset, thereby incurring a lower price impact and in turn higher profits.

4.2 Information Sharing Between Imperfectly Informed Investors

In the baseline model, we assume that H has perfect information about the asset fundamental. In this section, we relax this assumption and consider the more general case in which information can be transmitted between imperfectly informed investors.

Assume that there are two investors, denoted by 1 and 2, who are endowed with private information about the asset fundamental of potentially different precision. Specifically, investor i, where $i \in \{1, 2\}$, receives private information as follows:

$$\tilde{y}_i = \tilde{v} + \tilde{e}_i, \quad \tilde{e}_i \sim N(0, \rho_i^{-1}) \text{ and } \rho_i \in (0, +\infty].$$

For instance, if $\rho_1 > \rho_2$, investor 1 is more informed about the fundamental than investor 2. The baseline model is nested by assuming an investor's information precision to infinity, $\rho_i = +\infty$, and letting the other investor's information precision be $\rho_j > 0$, where $i, j \in \{1, 2\}$ and $i \neq j$. On $t = 0$, investor i decides whether or not to share her information to the other investor. Specifically, investor i's shared information is as follows:

$$\tilde{s}_i = \tilde{y}_i + \tilde{\varepsilon}_i, \text{ where } \tilde{\varepsilon}_i \sim N(0, \tau_i^{-1}) \text{ and } \tau_i \in [0, +\infty].$$

By choosing her information-sharing strategy τ_i, investor i maximizes her unconditional expected trading profits. All the other setups remain the same as in the baseline model.

The following proposition summarizes the equilibrium information sharing in this extended economy and Figure 2 graphically illustrates it.

Proposition 5 (Information sharing between imperfectly informed investors)**.** *Consider two investors endowed with private information with possibly different precision. Assume that $\rho_i \geq \rho_j$, where $i, j \in \{1, 2\}$ and $i \neq j$. The following must constitute an equilibrium.*

(1) *Investor i never shares her information, i.e., $\tau_i^* = 0$;*

(2) *If $\rho_i \geq \hat{\rho}_i \equiv 2(\rho_j + 1)$, investor j fully shares her information, i.e., $\tau_j^* = +\infty$; otherwise, investor j does not share her information, i.e., $\tau_j^* = 0$.*

Proposition 5 shows that the more informed investor never shares her information,

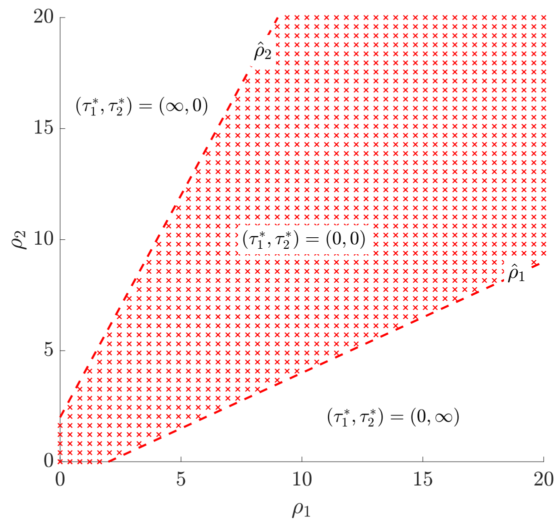


Figure 2: Information sharing between partially informed investors

whereas the less informed one shares her information only if her counterparty investor has sufficiently precise information about the fundamental. Figure 2 plots the two investors' information-sharing behavior against the precision of their endowed information ρ_1 and ρ_2; this pattern holds regardless of the value of σ_u. Consistent with Proposition 5, if the precision of investor i's endowed information exceeds some threshold ($\rho_i > \hat{\rho}_i$), the other investor j would like to share all her information with investor i. Otherwise, if the two investors' information precision levels are close, there is no information sharing between them.

Recall that in the baseline model L shares her information because of the trading-against-error effect, and this effect crucially relies on the fact that the more informed investor is able to sift the error in the received information. Since this error sifting is feasible only when the more informed investor possesses sufficiently precise information about the fundamental, information sharing only occurs under the same circumstance. To illustrate the mechanism more transparently, we examine the two investors' optimal trading strategies. Without loss of generality, assume that $\rho_1 \geq \rho_2$. Suppose that investor 2 shares her information with investor 1. After information sharing, the two investors' trading strategies are respectively as follows: $\tilde{x}_1 = \alpha_y \tilde{y}_1 + \alpha_2 \tilde{y}_2$ and $\tilde{x}_2 = \beta_2 \tilde{y}_2$, with

$$\alpha_y = \frac{\rho_1}{2\lambda(1+\rho_1+\rho_2)} > 0,$$
$$\alpha_2 = \frac{\rho_2(2+2\rho_2-\rho_1)}{6\lambda(1+\rho_1)(1+\rho_1+\rho_2)} < 0 \text{ iff } \rho_1 > \hat{\rho}_1 \equiv 2(1+\rho_2),$$
$$\beta_y = \frac{\rho_2}{3\lambda(1+\rho_2)} > 0.$$

First, consistent with Lemma 1, an investor tends to trade alongside her endowed information; that is, $\alpha_y, \beta_y > 0$. Second, investor 1 (the more informed investor) trades against the information shared by investor 2 (the less informed investor) if and only if investor 1 owns sufficiently precise information about the fundamental, namely, $\alpha_2 < 0$ if and only if $\rho_1 > \hat{\rho}_1 \equiv 2(1+\rho_2)$. If, however, investor 1's own information is not that precise but investor 2 still shares her information, then instead of trading against the shared information, investor 1 trades alongside the shared information (i.e., $\alpha_2 > 0$ if $\rho_1 < \hat{\rho}_1$). In this case, the information shared by investor 2 helps investor 1 better forecast the fundamental, thereby eroding investor 2's competitive advantage and gaining investor 1 higher profits at the expense of investor 2. Therefore, in this case investor 2 should refrain from sharing any of her information.

4.3 H: "I am not listening"

Before proceeding, we make the following additional assumptions in the subsequent Sections 4.3–4.6.

Assumption 1. *The perfectly informed investors (with private information $\tilde{v}$) do not share their information.*

Assumption 2. *When making information-sharing decisions, a coarsely informed investor shares either all or none of her information.*

Assumption 1 allows us to focus on the information-sharing behavior of coarsely informed investors and we numerically verify this assumption across these extensions. Assumption

2 helps simplify the derivation and we believe that this assumption does not change the equilibrium outcomes.

In the baseline model, although H becomes worse off after being shared with information, he cannot commit to not using it. In this section, we relax this assumption and allow H to commit to not using this information. We show that as long as there is a large number of well informed investors it can be an equilibrium in which all of them trade against the shared information, despite the fact that they would be better off had they committed to not using the information. In other words, the well informed investors can be trapped in a prisoner's dilemma.

In the extended economy, there are two groups of investors: (i) a number M of perfectly informed investors, denoted by $H_1, ..., H_M$, where $M \geq 1$ is an integer, who privately observe $\tilde{v}$; and (ii) L who only privately observes a noisy signal $\tilde{y}$ about the asset fundamental as specified by (1). In addition to Assumptions 1 and 2, we further assume that L's shared information (if any) is observable to all Hs. All other setups remain the same as in the baseline model. The following proposition summarizes the equilibrium outcomes in this extended economy.

Proposition 6. *Consider that there are a number M of Hs and one L and each H can commit not to using the shared information. The following statements must be true.*

(i) *When $M > 3$, the following cannot be sustained as an equilibrium: L shares her information and every H commits to not using the shared information.*

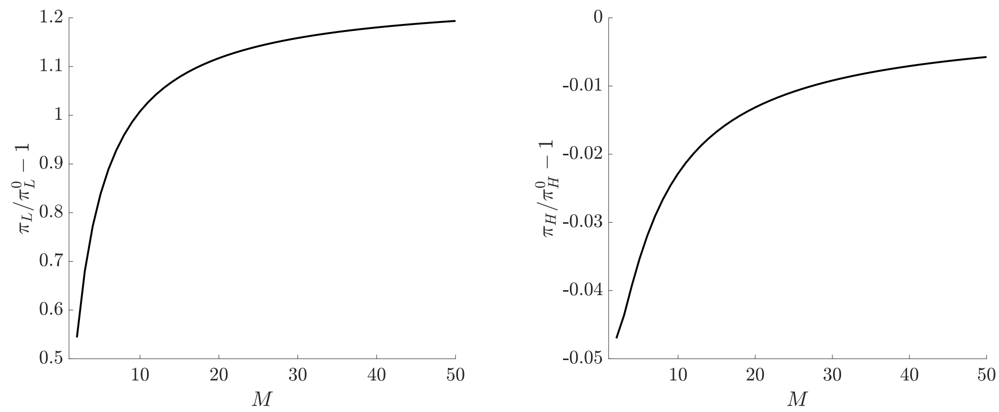
(ii) *There exists a constant $\hat{M} > 0$ such that when $M > \hat{M}$, the following equilibrium always exists: L shares her information and all Hs use L's shared information; however, Hs' profits would be higher had they all committed to not using the shared information.*

In the baseline model with one H, we have shown that H would be better off by not trading against the shared information. In other words, if H has commitment power, he would commit to not using the shared information. Will this continue to hold if there are multiple Hs? Part (i) of Proposition 6 suggests that it is not when there are more than three Hs. The intuition is as follows. When there is a single H, he fully internalizes the negative effect of trading against the shared information, optimally refraining from using it. However, when there are multiple of them, one H can deviate to using the information and enjoying its incremental value without fully accounting for its impact on L's trading and market makers' price setting.

Moreover, in the presence of multiple Hs, it is always an equilibrium in which every H uses L's shared information. We can analytically prove this when M is sufficiently large, as given by Part (ii) of Proposition 6. Meanwhile, these Hs are trapped in a prisoner's dilemma as they would be better off if they all committed to not using L's shared information, which couldn't be sustained in equilibrium as shown by Part (i) of Proposition 6. We next numerically discuss that this prisoner's dilemma type of equilibrium should be a robust feature of the multi-H economy in Figure 3.

First, as the number of Hs increases, L has a stronger strategic motive to share her private information. To see this, we compute the profit change of L from the benchmark economy without information sharing to the hypothetical extended economy in which L genuinely shares her information and all Hs trade against it, i.e., $\frac{\pi_L}{\pi_L^0} - 1$; see Panel (a) of Figure 3. We find that consistently for all values of M, sharing information improves L's profits. Further, as M grows, the profit improvement ratio keeps increasing; that is, L has a stronger information-sharing motive. This is because with more Hs trading against the shared information, L's informed order flow can be offset more; consequently, L trades more aggressively on her own information and makes higher profits. In fact, it can be shown that if M goes to infinity, L's profit improvement ratio approaches $\frac{3+2\rho}{1+\rho^2}$, which is the upper limit of the profit improvement ratio for L via information sharing. For example, if L' information is very coarse (i.e., $\rho \to 0$), this limit reaches 3; that is, by sharing her information to a sufficiently large number of Hs, L's profits can triple in the best scenario.

Second, as M increases, Hs, though worse off after information sharing, incur fewer losses. Panel (b) of Figure 3 plots the profit change of each H from the benchmark economy without information sharing to the hypothetical extended economy in which L genuinely shares her information and all Hs trade against it, i.e., $\frac{\pi_H}{\pi_H^0} - 1$. We find that for $M \geq 2$ all Hs' profits drop after they trade against the shared information. In addition, while Hs keep suffering a loss after information sharing, the loss decreases in M. This is because in the presence of



(a) L's profit change (b) An H's profit change

Figure 3: Multiple Hs ($\rho = 1$)

multiple Hs, although as in the baseline model L trades more aggressively after sharing her information, the incremental aggressiveness is smaller if L is faced with a large number of rival Hs. Therefore, each H's loss decreases.

4.4 Multiple Ls and Hs

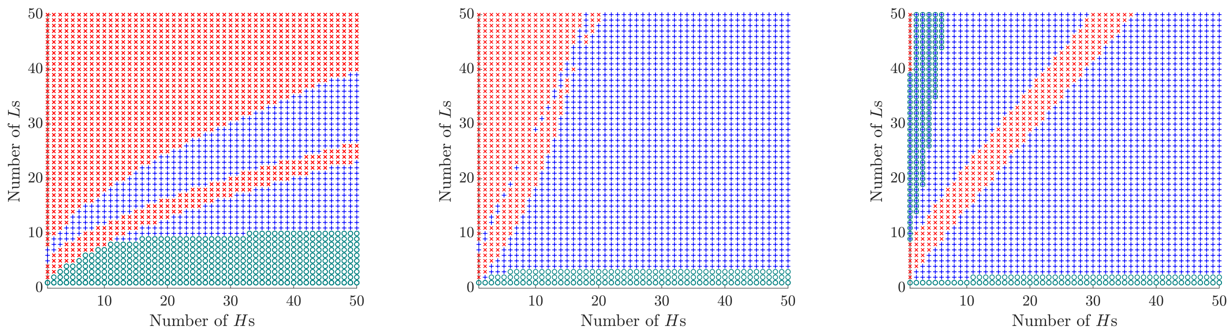
In the baseline model, there is only one pair of L-H investors and we show that information flows from L to H due to the novel trading-against-error effect. One natural conjecture is that the presence of multiple Ls will weaken their information-sharing incentives due to the competition. In this section, we extend the baseline model by considering a market with multiple Ls and Hs and examine the prevalence of information sharing.

Assume that there are two groups of investors: (i) a number M of Hs, denoted by H_1,...,H_M, and (ii) a number N of Ls, denoted by L_1,...,L_N, where M and N are integers. All Hs observe $\tilde{v}$ whereas investor L_n, where $n \in \{1, ..., N\}$, only observes a noisy signal:

$$\tilde{y}_n = \tilde{v} + \tilde{\eta} + \tilde{e}_n, \text{ with } \tilde{\eta} \sim N(0, \xi^{-1}) \text{ and } \tilde{e}_n \sim N(0, \rho^{-1}). \tag{19}$$

The signal structure (19) introduces common noise $\tilde{\eta}$ into Ls' information and thus allows for information correlation among their information. One can interpret that the common noise represents the sentiment among the coarsely informed investors. In addition to Assumptions

1 and 2, we assume that once an L shares her information, this signal becomes observable to all Hs and the other peer Ls. All other model setups develop in the same way as in the baseline model and the baseline model is nested by setting $M = N = 1$ and $\xi = \infty$.



(a) $\xi = 50$ (b) $\xi = 5$ (c) $\xi = 0.5$

This figure plots the regimes of Ls' information-sharing behavior in the parameter space of (M, N) for different values of ξ in different panels. The parameter values are $\sigma_u = 1$ and $\rho = 1$. Denote N_1 the number of Ls share their information in equilibrium. We use "x" to indicate that none of Ls shares their private information (i.e., $N_1^* = 0$), "o" to indicate that all Ls share their private information (i.e., $N_1^* = N$), and "+" to indicate that a partial fraction of Ls share their information (i.e., $0 < N_1^* < N$).

Figure 4: Multiple Ls and Hs

We use numerical analyses to demonstrate that information sharing remains a prevalent phenomenon despite the presence of a large number of Ls. Denote N_1 the number of Ls that share their private information. Figure 4 plots three types of Ls' information-sharing behavior in the parameter space of M and N for different values of ξ: (i) None of Ls shares her information (i.e., $N_1^* = 0$, marked by "x"), (ii) all Ls share their information (i.e., $N_1^* = N$, marked by "o"), and (iii) only a fraction of Ls share their information (i.e., $0 < N_1^* < N$, marked by "+").

We find that as conjectured in the beginning of this section, Panels (a) and (b) of Figure 4 show that when there are a large number of Ls but a small number of Hs, none of Ls is willing to share her information in equilibrium. This is intuitive because in the presence of multiple Ls, revealing private information to other peer Ls dissipates an L-investor's informational advantage and with a small number of Hs, the trading-against-error effect is diminished; both forces discourage Ls from sharing their information.

However, as shown in Panel (c) of Figure 4, Ls' information-sharing incentives can be restored when the common noise $\tilde{\eta}$ is important (i.e., low ξ). Now, sharing information and inviting Hs to trade against the common noise $\tilde{\eta}$ becomes the common interest among Ls, thereby sustaining the coordination-type equilibrium in which several Ls are willing to share private information despite the potential loss of informational advantage.

4.5 Public Shared Information

The insight in the baseline model hinges on the fact that rational investors can privately communicate with each other; that is, the shared information is not revealed to the public (market makers). However, in reality, information may be leaked during the communication process. In this section, we allow the shared information to be public and show that our key insights remain robust as long as H has superior ability to process the shared information than market makers.

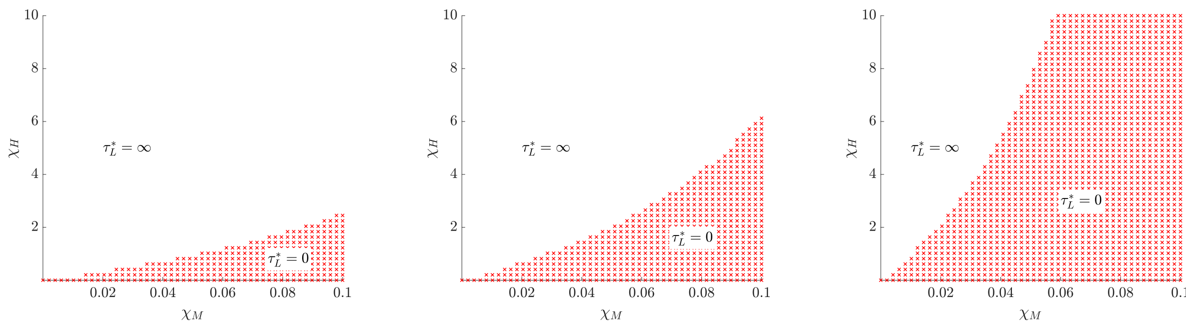
Consider the following extended economy. In addition to Assumptions 1 and 2, we assume that once L shares her private information, the information becomes observable to market makers as well. Following Myatt and Wallace (2002), we introduce "receiver noise" to capture receivers' different capabilities in interpreting the same information. Specifically, if L shares her information $\tilde{y}$, H and market makers respectively observe

$$\tilde{q}_H = \tilde{y} + \tilde{\zeta}_H \text{ and } \tilde{q}_M = \tilde{y} + \tilde{\zeta}_M,$$

where $\tilde{\zeta}_H \sim N(0, \chi_H^{-1})$ and $\tilde{\zeta}_M \sim N(0, \chi_M^{-1})$. The random variables $\{\tilde{v}, \tilde{y}, \tilde{\zeta}_H, \tilde{\zeta}_M\}$ are mutually independent. Note that due to the receiver noise, L should not be able to know $\tilde{q}_H$ or $\tilde{q}_M$. In the baseline model, we have $\chi_H = \infty$ (i.e., H can perfectly interpret L's shared information) and $\chi_M = 0$ (i.e., market makers cannot process L's shared information even if they observes it).

We conduct extensive numerical analysis for this extended economy. We use Figure 5 to characterize L's information-sharing behavior. The shaded area indicates the region in which L does not share her information ($\tau_L^* = 0$) and the blank area is the one in which L fully shares ($\tau_L^* = \infty$).

We find that if H has superior ability than market makers in interpreting the shared information (i.e., low χ_M and high χ_H), L is willing to share her information despite the



(a) $\rho = 3$ (b) $\rho = 5$ (c) $\rho = 10$

Figure 5: Public shared information ($\sigma_u = 1$)

potential information leakage to market makers. Again, by sharing her private information and inviting H to trade against it, L has her order flow partially offset and obtains a better execution price. The condition of low χ_M and high χ_H is well expected in reality due to the specialization of different agents: Market makers focus on making the market whereas strategic investors specialize in collecting information to inform trading. Therefore, information sharing by the coarsely informed investor remains a prevalent phenomenon in this extended economy. Moreover, as shown in Figure 5, as L owns more precise information (ρ increases), she is less likely to share her information. This is because with the less error in the endowed information, L benefits less from the trading-against-error effect in information sharing.

4.6 Other Extensions

We have also considered several other extensions to further examine the robustness of our results. The details can be found in the supplementary online appendix.

First, in Section S4 of the online appendix, when L has to costly acquire private information, we find that she is still willing to fully share her acquired information. Moreover, information sharing can have an ambiguous effect on L's information acquisition. On the one hand, relative to the benchmark without information sharing, by sharing information L can make higher profits and thus afford to acquire more information to inform her trading decisions. On the other hand, after acquiring more precise information, L's private information becomes less differential from H's, suggesting lower trading profits from the trading-against-error-effect; this force depresses L's information-acquisition incentives. The equilibrium amount of acquired information is shaped by the trade-off.

Second, how does information flow when there are a group of rational investors with information of different precisions? Section S5 of the online appendix studies the simplest case in which there are one perfectly informed investor H and two coarsely informed investors L_1 and L_2. We find that the least informed investor always has the strongest incentive to share her information. This is intuitive as the least informed investor can potentially gain the most from the trading-against-error effect.

5 Applications

5.1 Why do Barking Dogs Bark? Communication on Social Media

Social media is landscape-shifting, with its relevance of financial markets only growing (SEC, 2012). While there is growing interest in analyzing and utilizing investment opinions expressed on social media, the evidence regarding their predictability is mixed. For instance, Antweiler and Frank (2004) and Das and Chen (2007) find that the volume of messages on message boards, such as Yahoo! or Raging Bull, is associated with stock return volatility. But they fail to detect strong relationship between opinion transmitted through the social media and stock returns. In fact, one common view is that due to their openness and lack of regulation, social media outlets provide uninformed actors an avenue to easily spread erroneous information among market participants.[4] However, more recently, Chen et al. (2014) find that the views expressed in Seeking Alpha articles and commentaries predict stock returns over the ensuing three months and earnings surprises; Jame et al. (2016) show that crowdsourced earnings forecasts on the Estimize platform provide incrementally value-relevant information to predict earnings; and Bartov, Faurel, and Mohanram (2018) document that tweets just prior to a firm's earnings announcement predict its earnings and announcement returns.

As commented by Antweiler and Frank (2004), we need first understand why people post

[4] See Frieder and Zittrain (2007) and Hanke and Hauser (2008) for related evidence.

messages on the social media outlet and to properly answer this question requires a theory of communication that contains a financial market. Our analysis offers such a theory and helps explain why the investment opinions on the social media can contain fundamental information, but at the same time can be very noisy. Specifically, each investor who shares her investment opinions on social media is represented by the coarsely informed investor L in our model, whereas the sophisticated investor who extracts investment signals from the social media (such as a hedge fund who actively analyzes tweets or r/wallstreetbets) is the well informed investor H. As such, the opinions expressed on social media can be seen as being transmitted from the coarsely informed investors to the well informed ones. Our theory thus rationalizes this information-sharing behavior as follows: By sharing noisy yet truthful information on the social media outlets, the mass investors invite the well informed investors to trade against their information, which partially offsets their informed order flow, and gains them a better execution price.[5]

Furthermore, extensions in Sections 4.4 and 4.5 enable us to better map our framework to the social media setting. As shown in 4.4, even if the size of social media investors is large and each investor has only very limited price impact, they would be willing to genuinely share their information to have sophisticated investors trade against the common noise $\tilde{\eta}$ in their information. In this sense, the common noise well represents the sentiment on social media. Section 4.5 shows that even if the shared investment opinions are public, as long as the uninformed market participants (e.g., market makers) are not as skilled as the well informed investors in analyzing the contents and extracting useful information, the social media investors' information-sharing incentives can be preserved.

Our model also sheds new light on the increasingly popular trading strategies based on the sentiment extracted from the social media. For example, a growing number of hedge funds are buying the data feeds from Dataminr, which applies advanced analytics to the entire Twitter "fire hose" to detect events likely to move the market ("How investors are using social media to make money," December 7, 2015, Fortune). Our theory suggests that for investors that are not well informed, such sentiment might better inform their trading decisions and

[5]While if asked, investors may communicate other reasons for their social media information sharing incentives, the reason why they don't suffer from the shared information is probably due to the strategic effects documented in our model.

increase trading profits. However, if the investors per se have been well informed about the fundamental of a firm, an industry, or the economy, then subscription to the data feeds and trade on them can backfire (see Proposition 3), or even create a prisoner's dilemma for these investors (see Proposition 6).

Finally, we develop empirical predictions of our model in the social media setting. Broadly speaking, there exist three types of information on social media that helps inform investment: (i) Information about the asset fundamental (i.e., $\tilde{v}$ in our model), (ii) information about the noise in social media investors' private information (i.e., $\tilde{e}$ in the baseline model or $\tilde{\eta} + \tilde{e}_n$ in Section 4.4), and (iii) information about the noise trading (i.e., $\tilde{u}$ in our model). Our theory predicts that social media investors share their noisy private signals, which is informative about the asset fundamental (type (i) information), but also contaminated by the noise (type (ii) information); and the purpose of the information sharing is to have the latter noise traded against.

To distinguish between different cases, first, if the social media messages contain mostly type (iii) information, they should only have short-term predictability followed by reversals because this information reflects a temporary shift in stock demand. Nevertheless, empirical evidence appears to fail to detect such reversals (e.g., Chen et al., 2014; Jame et al., 2016; Gu and Kurov, 2020).

Second, the distinction between types (i) and (ii) information is more subtle. While both existing theories on information sharing and ours predict that both types of information will be present in the social media outlet, the former argue that information transmits from the investors who know the fundamental better to the less informed ones, whereas the latter predicts the opposite.[6] The corresponding implications are thus different: According to existing theories, investors using the social media information aim to learn the fundamental part $\tilde{v}$ and thus always trade *alongside* what the extracted signal suggests; by contrast, in our theory, investors using the social media information target the noise component and their

[6]It is important to point out that one difficulty of applying the existing theories to the social media setting is that the information sharers should be well-known informed investors. For instance, for the short-and-disclose strategy (e.g., Ljungqvist and Qian, 2016) and market-manipulation strategy (e.g., Benabou and Laroque, 1992) to work, the necessary condition is the investors' reputation of being well informed as well as the associated credibility. This condition, however, is clearly not true for the mass investors active in the social media outlet.

trading should be *against* the overall extracted signal. Empirically, one may investigate the trading behavior of investors posting on the social media and investors using the social media information and test if the two investor groups' trade are negatively or positively correlated.

5.2 Communication and Information Network

Word of mouth communication plays an important role among professional investors in information transmission in financial markets (e.g., Shiller and Pound, 1986; Hong, Kubik, and Stein, 2005; Luo, 2018). Given the enormous amount of resources spent on acquiring information, why do investors share their valuable private information? Who shares information with whom?

The answers to these questions are fundamental to our understanding of information networks and their implications for financial markets. Our theory offers a parsimonious framework to answer the two questions. Specifically, even in possession of only coarse information, an investor would like to share her information with the well informed investors to induce them to offset her informed order flow. The more genuine the shared information, the higher the benefits for the coarsely informed investor. Therefore, the focal coarsely informed investor is willing to truthfully share her information. Meanwhile, the direction of the information flow is unique here in that it transmits from the less informed side to the more informed one, whereas in the common explanations, the information flows in the opposite direction. This novel insight thus helps explain why information sharing can be such a widespread phenomenon in financial markets; that is, even coarsely informed investors would like to share their investment opinions.

Furthermore, as shown by Proposition 3, while the coarsely informed investor gains from sharing her information, the well informed investor loses from trading against the shared information. Therefore, if the well informed investor can choose, he might only communicate with other well informed investors but refuse to talk to the coarsely informed investors. According to our theory, as the very basic component of an information network, a pair of investors forms a stable relationship only if it consists of investors with similar information quality. Consistently, casual observations suggest that professional fund managers usually establish a core group of friends of similar background and the community is relatively stable

(Cohen, Frazzini, and Malloy, 2008, 2010; Cohen and Malloy, 2010).

Meanwhile, Section 4.3 suggests that when there are multiple well informed investors, all of them may end up reading and trading against the information shared by the coarsely informed investor. This is a prisoner's dilemma because they could have been better off if they together refused the shared information. Interestingly, this suggests that for the increasingly popular investment conferences (Luo, 2018), even though the expressed investment ideas might not be that great, an well informed investor (e.g., a prestigious fund manager) attending it can induce other well informed ones to participate because otherwise the non-participants are left at an informational disadvantage. This result potentially explains why such investment conferences are gaining popularity.

6 Conclusion

This paper proposes that an investor with imperfect information may voluntarily reveal her information to a well informed investor. Being able to sift the error from the fundamental, the more informed investor trades against the shared information. In this way, the investor that shares her information can have her informed order flows partially offset and gain better execution price (trading-against-error effect). By contrast, the well informed investor never shares his information because doing so only dissipates his informational advantage and erodes his profits accordingly. Further, after information sharing, the less informed investor becomes less concerned about the error in her information and trades more aggressively on it. The well informed investor is forced to trade less aggressively despite the superior information; coupled with market makers' steeper pricing schedule, the well informed investor makes fewer profits. Moreover, market liquidity worsens whereas both market efficiency and trading volume increase after information sharing. Overall, our model offers a novel explanation for why investors share their information in financial markets, and our explanation is unique in that it is the less informed investor that releases her information to the more informed investor.

Appendix: Proofs

Proof of Lemma 1

See the main text.

Proof of Proposition 1-2 and Corollary 1-2

Most of the proof is presented in the main text. We here only verify that H does not share any information and L fully shares, i.e., $\tau_H^* = 0$ and $\tau_L^* = +\infty$. Equation (11) can be rewritten as the following:

$$\frac{\partial \pi_L(\tau_H, \tau_L)}{\partial \tau_L} = \rho^2 \sigma_u \times \frac{\left(\begin{array}{r} 4\tau_H\left(2445\rho^2+6374\rho+4128\right)\rho^2\tau_L+4\tau_H\left(2856\rho^2+6907\rho+4128\right)\rho\tau_L^2 \\ +64\tau_H\left(69\rho^2+155\rho+86\right)\tau_L^3+4\tau_H\left(693\rho^2+1947\rho+1376\right)\rho^3 \\ +1472\tau_H^3\left(\tau_L+\rho\right)^3+16\tau_H^2\left(\tau_L+\rho\right)^2\left((276\rho+310)\tau_L+\rho(219\rho+310)\right) \\ +\left(2772\rho^3+10803\rho^2+14072\rho+6048\right)\rho^2\tau_L \\ +4\left(876\rho^3+3187\rho^2+3823\rho+1512\right)\rho\tau_L^2+32(\rho+1)^2(46\rho+63)\tau_L^3 \\ +9\left(81\rho^3+335\rho^2+476\rho+224\right)\rho^3 \end{array}\right)}{\left(\begin{array}{r} \left(\begin{array}{r} 6\rho\tau_L\left(8\tau_H\left(8\tau_H+14\rho+17\right)+\rho(48\rho+113)+72\right) \\ +24\tau_L^2\left(\tau_H+\rho+1\right)\left(8\tau_H+8\rho+9\right) \\ +24\rho^2\tau_H\left(8\tau_H+12\rho+17\right)+54(\rho(2\rho+5)+4)\rho^2 \end{array}\right)^{3/2} \\ \times\left(4\tau_H\left(\tau_L+\rho\right)+4(\rho+1)\tau_L+\rho(3\rho+4)\right)^2 \end{array}\right)} > 0.$$

Therefore, L optimally chooses $\tau_L^* = +\infty$. Furthermore, given τ_L, we can derive that:

$$\frac{\partial \pi_H(\tau_H, \tau_L)}{\partial \tau_H} = -2\sigma_u (\tau_L + \rho)^2$$

$$\times \frac{\begin{pmatrix} 4\tau_H \begin{pmatrix} (1713\rho^2 + 4450\rho + 2856)\rho^2\tau_L + (2184\rho^2 + 5021\rho + 2856)\rho\tau_L^2 \\ +8(114\rho^2 + 233\rho + 119)\tau_L^3 + (441\rho^2 + 1293\rho + 952)\rho^3 \end{pmatrix} \\ +1216\tau_H^3 (\tau_L + \rho)^3 + 16\tau_H^2 (\tau_L + \rho)^2 ((228\rho + 233)\tau_L + \rho(159\rho + 233)) \\ +2(882\rho^3 + 3495\rho^2 + 4532\rho + 1944)\rho^2\tau_L + 4(636\rho^3 + 2225\rho^2 + 2561\rho + 972)\rho\tau_L^2 \\ +16(\rho+1)^2(76\rho + 81)\tau_L^3 + 9(45\rho^3 + 202\rho^2 + 292\rho + 144)\rho^3 \end{pmatrix}}{\begin{pmatrix} 3\begin{pmatrix} \rho\tau_L (8\tau_H (8\tau_H + 14\rho + 17) + \rho(48\rho + 113) + 72) \\ +4\tau_L^2 (\tau_H + \rho + 1)(8\tau_H + 8\rho + 9) \\ +4\rho^2\tau_H (8\tau_H + 12\rho + 17) + 9(\rho(2\rho + 5) + 4)\rho^2 \end{pmatrix}^{3/2} \\ \times (4\tau_H (\tau_L + \rho) + 4(\rho + 1)\tau_L + \rho(3\rho + 4))^2 \end{pmatrix}} < 0.$$

Therefore, investor H will not share any of his information: $\tau_H^* = 0$.

Proof of Proposition 3

Based on Corollaries 1-2, we know that

$$\pi_H^* - \pi_H^0 = \frac{1}{6}\sigma_u \left(\frac{9 + 4\rho}{\sqrt{9 + 17\rho + 8\rho^2}} - \frac{6(2+\rho)^2}{(4 + 3\rho)\sqrt{4 + 5\rho + 2\rho^2}} \right) < 0,$$

where the inequality follows because

$$\left((9 + 4\rho)(4 + 3\rho)\sqrt{4 + 5\rho + 2\rho^2}\right)^2 - \left(6(2+\rho)^2\sqrt{9 + 17\rho + 8\rho^2}\right)^2$$
$$= -\rho\left(1296 + 3044\rho + 2611\rho^2 + 970\rho^3 + 132\rho^4\right) < 0.$$

Similarly,

$$\pi_L^* - \pi_L^0 = \frac{1}{3}\rho\sigma_u \left(\frac{2}{\sqrt{9 + 17\rho + 8\rho^2}} - \frac{3(1+\rho)}{(4 + 3\rho)\sqrt{4 + 5\rho + 2\rho^2}} \right) > 0,$$

where the inequality follows because

$$\left(2(4+3\rho)\sqrt{4+5\rho+2\rho^2}\right)^2-\left(3(1+\rho)\sqrt{9+17\rho+8\rho^2}\right)^2$$
$$=175+389\rho+293\rho^2+75\rho^3>0.$$

Furthermore, according to Corollaries 1-2,

$$\lambda^*-\lambda^0=\frac{1}{6\sigma_u}\left(\frac{\sqrt{9+17\rho+8\rho^2}}{1+\rho}-\frac{6\sqrt{4+5\rho+2\rho^2}}{4+3\rho}\right)>0,$$

where the inequality holds because

$$\left((4+3\rho)\sqrt{9+17\rho+8\rho^2}\right)^2-\left(6(1+\rho)\sqrt{4+5\rho+2\rho^2}\right)^2 = \rho\left(21\rho^2+41\rho+20\right) > 0.$$

For market efficiency, in the benchmark economy without information sharing,

$$(m^0)^{-1}=1-\frac{(\alpha_v^0+\beta_y^0)^2}{\alpha_v^0+\beta_y^0)^2+(\beta_y^0)^2/\rho+\sigma_u^2}=\frac{2+\rho}{4+3\rho},$$

and in equilibrium,

$$(m^*)^{-1}=1-\frac{(\alpha_v^*+\beta_y^*-\alpha_L^*)^2}{(\alpha_v^*+\beta_y^*-\alpha_L^*)^2+(\beta_y^{*0}-\alpha_L^*)^2/\rho+\sigma_u^2}=\frac{32\rho^2+93\rho+36}{153\rho^2+225\rho+72}.$$

A direct comparison yields $m^*>m^0$.

Finally, we discuss trading volume. The trading volume of H in the benchmark economy and that in equilibrium are respectively $TV_H^0=\frac{(2+\rho)\sigma_u}{\sqrt{2\pi}\sqrt{4+5\rho+2\rho^2}}$ and $TV_H^*=\frac{\sqrt{9+4\rho}\sigma_u}{\sqrt{2\pi}\sqrt{9+8\rho}}$, and it can be shown that $TV_H^*<TV_H^0$. Similarly, the trading volume of L in the benchmark economy and that in equilibrium are respectively $TV_L^0=\frac{\sqrt{\rho(1+\rho)}\sigma_u}{\sqrt{2\pi}\sqrt{4+5\rho+2\rho^2}}$ and $TV_L^*=\frac{\sqrt{\rho(1+\rho)}\sigma_u}{\sqrt{2\pi}\sqrt{(9+8\rho)(1+\rho)}}$, and it can be shown that $TV_L^*>TV_L^0$. Further, the trading volume of market makers in the benchmark economy and that in equilibrium are respectively $TV_M^0=\frac{\sqrt{(1+\rho)(4+3\rho)}\sigma_u}{\sqrt{\pi}\sqrt{4+5\rho+2\rho^2}}$ and $TV_M^*=\frac{\sqrt{3(6+11\rho)}\sigma_u}{\sqrt{2\pi}\sqrt{9+8\rho}}$, and it can be shown that $TV_M^*>TV_M^0$. Last, to prove the total trading volume increases after information sharing ($TV^*>TV^0$), given that the trading volume of market makers increases ($TV_M^* > TV_M^0$), it suffices to show that $TV_H^* + TV_L^* > TV_H^0 + TV_L^0$. Define

$$f(\rho) \equiv (TV_H^* + TV_L^*) - \left(TV_H^0 + TV_L^0\right) = \frac{\sigma_u}{\sqrt{2\pi}} \left(\frac{2\sqrt{\rho} + \sqrt{9+4\rho}}{\sqrt{9+8\rho}} - \frac{2+\rho+\sqrt{\rho(1+\rho)}}{\sqrt{4+5\rho+2\rho^2}} \right).$$

Solving $f(\rho) = 0$ yields the unique real root $\rho = 0$. This suggests that for all $\rho \in (0, +\infty)$, $f(\rho)$ shares the same sign. With a randomly picked positive number, i.e., $\rho = 1$, we find that $f(1) = \frac{\sigma_u}{\sqrt{2\pi}} \left(\frac{2+\sqrt{13}}{\sqrt{17}} - \frac{3+\sqrt{2}}{\sqrt{11}} \right) \approx 0.0286 \cdot \frac{\sigma_u}{\sqrt{2\pi}} > 0$. Therefore, $f(\rho) > 0$ for $\rho > 0$; that is, the total trading volume increases after information sharing.

Proof of Proposition 4

We first prove part (1) of this proposition. Upon the realizations of H's and L's private signals: $v = \tilde{v}$ and $y = \tilde{y}$, suppose that neither investor shares information. Following the standard procedure, we can show the two investors' conditional expected trading profits as follows:

$$E[\pi_L^{(NS,NS)} | \tilde{y} = y] = \frac{\rho^2 \sigma_u}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}} y^2,$$
$$E[\pi_H^{(NS,NS)} | \tilde{v} = v] = \frac{(2+\rho)^2 \sigma_u}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}} v^2.$$

Now suppose that L deviates to sharing her observation of y. Then (i) H updates his belief and trades accordingly, and (ii) MMs know that L shares her info (but not the specific realization) and adjust the pricing rule accordingly. L's profit becomes

$$E[\pi_L^{(S,NS)} | \tilde{y}] = \frac{2\rho^2 \sigma_u \sqrt{1+\rho}}{3(1+\rho)^2 \sqrt{9+8\rho}} \tilde{y}^2.$$

Since $E[\pi_L^{(S,NS)} | \tilde{y}] > E[\pi_L^{(NS,NS)} | \tilde{y}]$ for any realization of $\tilde{y}$, L always has incentive to deviate.

For part (2) of the proposition, suppose that upon observing the realization of his private signal, H shares it. Then, regardless of L's sharing decision, we can show H's conditional expected trading profits as follows:

$$E[\pi_H^{(S,S)}|\tilde{v}=v] = E[\pi_H^{(S,NS)}|\tilde{v}=v] = \frac{\sigma_u}{3\sqrt{2}}v^2.$$

Now we study H's deviation in the following two cases. First, consider that in the conjectured equilibrium L does not share her information. Then after deviation, both market makers and L do not update beliefs about H's private signal. We then compute H's conditional expected trading profits after deviation as follows:

$$E'[\pi_H^{(NS,NS)}|\tilde{v}=v] = \frac{(2+\rho)^2\sigma_u}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}}\tilde{v}^2.$$

Since $E'[\pi_H^{(NS,NS)}|\tilde{v}=v] > E[\pi_H^{(S,NS)}|\tilde{v}=v]$ for any realization of $\tilde{v}$, H has incentive to deviate if L does not share her information.

Second, consider that L also shares her information in the conjectured equilibrium. Then after H's deviation, again both market makers and L do not update beliefs about H's private signal. We can compute H's conditional expected profit after deviation as

$$E'[\pi_H^{(NS,S)}|\tilde{v}=v] = \frac{(\rho+(3+2\rho)^2\tilde{v}^2)\sigma_u\sqrt{1+\rho}}{6(1+\rho)^2\sqrt{9+8\rho}}.$$

Since $E'[\pi_H^{(NS,S)}|\tilde{v}=v] > E[\pi_H^{(S,S)}|\tilde{v}=v]$ for any realization of $\tilde{v}$, H has incentive to deviate to not sharing his information. Overall, regardless of L's sharing strategy, H always has incentives to deviate to not sharing his information. So the conjectured equilibrium involving H sharing information cannot be sustained.

We finally prove part (3) of this proposition in the following two steps. Suppose that upon observing the realization of their private signals, L fully shares her information whereas H does not. Then, the two investors' conditional expected profits can be computed as follows:

$$E[\pi_L^{(S,NS)}|\tilde{y}=y] = \frac{2\rho^2\sigma_u\sqrt{1+\rho}}{3(1+\rho)^2\sqrt{9+8\rho}}y^2,$$
$$E[\pi_H^{(S,NS)}|\tilde{v}=v] = \frac{(\rho+(3+2\rho)^2v^2)\sigma_u\sqrt{1+\rho}}{6(1+\rho)^2\sqrt{9+8\rho}}.$$

First, given that L fully shares her information, we show that H will not deviate from non-sharing for any given realization of $\tilde{v}$. Consider that H deviates to sharing his observation of v. Then (i) L naturally updates her belief about the asset fundamental and trades accordingly, and and (ii) MMs know that H shares his info. H's profits become the following:

$$E[\pi_H^{(S,S)}|\tilde{v}] = \frac{\sigma_u}{3\sqrt{2}}\tilde{v}^2$$

Since $E[\pi_H^{(S,NS)}|\tilde{v}] - E[\pi_H^{(S,S)}|\tilde{v}] = \frac{\sigma_u^2}{6}\left(\frac{\rho}{(1+\rho)^{3/2}\sqrt{9+8\rho}} + \left(\frac{(3+2\rho)^2}{(1+\rho)^{3/2}\sqrt{9+8\rho}} - \sqrt{2}\right)\tilde{v}^2\right) > 0$ for any $\tilde{v}$, H will not deviate from his non-sharing strategy.

Second, given that H does not share his information, we will show that L will not deviate from sharing her information for any realization of $\tilde{y}$. Consider that L deviates to not sharing her information. Then, if (i) H holds passive beliefs along this off-equilibrium path, i.e., believing that L's private information $\tilde{y}$ still follows the original distribution; and (ii) MMs can observe the information-sharing behavior per se but also hold passive beliefs (not updating about the distribution of L's private signal), then her profit becomes

$$E[\pi_L^{(NS,NS)}|\tilde{y}] = \frac{\rho^2\sigma_u}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}}\tilde{y}^2.$$

And we can that for any $\tilde{y}$, $E[\pi_L^{(NS,NS)}|\tilde{y}] < E[\pi_L^{(S,NS)}|\tilde{y}]$, so L will not deviate from sharing her information. Taken together, that L fully shares her information whereas H never shares his regardless of the realizations of their private signals can be sustained as an equilibrium.

Proof of Proposition 5

Without loss of generality, we consider the case in which investor 1 is endowed with information of higher precision than investor 2; that is, $\rho_1 \geq \rho_2$. We conjecture a linear pricing rule for market makers $\tilde{p} = \lambda\tilde{\omega}$ and linear trading strategies for the two investors: $\tilde{x}_1 = \alpha_y\tilde{y}_1 + \alpha_1\tilde{s}_1 + \alpha_2\tilde{s}_2$ and $\tilde{x}_2 = \beta_y\tilde{y}_2 + \beta_1\tilde{s}_1 + \beta_2\tilde{s}_2$.

After information sharing, investor 1's information set is $\mathcal{F}_1 = \{\tilde{y}_1, \tilde{s}_1, \tilde{s}_2\}$. With the information set, investor 1's posterior beliefs about the fundamental value, investor 2's information, and noise trading are

$$E(\tilde{v}|\mathcal{F}_1) = \frac{\rho_1(\rho_2+\tau_2)\tilde{y}_1 + \rho_2\tau_2\tilde{s}_2}{(\rho_1+1)\tau_2 + \rho_2(\rho_1+\tau_2+1)},$$
$$E(\tilde{y}_2|\mathcal{F}_1) = \frac{\rho_1\rho_2\tilde{y}_1 + (\rho_1+\rho_2+1)\tau_2\tilde{s}_2}{(\rho_1+1)\tau_2 + \rho_2(\rho_1+\tau_2+1)},$$
$$E(\tilde{u}|\mathcal{F}_1) = 0.$$

Then investor 1's conditional trading profits can be expressed as follows:

$$\pi_1 = \tilde{x}_1\left(E(\tilde{v}|\mathcal{F}_1) - \lambda\left(\tilde{x}_1 + \beta_y E(\tilde{y}_2|\mathcal{F}_1)\right) + \beta_1\tilde{s}_1 + \beta_2\tilde{s}_2\right). \quad \text{(A1)}$$

Maximizing investor 1's profits yields her trading strategy: $\tilde{x}_1 = \alpha_y\tilde{y}_1 + \alpha_1\tilde{s}_1 + \alpha_2\tilde{s}_2$, with

$$\alpha_y = \frac{\rho_1(\tau_2+\rho_2(1-\lambda\beta_y))}{2\lambda((\rho_1+1)\tau_2+\rho_2(\rho_1+\tau_2+1))},$$
$$\alpha_1 = -\frac{\beta_1}{2},$$
$$\alpha_2 = -\frac{\beta_2}{2} - \frac{\tau_2(\lambda(\rho_1+\rho_2+1)\beta_y - \rho_2)}{2\lambda((\rho_1+1)\tau_2+\rho_2(\rho_1+\tau_2+1))}.$$

Similarly,we can derive the trading strategy of investor 2 $\tilde{x}_2 = \beta_y\tilde{y}_2 + \beta_1\tilde{s}_1 + \beta_2\tilde{s}_2$, with

$$\beta_y = \frac{\rho_2(\tau_1+\rho_1(1-\lambda\alpha_y))}{2\lambda((\rho_2+1)\tau_1+\rho_1(\rho_2+\tau_1+1))},$$
$$\beta_1 = -\frac{\alpha_1}{2} - \frac{\tau_1(\lambda(\rho_1+\rho_2+1)\alpha_y - \rho_1)}{2\lambda((\rho_2+1)\tau_1+\rho_1(\rho_2+\tau_1+1))},$$
$$\beta_2 = -\frac{\alpha_2}{2}.$$

With the two investors best-response trading strategies we can derive their optimal trading rules as functions of $\{\tau_1, \tau_2, \lambda\}$. Further, in the pricing rule, $\lambda > 0$ is determined by:

$$\lambda = \frac{\sqrt{\begin{array}{c} \left(\begin{array}{c} 18\rho_2^4 + (48\tau_1 + 48\tau_2 + 45)\rho_2^3 + \left(32\tau_1^2 + 4(28\tau_2 + 17)\tau_1 + 32\tau_2^2 + 113\tau_2 + 36\right)\rho_2^2 \\ +4\tau_2\left(16\tau_1^2 + 2(8\tau_2 + 17)\tau_1 + 17\tau_2 + 18\right)\rho_2 + 4\left(8\tau_1^2 + 17\tau_1 + 9\right)\tau_2^2 \end{array}\right)\rho_1^4 \\ + \left(\begin{array}{c} (48\tau_1 + 48\tau_2 + 45)\rho_2^4 + 8\left(8\tau_1^2 + (28\tau_2 + 23)\tau_1 + 8\tau_2^2 + 23\tau_2 + 9\right)\rho_2^3 \\ +4\left(24(2\tau_2 + 1)\tau_1^2 + 2\left(24\tau_2^2 + 64\tau_2 + 17\right)\tau_1 + 41\tau_2^2 + 52\tau_2 + 9\right)\rho_2^2 \\ +8\tau_2\left(8(2\tau_2 + 3)\tau_1^2 + (41\tau_2 + 34)\tau_1 + 17\tau_2 + 9\right)\rho_2 \\ +4\left(24\tau_1^2 + 34\tau_1 + 9\right)\tau_2^2 \end{array}\right)\rho_1^3 \\ + \left(\begin{array}{c} \left(32\tau_1^2 + (112\tau_2 + 113)\tau_1 + 4\left(8\tau_2^2 + 17\tau_2 + 9\right)\right)\rho_2^4 \\ +4\left((48\tau_2 + 41)\tau_1^2 + 4\left(12\tau_2^2 + 32\tau_2 + 13\right)\tau_1 + 24\tau_2^2 + 34\tau_2 + 9\right)\rho_2^3 \\ +4\left(\left(48\tau_2^2 + 113\tau_2 + 24\right)\tau_1^2 + \left(113\tau_2^2 + 134\tau_2 + 17\right)\tau_1 + \tau_2(24\tau_2 + 17)\right)\rho_2^2 \\ +8\tau_2\left(12(3\tau_2 + 2)\tau_1^2 + (41\tau_2 + 17)\tau_1 + 4\tau_2\right)\rho_2 + 4\tau_1(24\tau_1 + 17)\tau_2^2 \end{array}\right)\rho_1^2 \\ +4\tau_1\left(\begin{array}{c} \left(\tau_1(16\tau_2 + 17) + 2\left(8\tau_2^2 + 17\tau_2 + 9\right)\right)\rho_2^4 \\ +2\left(24\tau_2^2 + 34\tau_2 + \tau_1\left(16\tau_2^2 + 41\tau_2 + 17\right) + 9\right)\rho_2^3 \\ +\left(2\tau_2(24\tau_2 + 17) + \tau_1\left(72\tau_2^2 + 82\tau_2 + 8\right)\right)\rho_2^2 + 16\tau_2\left(\tau_2 + \tau_1(3\tau_2 + 1)\right)\rho_2 + 8\tau_1\tau_2^2 \end{array}\right)\rho_1 \\ +4\rho_2(\rho_2 + 1)\tau_1^2\left(\left(8\tau_2^2 + 17\tau_2 + 9\right)\rho_2^2 + \tau_2(16\tau_2 + 17)\rho_2 + 8\tau_2^2\right) \end{array}}}{3\sigma_u\left(\begin{array}{c} \left(3\rho_2^2 + 4(\tau_1 + \tau_2 + 1)\rho_2 + 4(\tau_1 + 1)\tau_2\right)\rho_1^2 \\ +4\left((\tau_1 + \tau_2 + 1)\rho_2^2 + (2\tau_2 + 2\tau_1(\tau_2 + 1) + 1)\rho_2 + (2\tau_1 + 1)\tau_2\right)\rho_1 \\ +4(\rho_2 + 1)\tau_1\left(\tau_2 + \rho_2(\tau_2 + 1)\right) \end{array}\right)}.$$

Inserting the optimal trading strategies into the two investors' profit functions yields their unconditional profits: $E[\pi_1] \equiv \pi_1(\tau_1, \tau_2; \rho_1, \rho_2)$ and $E[\pi_2] \equiv \pi_2(\tau_1, \tau_2; \rho_1, \rho_2)$. We then show in the next two steps that the following can be sustained in equilibrium: $\tau_1^* = 0$, and if $\rho_1 \geq 2(\rho_2 + 1)$, $\tau_2^* = \infty$,and otherwise if $\rho_1 < 2(\rho_2 + 1)$, $\tau_2^* = 0$.

Step 1. Given $\tau_1 = 0$, we show that if $\rho_1 \geq 2(\rho_2 + 1)$, $\tau_2^* = \infty$, and otherwise if $\rho_1 < 2(\rho_2 + 1)$, $\tau_2^* = 0$.

$$\frac{\partial \pi_2(\tau_2; \rho_1, \rho_2)}{\partial \tau_2}|_{\tau_1 = 0} = (\rho_1 - 2(\rho_2 + 1))\,\rho_1^3 \rho_2^2 \sigma_u \frac{\Psi_1}{\Psi_2},$$

where $\Psi_1, \Psi_2 > 0$ and are given in the supplementary online appendix. Therefore, if $\rho_1 > 2(1 + \rho_2)$, $\frac{\partial \pi_2(\tau_1, \tau_2; \rho_1, \rho_2)}{\partial \tau_2}|_{\tau_1 = 0} > 0$, so investor 2 fully shares her information. If, instead, $\rho_1 < 2(1 + \rho_2)$, $\frac{\partial \pi_2(\tau_1, \tau_2; \rho_1, \rho_2)}{\partial \tau_2}|_{\tau_1 = 0} < 0$ and investor 2 does not share any information. If $\rho_1 = 2(1 + \rho_2)$, investor 2 is indifferent.

Step 2. We confirm that given investor 2's best response characterized in Step 1, investor 1 will not deviate from $\tau_1 = 0$. When $\tau_2 = 0$, we obtain that

$$\frac{\partial \pi_1(\tau_1, \tau_2; \rho_1, \rho_2)}{\partial \tau_1}|_{\tau_2=0} = \rho_1^2 \rho_2 \sigma_u (\rho_2 - 2 - 2\rho_1) \frac{\Psi_3}{\Psi_4}$$

where $\Psi_3, \Psi_4 > 0$ and are given in the online supplementary online appendix. Since $\rho_2 < \rho_1$, $\frac{\partial \pi_1(\tau_1,\tau_2;\rho_1,\rho_2)}{\partial \tau_1}|_{\tau_2=0} < 0$ always holds. Therefore, when $\tau_2 = 0$, investor 1 will not deviate from $\tau_1 = 0$. Moreover, when $\tau_2 = \infty$, we can show that $\frac{\partial \pi_1(\tau_1,\tau_2;\rho_1,\rho_2)}{\partial \tau_1}|_{\tau_2=\infty} = \Psi_5 < 0$, where Ψ_5 is given in the supplementary online appendix. Therefore, investor 1 will not deviate either.

Proof of Proposition 6

We first examine the equilibrium when Hs do not share their information (Assumption 1) and L fully shares her information. In this way, we focus on Hs' reading shared information. Assume that among the H investors, M_1 of them choose to use the information shared by investor L. We consider the following symmetric linear trading strategies; that is, the H investor that uses the shared information demands $\tilde{x}_i = \alpha_{v_1}\tilde{v} + \alpha_{L_1}\tilde{y}$ units of the risky asset, where $i \in \{1, ..., M_1\}$, and the H investor that commits to not using the shared information demands $\tilde{x}_k = \alpha_{v_2}\tilde{v}$ units of the risky asset, where $k \in \{M_1 + 1, ..., M\}$. The L investor demands $x_L = \beta\tilde{y}$ units of the risky asset. We also consider a linear pricing rule for market makers $\tilde{p} = \lambda\tilde{\omega}$.

Consider the H investor $i \in \{1, ..., M_1\}$ that uses the shared information. With the information set $\{\tilde{v}, \tilde{y}\}$, his conditional expected profits are as follows:

$$E[\tilde{x}_i(\tilde{v} - \tilde{p})] = \tilde{x}_i \left(\tilde{v} - \lambda \left(\tilde{x}_i + (M_1 - 1)(\alpha_{v_1}\tilde{v} + \alpha_{L_1}\tilde{y}) + (M - M_1)\alpha_{v_2}\tilde{v} + \beta\tilde{y}\right)\right).$$

Maximizing the profits yields the investor H_i's optimal trading rule $\tilde{x}_i = \alpha_{v_i}\tilde{v} + \alpha_{L_i}\tilde{y}$ with

$$\alpha_{v_i} = \frac{1}{2\lambda}\left(1 - (M_1 - 1)\lambda\alpha_{v_1} - (M - M_1)\lambda\alpha_{v_2}\right) \text{ and } \alpha_{L_i} = -\frac{1}{2}\left((M_1 - 1)\alpha_{L_1} + \beta\right). \quad \text{(A2)}$$

For the H investor $k \in \{M_1 + 1, ..., M\}$ that commits to not using the shared information, with the information set $\{\tilde{v}\}$, his conditional expected profits are as follows:

$$E[\tilde{x}_k(\tilde{v}-\tilde{p})] = \tilde{x}_k\left(\tilde{v} - \lambda\left(\tilde{x}_k + M_1(\alpha_{v_1}\tilde{v} + \alpha_{L_1}\tilde{v}) + (M - M_1 - 1)\alpha_{v_2}\tilde{v} + \beta\tilde{v}\right)\right).$$

Maximizing the profits yields the investor H_k's optimal trading rule $\tilde{x}_k = \alpha_{v_k}\tilde{v}$ with

$$\alpha_{v_k} = \frac{1}{2\lambda}\left(1 - \beta\lambda - M_1\lambda(\alpha_{L_1} + \alpha_{v_1}) - (M - M_1 - 1)\lambda\alpha_{v_2}\right). \tag{A3}$$

For investor L, her conditional expected trading profits are as follows:

$$E[\tilde{x}_L(\tilde{v}-\tilde{p})] = \tilde{x}_L\left(\frac{\rho}{1+\rho}\tilde{y} - \lambda\left(\tilde{x}_L + M_1\left(\alpha_{v_1}\frac{\rho}{1+\rho}\tilde{y} + \alpha_{L_1}\tilde{y}\right) + (M - M_1)\alpha_{v_2}\frac{\rho}{1+\rho}\tilde{y}\right)\right).$$

Maximizing investor L's profits yields her optimal trading strategy $\tilde{x}_L = \beta\tilde{y}$, with

$$\beta = -\frac{M_1}{2}\alpha_{L_1} + \frac{\rho}{2\lambda(1+\rho)}\left(1 - M_1\lambda\alpha_{v_1} - (M - M_1)\lambda\alpha_{v_2}\right). \tag{A4}$$

Imposing symmetric equilibrium $\alpha_{v_i} = \alpha_{v_1}$, $\alpha_{L_i} = \alpha_{L_1}$, and $\alpha_{v_k} = \alpha_{v_2}$, the interaction of the reaction functions (A2)-(A4) yields the optimal trading strategies as specified below:

$$\alpha_{v_1} = \frac{(2+M_1)(1+\rho)}{\lambda\left((1+M)(2+M_1) + (2+M)(1+M_1)\rho\right)}, \tag{A5}$$

$$\alpha_{L_1} = -\frac{\rho}{\lambda\left((1+M)(2+M_1) + (2+M)(1+M_1)\rho\right)}, \tag{A6}$$

$$\alpha_{v_2} = \frac{2 + M_1 + \rho + M_1\rho}{\lambda\left((1+M)(2+M_1) + (2+M)(1+M_1)\rho\right)}, \tag{A7}$$

$$\beta = \frac{(1+M_1)\rho}{\lambda\left((1+M)(2+M_1) + (2+M)(1+M_1)\rho\right)}. \tag{A8}$$

Using the weak efficiency rule, market makers' optimal pricing rule is as follows:

$$\lambda = \frac{M_1(\alpha_{v_1} + \alpha_{L_1}) + (M - M_1)\alpha_{v_2} + \beta}{(M_1(\alpha_{v_1} + \alpha_{L_1}) + (M - M_1)\alpha_{v_2} + \beta)^2 + (M_1\alpha_{L_1} + \beta)^2/\rho + \sigma_u^2}.$$

Inserting the optimal trading strategies into λ we can express the equilibrium pricing rule as follows $\tilde{p} = \lambda\tilde{\omega}$ with

$$\lambda = \frac{\sqrt{M(2+M_1+\rho+M_1\rho)^2+\rho+\rho^2+M_1\rho(3+M_1+(2+M_1)\rho)}}{((1+M)(2+M_1)+(2+M)(1+M_1)\rho)\,\sigma_u}. \tag{A9}$$

Now, inserting the optimal trading rules (A5)-(A8) and the optimal pricing rule (A9) into the investors' expected trading profits and taking expectations yields their respective unconditional profits as follows:

$$\pi_i(M_1, M) = (\rho+1)\left(M_1^2(\rho+1)+2M_1(\rho+2)+\rho+4\right)\sigma_u\Gamma^{-1}, \tag{A10}$$

$$\pi_k(M_1, M) = (M_1\rho+M_1+\rho+2)^2\sigma_u\Gamma^{-1}, \tag{A11}$$

$$\pi_L(M_1, M) = (M_1+1)^2\rho(\rho+1)\sigma_u\Gamma^{-1}, \tag{A12}$$

where

$$\Gamma = ((M+2)(M_1+1)\rho+(M+1)(M_1+2)) \times \sqrt{M(M_1\rho+M_1+\rho+2)^2+\rho(M_1((M_1+2)\rho+M_1+3)+\rho+1)}.$$

Proof of Part (i) We discuss if $M_1 = 0$ (that all Hs commit not to using L's shared information) is an equilibrium. When $M_1 = 0$, based on equations (A11), the profits of Hs are as follows:

$$\pi_k(0, M) = \frac{(2+\rho)^2\sigma_u}{(2+2\rho+M(2+\rho))\sqrt{M(3+2\rho)^2+\rho(5+4\rho)}}.$$

If one H deviates to receiving the shared information, according to equation (A10), her profits will become

$$\pi_i(1, M) = \frac{(1+\rho)(9+4\rho)\sigma_u}{(3+4\rho+M(3+2\rho))\sqrt{M(3+2\rho)^2+\rho(5+4\rho)}}.$$

Therefore, $M_1 = 0$ is not an equilibrium if $\frac{\pi_i(1,M)}{\pi_k(0,M)} > 1$. Further, we know that

$$\frac{\partial}{\partial M}\left(\frac{\pi_i(1,M)}{\pi_k(0,M)}\right) = \frac{\rho\pi_i(1,M)}{\pi_k(0,M)} \frac{\begin{pmatrix} M^2\left(18\rho^4+121\rho^3+303\rho^2+335\rho+138\right) \\ +M\left(56\rho^4+299\rho^3+577\rho^2+469\rho+132\right) \\ +2(\rho+1)^2\left(24\rho^2+56\rho+33\right)\end{pmatrix}}{\begin{pmatrix} 2(M(\rho+2)+2(\rho+1))\left(M(\rho+2)^2+\rho(\rho+1)\right) \\ \times(M(2\rho+3)+4\rho+3)\left(M(2\rho+3)^2+\rho(4\rho+5)\right)\end{pmatrix}} > 0$$

and when $M = 4$,

$$\frac{\pi_i(1,4)}{\pi_k(0,4)} = \frac{2(\rho+1)(3\rho+5)(4\rho+9)\sqrt{5\rho^2+17\rho+16}}{3(\rho+2)^2(4\rho+5)\sqrt{20\rho^2+53\rho+36}} > 1,$$

where the inequality holds because

$$\begin{aligned}\left(2(\rho+1)(3\rho+5)(4\rho+9)\sqrt{5\rho^2+17\rho+16}\right)^2 - \left(3(\rho+2)^2(4\rho+5)\sqrt{20\rho^2+53\rho+36}\right)^2 \\ = \rho\left(240\rho^6+2744\rho^5+13071\rho^4+33004\rho^3+46336\rho^2+34132\rho+10260\right) > 0.\end{aligned}$$

Therefore, if $M > 4$, we must have $\frac{\pi_i(1,M)}{\pi_k(0,M)} > 1$; that is, $M_1 = 0$ cannot be sustained an equilibrium. Note that when $M \leq 2$, we can show that $\pi_i(1,M) < \pi_k(0,M)$; thus, $M_1 = 0$ can be an equilibrium.

Proof of Part (ii) To prove the second part of the proposition, we first characterize the condition under which that L shares information and all Hs use the shared information can be sustained in equilibrium. We then prove that in this equilibrium Hs are trapped in a prisoner's dilemma.

First, we discuss when can $M_1 = M$ (that all Hs receive L's shared information) be an equilibrium. When $M_1 = M$, based on equation (A10), the profits of Hs are as follows:

$$\pi_i(M,M) = \frac{\left(M^2(\rho+1)+2M(\rho+2)+\rho+4\right)\sigma_u}{(M^2+3M+2)\sqrt{\rho\left(M^2(\rho+1)+M(2\rho+3)+\rho+1\right)+M(M\rho+M+\rho+2)^2}}.$$

If one H deviates to not using the shared information, according to equation (A11), his profits will become

$$\pi_k(M-1,M) = \frac{(M\rho+M+1)^2\sigma_u}{(M^2(\rho+1)+2M(\rho+1)+1)\sqrt{M^3(\rho+1)^2+M^2(\rho^2+3\rho+2)+M(\rho+1)-\rho}}.$$

Therefore, $M_1 = M$ is an equilibrium if $\frac{\pi_k(M-1,M)}{\pi_i(M,M)} < 1$. Further,

$$\frac{\partial}{\partial M}\left(\frac{\pi_k(M-1,M)}{\pi_i(M,M)}\right) = \frac{\rho\pi_k(M-1,M)}{\pi_i(M,M)} \times \frac{\begin{pmatrix} 4M^{10}(\rho+1)^4 + M^9(\rho+1)^3(23\rho+31) + M^8(\rho+1)^2\left(29\rho^2+89\rho+64\right) \\ -M^7(\rho+1)^2\left(97\rho^2+244\rho+143\right) - M^6\left(397\rho^4+2092\rho^3+4005\rho^2+3326\rho+1016\right) \\ -M^5\left(624\rho^4+3715\rho^3+7907\rho^2+7213\rho+2397\right) - M^4\left(532\rho^4+3590\rho^3+8551\rho^2+8651\rho+3166\right) \\ -M^3\left(258\rho^4+1966\rho^3+5321\rho^2+6128\rho+2551\right) - M^2\left(68\rho^4+562\rho^3+1785\rho^2+2484\rho+1246\right) \\ -M\left(8\rho^4+62\rho^3+260\rho^2+523\rho+340\right) - 2\left(3\rho^2+25\rho+20\right) \end{pmatrix}}{\begin{pmatrix} 2\left(M^2+3M+2\right)(M\rho+M+1) \times \left(M^2(\rho+1)+2M(\rho+2)+\rho+4\right) \\ \times\left(M^3(\rho+1)+M^2(3\rho+4)+M(3\rho+4)+\rho\right) \\ \times\left(M^3(\rho+1)^2+M^2\left(\rho^2+3\rho+2\right)+M(\rho+1)-\rho\right) \end{pmatrix}}.$$

So, when M is sufficiently large, $\frac{\partial}{\partial M}\left(\frac{\pi_k(M-1,M)}{\pi_i(M,M)}\right) > 0$. We further know that $\lim_{M\to\infty}\frac{\pi_k(M-1,M)}{\pi_i(M,M)} = 1$ and $\lim_{M\to 1}\frac{\pi_k(M-1,M)}{\pi_i(M,M)} = \frac{6(\rho+2)^2\sqrt{8\rho^2+17\rho+9}}{(3\rho+4)(4\rho+9)\sqrt{2\rho^2+5\rho+4}} > 1$. Thus, there must exist $\hat{M}_1 > 0$ such that when $M > \hat{M}_1$, $\frac{\pi_k(M-1,M)}{\pi_i(M,M)} < 1$.

Finally, we need to make sure that L will not deviate to not sharing information to sustain the equilibrium in which all Hs use the shared information. Following the similar derivation we did in the beginning of the proof, we can derive that if L deviates to not sharing, her unconditional expected profits are

$$\pi_L^{deviate} = \frac{\rho(\rho+1)\sigma_u}{(M(\rho+2)+2(\rho+1))\sqrt{M(\rho+2)^2+\rho(\rho+1)}}.$$

When L shares her information and all Hs use the information, L's expected profits are

$$\pi_L = \frac{(M+1)(M(\rho+2)+2(\rho+1))\sqrt{M(\rho+2)^2+\rho(\rho+1)}}{(M+2)(\rho+1)\sqrt{\rho\left(M^2(\rho+1)+M(2\rho+3)+\rho+1\right)+M(M\rho+M+\rho+2)^2}}.$$

Define $f(M) \equiv \frac{\pi_L}{\pi_L^{deviate}}$. We can further show that $f'(M) > 0$ and $f(1) = \frac{2(3\rho+4)\sqrt{2\rho^2+5\rho+4}}{3(\rho+1)\sqrt{8\rho^2+17\rho+9}} > 1$. Therefore, for $M \geq 1$, $\pi_L > \pi_L^{deviate}$ always holds. In other words, that L shares her information and all Hs use the shared information can be sustained in equilibrium.

Prisoner's dilemma for Hs when in equilibrium $M_1 = M$ Finally, we show that while $M_1 = M$ can be sustained as an equilibrium for large $M > \hat{M}$, it is always dominated by $M_1 = 0$ in terms of Hs' profits. That is, Hs would have been better off if they committed not to receiving the shared information.

An H investor's profits when all Hs do not use L's shared information and when all Hs use the shared information are respectively as follows:

$$\pi_i(0, M) = \frac{(\rho+2)^2\sigma_u}{(M(\rho+2)+2\rho+2)\sqrt{M(\rho+2)^2+\rho(\rho+1)}},$$
$$\pi_k(M, M) = \frac{\left(M^2(\rho+1)+2M(\rho+2)+\rho+4\right)\sigma_u}{\left(M^2+3M+2\right)\sqrt{\rho\left(M^2(\rho+1)+M(2\rho+3)+\rho+1\right)+M(M\rho+M+\rho+2)^2}}.$$

We know that

$$\frac{\partial}{\partial M}\frac{\pi_i(0, M)}{\pi_k(M, M)} = \rho(\rho+2)^2$$
$$\times\frac{\begin{pmatrix} -M^7(\rho+1)^2\left(4\rho^2+15\rho+14\right) - M^6\left(22\rho^4+119\rho^3+242\rho^2+217\rho+72\right) \\ -M^5\left(33\rho^4+165\rho^3+314\rho^2+268\rho+86\right) + M^4\left(37\rho^4+185\rho^3+368\rho^2+336\rho+116\right) \\ +M^3\left(181\rho^4+805\rho^3+1372\rho^2+1060\rho+312\right) + M^2\left(237\rho^4+949\rho^3+1380\rho^2+844\rho+176\right) \\ +2M\rho\left(70\rho^3+247\rho^2+293\rho+116\right)+32\rho(\rho+1)^3 \end{pmatrix}}{\begin{pmatrix} 2(M(\rho+2)+2(\rho+1))^2\left(M^2(\rho+1)+2M(\rho+2)+\rho+4\right)^2 \\ \times\sqrt{\rho\left(M^2(\rho+1)+M(2\rho+3)+\rho+1\right)+M(M\rho+M+\rho+2)^2}\times\left(M(\rho+2)^2+\rho(\rho+1)\right)^{3/2} \end{pmatrix}}.$$

So, there exists a constant $\hat{M}_2 > 0$ such that when $M > \hat{M}_2$, $\frac{\partial}{\partial M}\frac{\pi_i(0,M)}{\pi_k(M,M)} < 0$. Further, as $M \to +\infty$, $\frac{\pi_i(0,M)}{\pi_k(M,M)} \to 1$. When $M = 2$,

$$\frac{\pi_i(0, 2)}{\pi_k(2, 2)} = \frac{6(\rho+2)^2\sqrt{27\rho^2+59\rho+32}}{(2\rho+3)(9\rho+16)\sqrt{3\rho^2+9\rho+8}} > 1,$$

where the inequality holds because

$$\left(6(\rho+2)^2\sqrt{27\rho^2+59\rho+32}\right)^2-\left((2\rho+3)(9\rho+16)\sqrt{3\rho^2+9\rho+8}\right)^2=$$
$$\rho\left(612\rho^4+4137\rho^3+10431\rho^2+11608\rho+4800\right)>0.$$

Therefore, when $M>\hat{M}_2$, $\frac{\pi_i(0,M)}{\pi_k(M,M)}>1$; that is, the H investors would be better off if all of them committed to not using the shared information. Overall, when $M>\max\{\hat{M}_1,\hat{M}_2\}$, part (ii) of the proposition holds.

References

Admati, Anat R., and Paul Pfleiderer. (1986). A monopolistic market for information. *Journal of Economic Theory*, 39(2), 400-438.

Admati, Anat R., and Paul Pfleiderer. (1988). Selling and trading on information in financial markets. *American Economic Review*, 78(2), 96-103.

Admati, Anat R., and Paul Pfleiderer. (1990). Direct and indirect sale of information. *Econometrica*, 901-928.

Allen, Franklin. (1990). The market for information and the origin of financial intermediation. *Journal of Financial Intermediation*, 1(1), 3-30.

Antweiler, Werner, and Murray Z. Frank. (2004). Is all that talk just noise? The information content of internet stock message boards. *Journal of Finance*, 59(3), 1259-1294.

Aragon, George O., and Philip E. Strahan. (2012). Hedge funds as liquidity providers: Evidence from the Lehman bankruptcy. *Journal of Financial Economics*, 103(3), 570-587.

Balasubramaniam, Swaminathan. (2021). Information sharing among strategic traders: The role of disagreement. Working Paper, Washington University in St. Louis.

Bartov, Eli, Lucile Faurel, and Partha S. Mohanram. (2018). Can Twitter help predict firm-level earnings and stock returns?. *The Accounting Review*, 93(3), 25-57.

Benabou, Roland, and Guy Laroque. (1992). Using privileged information to manipulate markets: Insiders, gurus, and credibility. *Quarterly Journal of Economics*, 107(3), 921-958.

Bernhardt, Dan, and Jianjun Miao. (2004). Informed trading when information becomes stale. *Journal of Finance*, 59(1), 339-390.

Biais, Bruno, and Laurent Germain. (2002). Incentive-compatible contracts for the sale of information. *Review of Financial Studies*, 15(4), 987-1003.

Cao, H. Henry, Richard K. Lyons, and Martin DD Evans. (2003). Inventory information. *Journal of Business*, 79, 325–364.

Cespa, Giovanni. (2008). Information sales and insider trading with long-lived information. *Journal of Finance*, 63(2), 639-672.

Chen, Hailiang, Prabuddha De, Yu Jeffrey Hu, and Byoung-Hyoun Hwang. (2014). Wisdom of crowds: The value of stock opinions transmitted through social media. *Review of Financial Studies*, 27(5), 1367-1403.

Cohen, Lauren, Andrea Frazzini, and Christopher Malloy. (2008). The small world of investing: Board connections and mutual fund returns. *Journal of Political Economy*, 116(5), 951-979.

Cohen, Lauren, Andrea Frazzini, and Christopher Malloy. (2010). Sell-side school ties. *Journal of Finance*, 65(4), 1409-1437.

Cohen, Lauren, and Christopher J. Malloy. (2010). The power of alumni networks. *Harvard Business Review* 88, no. 10.

Das, Sanjiv R., and Mike Y. Chen. (2007). Yahoo! for Amazon: Sentiment extraction from small talk on the web. *Management science*, 53(9), 1375-1388.

De Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann. (1990). Noise trader risk in financial markets. *Journal of Political Economy*, 98(4), 703-738.

DeMarzo, Peter M., Dimitri Vayanos, and Jeffrey Zwiebel. (2003). Persuasion bias, social influence, and unidimensional opinions. *Quarterly Journal of Economics*, 118(3), 909-968.

Farboodi, Maryam, and Laura Veldkamp (2020). Long run growth of financial data technology. Forthcoming at *American Economic Review*.

Fishman, Michael J., and Kathleen M. Hagerty (1995). The incentive to sell financial market information. *Journal of Financial Intermediation*, 4(2), 95-115.

Foucault, Thierry, and Laurence Lescourret. (2003). Information sharing, liquidity and transaction costs. *Finance*, 24, 45-78.

Frieder, Laura, and Jonathan Zittrain. (2007). Spam works: Evidence from stock touts and corresponding market activity. Hastings Communications and Entertainment Law Journal 30:479–520.

Ganguli, Jayant Vivek, and Liyan Yang. (2009). Complementarities, multiplicity, and supply information. *Journal of the European Economic Association*, 7(1), 90-115.

García, Diego, and Francesco Sangiorgi. (2011). Information sales and strategic trading. *Review of Financial Studies*, 24(9), 3069-3104.

Glode, Vincent, Christian C. Opp, and Xingtan Zhang(2018). Voluntary disclosure in bilateral transactions. *Journal of Economic Theory*, 175, 652-688.

Grossman, Sanford J. (1981). The informational role of warranties and private disclosure about product quality. *Journal of Law and Economics*, 24(3), 461-483.

Grossman, Sanford J., and Oliver D. Hart. (1980). Disclosure laws and takeover bids. *Journal of Finance*, 35(2), 323-334.

Gu, Chen, and Alexander Kurov. (2020). Informational role of social media: Evidence from Twitter sentiment. *Journal of Banking and Finance*, Forthcoming.

Hanke, Michael, and Florian Hauser. (2008). On the effects of stock spam e-mails. Journal of Financial markets, 11(1), 57-83.

Han, Bing, and David A. Hirshleifer. (2016). Social transmission bias and active investing. Working Paper, University of Toronto and University of California, Irvine.

Hong, Harrison, Jeffrey D. Kubik, and Jeremy C. Stein. (2005). Thy neighbor's portfolio: Word-of-mouth effects in the holdings and trades of money managers. *Journal of Finance*, 60(6), 2801-2824.

Indjejikian, R., Lu, Hai, and Yang, Liyan. (2014). Rational information leakage. *Management Science*, 60(11), 2762-2775.

Jame, Russell, Rick Johnston, Stanimir Markov, and Michael C. Wolfe. (2016). The value of crowdsourced earnings forecasts. *Journal of Accounting Research*, 54(4), 1077-1110.

Jylhä, Petri, Kalle Rinne, and Matti Suominen. (2014). Do hedge funds supply or demand liquidity?. *Review of Finance*, 18(4), 1259-1298.

Kaniel, Ron, Gideon Saar, and Sheridan Titman. (2008). Individual investor trading and stock returns. *Journal of Finance*, 63(1), 273-310.

Kim, Oliver, and Robert E. Verrecchia. (1991). Trading volume and price reactions to public announcements. *Journal of Accounting Research* 29(2): 302-321.

Kovbasyuk, Sergey, and Marco Pagano (2015). Advertising arbitrage. Working Paper, Einaudi Institute for Economics and Finance (EIEF) and University of Naples Federico II.

Kyle, A. S. (1985). Continuous auctions and insider trading. *Econometrica* 1315-1335.

Lee, S. (2013). Active investment, liquidity externalities, and markets for information. Working Paper, Santa Clara University.

Liu, Ying. (2017). Why do large investors disclose their information? Working Paper, Shanghai University of Finance and Economics.

Ljungqvist, Alexander, and Wenlan Qian. (2016). How constraining are limits to arbitrage?. *Review of Financial Studies*, 29(8), 1975-2028.

Luo, Patrick. (2018). Talking Your Book: Evidence from Stock Pitches at Investment Conferences. Working Paper, Harvard University.

Marmora, Paul, and Oleg Rytchkov (2018). Learning about noise. *Journal of Banking and Finance*, 89, 209-224.

Shiller, Robert J., and John Pound. (1986). Survey evidence on diffusion of interest among institutional investors (No. 794). Cowles Foundation for Research in Economics, Yale University.

Sobel, Joel. (1985). A theory of credibility. *Review of Economic Studies*, 52(4), 557-573.

Tumarkin, Robert, and Robert F. Whitelaw. (2001). News or noise? Internet postings and stock prices. *Financial Analysts Journal*, 57(3), 41-51.

McAfee, R. Preston, and Marius Schwartz. (1994). Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity. *American Economic Review*, 210-230.

Madrigal, Vicente.(1996). Non-fundamental speculation. *Journal of Finance*, 51(2), 553-578.

Milgrom, Paul R. (1981). Good news and bad news: Representation theorems and applications. *Bell Journal of Economics*, 380-391.

Mitchell, Mark, and Todd Pulvino. (2012). Arbitrage crashes and the speed of capital. *Journal of Financial Economics*, 104(3), 469-490.

Myatt, David P., and Chris Wallace. (2012). Endogenous information acquisition in coordination games. *Review of Economic Studies*, 79(1), 340-374.

Naik, Narayan Y. (1997). Multi-period information markets. *Journal of Economic Dynamics and Control*, 21(7), 1229-1258.

Pasquariello, Paolo, and Yifei Wang. Speculation with information disclosure. (2016) Working Paper, University of Michigan.

Schmidt, Daniel. (2018). Stock market rumors and credibility. Forthcoming at *Review of Financial Studies.*

SEC (2012). Investment adviser use of social media. *National Examination Risk Alert*, 2:1-7.

Van Bommel, Jos. (2003). Rumors. *Journal of Finance*, 58(4), 1499-1520.

Verrecchia, Robert E. (1982). Information acquisition in a noisy rational expectations economy. *Econometrica* 1415-1430.

Vives, Xavier. (2010). Information and learning in markets: the impact of market microstructure. Princeton University Press.

Zuckerman, Roy. (2011). Synchronized arbitrage and the value of public announcements. Working Paper, Rutgers Business School.

Supplementary Online Appendix

S1 Ψ values in the proof of Proposition 5

$$\Psi_1 = \rho_1^4 \begin{pmatrix} 9\rho_2^5 (308\tau_2 + 335) + 3\rho_2^4 \left(1168\tau_2^2 + 3601\tau_2 + 1428\right) \\ +4\rho_2^3 \left(368\tau_2^3 + 3187\tau_2^2 + 3518\tau_2 + 504\right) \\ +4\rho_2^2\tau_2 \left(1240\tau_2^2 + 3823\tau_2 + 1512\right) \\ +32\rho_2\tau_2^2 (172\tau_2 + 189) + 729\rho_2^6 + 2016\tau_2^3 \end{pmatrix}$$
$$+ 2\rho_1^3 \begin{pmatrix} 9\rho_2^6 (350\tau_2 + 347) + 9\rho_2^5 \left(672\tau_2^2 + 1903\tau_2 + 881\right) \\ +\rho_2^4 \left(3424\tau_2^3 + 27602\tau_2^2 + 34717\tau_2 + 8658\right) \\ +4\rho_2^3 \left(3432\tau_2^3 + 11857\tau_2^2 + 7777\tau_2 + 864\right) \\ +2\rho_2^2\tau_2 \left(10320\tau_2^2 + 18121\tau_2 + 5184\right) + 32\rho_2\tau_2^2 (431\tau_2 + 324) + 405\rho_2^7 + 3456\tau_2^3 \end{pmatrix}$$
$$+12 (\rho_2 + 1) \rho_1^2 \begin{pmatrix} 3\rho_2^6 (98\tau_2 + 101) + 3\rho_2^5 \left(380\tau_2^2 + 873\tau_2 + 395\right) \\ +\rho_2^4 \left(976\tau_2^3 + 5714\tau_2^2 + 6265\tau_2 + 1482\right) + 4\rho_2^3 \left(894\tau_2^3 + 2488\tau_2^2 + 1471\tau_2 + 162\right) \\ +2\rho_2^2\tau_2 \left(2436\tau_2^2 + 3661\tau_2 + 972\right) + 8\rho_2\tau_2^2 (365\tau_2 + 243) + 648\tau_2^3 \end{pmatrix}$$
$$+ 8 (\rho_2 + 1)^2 \rho_1 \begin{pmatrix} 3\rho_2^5 \left(212\tau_2^2 + 431\tau_2 + 219\right) + \rho_2^4 \left(1096\tau_2^3 + 4634\tau_2^2 + 4729\tau_2 + 1170\right) \\ +4\rho_2^3 \left(912\tau_2^3 + 2110\tau_2^2 + 1129\tau_2 + 90\right) \\ +2\rho_2^2\tau_2 \left(2184\tau_2^2 + 2761\tau_2 + 540\right) + 8\rho_2\tau_2^2 (272\tau_2 + 135) + 360\tau_2^3 \end{pmatrix}$$
$$+ 32\rho_2 (\rho_2 + 1)^3 (\rho_2 (\tau_2 + 1) + \tau_2)^2 (\rho_2 (76\tau_2 + 81) + 76\tau_2),$$

$$\Psi_2 = 6 \left(\rho_2^2 (3\rho_1 + 4\tau_2 + 4) + 4\rho_2 (\rho_1 (\tau_2 + 1) + 2\tau_2 + 1) + 4 (\rho_1 + 1) \tau_2\right)^2 \rho_1^3$$
$$\begin{pmatrix} \rho_1^2 \left(\rho_2^3 (48\tau_2 + 45) + \rho_2^2 \left(32\tau_2^2 + 113\tau_2 + 36\right) + 4\rho_2\tau_2 (17\tau_2 + 18) + 18\rho_2^4 + 36\tau_2^2\right) \\ +\rho_1 \begin{pmatrix} \rho_2^4 (48\tau_2 + 45) + 8\rho_2^3 \left(8\tau_2^2 + 23\tau_2 + 9\right) \\ +4\rho_2^2 \left(41\tau_2^2 + 52\tau_2 + 9\right) + 8\rho_2\tau_2 (17\tau_2 + 9) + 36\tau_2^2 \end{pmatrix} \\ +4\rho_2 (\rho_2 + 1) \left(\rho_2^2 \left(8\tau_2^2 + 17\tau_2 + 9\right) + \rho_2\tau_2 (16\tau_2 + 17) + 8\tau_2^2\right) \end{pmatrix}^{3/2}.$$

$$\Psi_3 = 2\rho_1^7 \left(18\rho_2^2 (98\tau_1 + 101) + 12\rho_2 \left(212\tau_1^2 + 431\tau_1 + 219\right) + 405\rho_2^3 + 16 (\tau_1 + 1)^2 (76\tau_1 + 81)\right)$$

$$+ \rho_1^6 \begin{pmatrix} 18\rho_2^3 (350\tau_1 + 347) + 36\rho_2^2 \left(380\tau_1^2 + 971\tau_1 + 496\right) \\ +8\rho_2 \left(1096\tau_1^3 + 5906\tau_1^2 + 7315\tau_1 + 2484\right) \\ +729\rho_2^4 + 32 \left(456\tau_1^3 + 1165\tau_1^2 + 952\tau_1 + 243\right) \end{pmatrix}$$

$$+\rho_1^5 \begin{pmatrix} 9\rho_2^4 (308\tau_1 + 335) + 18\rho_2^3 \left(672\tau_1^2 + 1903\tau_1 + 881\right) + 32 \left(1140\tau_1^3 + 2330\tau_1^2 + 1428\tau_1 + 243\right) \\ +12\rho_2^2 \left(976\tau_1^3 + 6854\tau_1^2 + 8884\tau_1 + 2667\right) + 8\rho_2 \left(5840\tau_1^3 + 18344\tau_1^2 + 15267\tau_1 + 3357\right) \end{pmatrix}$$

$$+ \rho_1^4 \begin{pmatrix} 3\rho_2^4 \left(1168\tau_1^2 + 3601\tau_1 + 1428\right) + 2\rho_2^3 \left(3424\tau_1^3 + 27602\tau_1^2 + 34717\tau_1 + 8658\right) \\ +12\rho_2^2 \left(4552\tau_1^3 + 15666\tau_1^2 + 12149\tau_1 + 2130\right) \\ +8\rho_2 \left(12760\tau_1^3 + 27036\tau_1^2 + 14841\tau_1 + 1890\right) + 32 \left(1520\tau_1^3 + 2330\tau_1^2 + 952\tau_1 + 81\right) \end{pmatrix}$$

$$+ 4\rho_1^3 \begin{pmatrix} \rho_2^4 \left(368\tau_1^3 + 3187\tau_1^2 + 3518\tau_1 + 504\right) + 2\rho_2^3 \left(3432\tau_1^3 + 11857\tau_1^2 + 7777\tau_1 + 864\right) \\ +6\rho_2^2 \left(4224\tau_1^3 + 8637\tau_1^2 + 3914\tau_1 + 324\right) \\ +8\rho_2 \left(3640\tau_1^3 + 5141\tau_1^2 + 1669\tau_1 + 90\right) + 8\tau_1 \left(1140\tau_1^2 + 1165\tau_1 + 238\right) \end{pmatrix}$$

$$+ 4\rho_1^2\tau_1 \begin{pmatrix} \rho_2^4 \left(1240\tau_1^2 + 3823\tau_1 + 1512\right) + \rho_2^3 \left(10320\tau_1^2 + 18121\tau_1 + 5184\right) \\ +6\rho_2^2 \left(3896\tau_1^2 + 4633\tau_1 + 972\right) + 4\rho_2 \left(4540\tau_1^2 + 3841\tau_1 + 540\right) + 8\tau_1 (456\tau_1 + 233) \end{pmatrix}$$

$$+ 32 (\rho_2 + 1) \rho_1\tau_1^2 \left(\rho_2^3 (172\tau_1 + 189) + \rho_2^2 (690\tau_1 + 459) + 54\rho_2 (12\tau_1 + 5) + 76\tau_1\right)$$

$$+ 288\rho_2 (\rho_2 + 1)^2 (7\rho_2 + 10) \tau_1^3.$$

$$\Psi_4 = 6\rho_2 \left(\rho_1^2 (3\rho_2 + 4\tau_1 + 4) + 4\rho_1 (\rho_2 (\tau_1 + 1) + 2\tau_1 + 1) + 4 (\rho_2 + 1) \tau_1\right)^2$$

$$\times \sqrt{\begin{matrix} \rho_1^4 \left(\rho_2 (48\tau_1 + 45) + 18\rho_2^2 + 32\tau_1^2 + 68\tau_1 + 36\right) \\ +\rho_1^3 \left(\rho_2^2 (48\tau_1 + 45) + 8\rho_2 \left(8\tau_1^2 + 23\tau_1 + 9\right) + 4 \left(24\tau_1^2 + 34\tau_1 + 9\right)\right) \\ +\rho_1^2 \left(\rho_2^2 \left(32\tau_1^2 + 113\tau_1 + 36\right) + 4\rho_2 \left(41\tau_1^2 + 52\tau_1 + 9\right) + 4\tau_1 (24\tau_1 + 17)\right) \\ +4\rho_1\tau_1 \left(\rho_2^2 (17\tau_1 + 18) + 2\rho_2 (17\tau_1 + 9) + 8\tau_1\right) + 36\rho_2 (\rho_2 + 1) \tau_1^2 \end{matrix}}$$

$$\times \begin{pmatrix} \rho_1^4 \left(\rho_2 (48\tau_1 + 45) + 18\rho_2^2 + 4 \left(8\tau_1^2 + 17\tau_1 + 9\right)\right) \\ +\rho_1^3 \left(\rho_2^2 (48\tau_1 + 45) + 8\rho_2 \left(8\tau_1^2 + 23\tau_1 + 9\right) + 4 \left(24\tau_1^2 + 34\tau_1 + 9\right)\right) \\ +\rho_1^2 \left(\rho_2^2 \left(32\tau_1^2 + 113\tau_1 + 36\right) + 4\rho_2 \left(41\tau_1^2 + 52\tau_1 + 9\right) + 4\tau_1 (24\tau_1 + 17)\right) \\ +4\rho_1\tau_1 \left(\rho_2^2 (17\tau_1 + 18) + 2\rho_2 (17\tau_1 + 9) + 8\tau_1\right) + 36\rho_2 (\rho_2 + 1) \tau_1^2 \end{pmatrix}.$$

$$\Psi_5 = -\frac{\rho_1^2(\rho_1+\rho_2+1)^2\sigma_u\left(\rho_1^2(76\rho_2+76\tau_1+81)+76\rho_1\left(\rho_2(2\tau_1+1)+\rho_2^2+\tau_1\right)+76\rho_2(\rho_2+1)\tau_1\right)}{12\left((\rho_1+\rho_2+1)\left(\begin{array}{c}\rho_1^3\left(\rho_2(16\tau_1+17)+8\rho_2^2+8\tau_1^2+17\tau_1+9\right)\\ +\rho_1^2\left(16\rho_2^2(2\tau_1+1)+\rho_2\left(24\tau_1^2+49\tau_1+8\right)+8\rho_2^3+\tau_1(16\tau_1+17)\right)\\ +8(\rho_2+1)\rho_1\tau_1\left(\rho_2(3\tau_1+2)+2\rho_2^2+\tau_1\right)+8\rho_2(\rho_2+1)^2\tau_1^2\end{array}\right)\right)^{3/2}}.$$

S2 Equilibrium Characterization in Section 4.4

Recall that we assume that (i) Hs do not share their information (Assumption 1); (ii) each L shares either all or none of her information (Assumption 2); and (iii) one L's shared information is observable to all Hs and the other peer Ls. Note that unlike Section 4.2, Hs cannot commit to not using the shared information. In this extended economy, we are interested in when it can be sustained as an equilibrium that at least one L shares her private information.

Suppose that that among the number N of L investors, N_1 of them choose to share their information: $\tau_{L_1} = ... = \tau_{L_{N_1}} = \infty$, and the rest do not share any information: $\tau_{L_{N_1+1}} = ... = \tau_{L_N} = 0$. We then conjecture the following linear symmetric trading strategies: Investor H_k demands $\tilde{x}_{H,k} = \alpha_v \tilde{v} + \alpha_L(\tilde{y}_1 + ... + \tilde{y}_{N_1})$, where $k \in \{1, ..., M\}$; investor L_i that shares her information demands $\tilde{x}_{L,i} = \beta(\tilde{y}_1 + ... + \tilde{y}_{N_1})$, where $i \in \{1, ..., N_1\}$; and investor L_j that does not share her information demands $\tilde{x}_{L,j} = \gamma_0 \tilde{y}_j + \gamma(\tilde{y}_1 + ... + \tilde{y}_{N_1})$, where $j \in \{N_1+1, ..., N\}$. Note that all Ls who share information have the same information set $\{\tilde{y}_1, ..., \tilde{y}_{N_1}\}$, and investor L_j who does not share information has her unique information $\tilde{y}_j$ in addition to the common information set $\{\tilde{y}_1, ..., \tilde{y}_{N_1}\}$. We also consider a linear pricing rule $\tilde{p} = \lambda\tilde{\omega}$, where the total order flow $\tilde{\omega} = \sum_{k=1}^{M} \tilde{x}_{H,k} + \sum_{i=1}^{N_1} \tilde{x}_{L,i} + \sum_{j=N_1+1}^{N} \tilde{x}_{L,j} + \tilde{u}$.

We next derive each investor's optimal trading strategy. For investor H_k, where $k \in \{1, ..., M\}$, given his information set $\{\tilde{v}, \tilde{y}_1, ..., \tilde{y}_{N_1}\}$, the conditional expected profits are as follows:

$$E\left[\tilde{x}_{H,k}\left(\tilde{v}-\tilde{p}\right)|\tilde{v},\tilde{y}_1,...,\tilde{y}_{N_1}\right]=\tilde{x}_{H,k}$$
$$\times\left(\tilde{v}-\lambda\left(\begin{array}{c}\tilde{x}_{H,k}+(M-1)\,\alpha_v\tilde{v}+(M-1)\,\alpha_L\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)+N_1\beta\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)\\+\left(N-N_1\right)\left(\gamma_0\tilde{v}+\gamma_0\frac{\frac{\rho}{1+\rho}\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)}{\xi+\frac{\rho}{1+\rho}N_1}+\gamma\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)\right)\end{array}\right)\right).$$

Maximizing the profits yields H_k's optimal trading strategy: $\tilde{x}_{H,k}=\alpha_{v,k}\tilde{v}+\alpha_{L,k}\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)$, where

$$\alpha_{v,k} = \frac{1-\left((M-1)\,\alpha_v+\left(N-N_1\right)\gamma_0\right)\lambda}{2\lambda}, \tag{S1}$$
$$\alpha_{L,k} = -\frac{1}{2}\left((M-1)\,\alpha_L+N\gamma+\frac{\left(N-N_1\right)\gamma_0\frac{\rho}{1+\rho}}{\xi+\frac{\rho}{1+\rho}N_1}+N_1\left(\beta-\gamma\right)\right). \tag{S2}$$

For L_i that shares her private information, where $i\in\{1,...,N_1\}$, under the information set $\{\tilde{y}_1,...,\tilde{y}_{N_1}\}$, her conditional expected profits are as follows:

$$E\left[\tilde{x}_{L,i}\left(\tilde{v}-\tilde{p}\right)|\tilde{y}_1,...,\tilde{y}_{N_1}\right]=\tilde{x}_{L,i}$$
$$\times\left(\frac{\frac{\rho\xi}{\rho+\xi}\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)}{1+\frac{\rho\xi}{\rho+\xi}N_1}-\lambda\left(\begin{array}{c}M\alpha_v\frac{\frac{\rho\xi}{\rho+\xi}\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)}{1+\frac{\rho\xi}{\rho+\xi}N_1}+M\alpha_L\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)\\\tilde{x}_i+\left(N_1-1\right)\beta\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)\\+\left(N-N_1\right)\left(\gamma_0\frac{\rho\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)}{\frac{\xi}{1+\xi}+N_1\rho}+\gamma\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)\right)\end{array}\right)\right).$$

Maximizing the profits yields L_i's optimal trading strategy: $\tilde{x}_{L,i}=\beta_i\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)$, with

$$\beta_i=\frac{1}{2\lambda}\left(\begin{array}{c}\left(1-M\alpha_v\lambda\right)\frac{\frac{\rho\xi}{\rho+\xi}}{1+\frac{\rho\xi}{\rho+\xi}N_1}-M\alpha_L\lambda\\-\left(N_1-1\right)\beta\lambda-\left(N-N_1\right)\lambda\left(\gamma+\gamma_0\frac{\rho}{\frac{\xi}{1+\xi}+N_1\rho}\right)\end{array}\right). \tag{S3}$$

For L_j that does not shares her info, where $j\in\{N_1+1,...,N\}$, under her information set $\{\tilde{y}_j, \tilde{y}_1, ..., \tilde{y}_{N_1}\}$, the conditional expected profits are as follows:

$$E\left[\tilde{x}_{L,j}\left(\tilde{v}-\tilde{p}\right)|\tilde{y}_j, \tilde{y}_1, ..., \tilde{y}_{N_1}\right] = \tilde{x}_{L,j} \times \left(\begin{array}{c} \frac{\frac{\rho\xi}{\rho+\xi}\left(\tilde{y}_j+\tilde{y}_1+...+\tilde{y}_{N_1}\right)}{1+\frac{\rho\xi}{\rho+\xi}(N_1+1)} \\ -\lambda \left(\begin{array}{c} M\alpha_v \frac{\frac{\rho\xi}{\rho+\xi}\left(\tilde{y}_j+\tilde{y}_1+...+\tilde{y}_{N_1}\right)}{1+\frac{\rho\xi}{\rho+\xi}(N_1+1)} + M\alpha_L\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right) + N_1\beta\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right) \\ +\tilde{x}_{L,j} + (N-N_1-1)\left(\gamma_0 \frac{\rho\left(\tilde{y}_j+\tilde{y}_1+...+\tilde{y}_{N_1}\right)}{\frac{\xi}{1+\xi}+(N_1+1)\rho} + \gamma\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)\right) \end{array} \right) \end{array} \right).$$

Maximizing the profits yields L_j's optimal trading strategy: $\tilde{x}_{L,j} = \gamma_{0,i}\tilde{y}_j + \gamma_i\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)$, where

$$\gamma_{0,i} = \frac{1}{2\lambda}\left(\left(1-M\alpha_v\lambda\right)\frac{\frac{\rho\xi}{\rho+\xi}}{1+\frac{\rho\xi}{\rho+\xi}\left(N_1+1\right)} - \left(N-N_1-1\right)\gamma_0\lambda\frac{\rho}{\frac{\xi}{1+\xi}+\left(N_1+1\right)\rho}\right), \quad \text{(S4)}$$

$$\gamma_i = \frac{1}{2}\left(\begin{array}{c} \frac{1}{\lambda}\frac{(1-M\alpha_v\lambda)\frac{\rho\xi}{\rho+\xi}}{1+\frac{\rho\xi}{\rho+\xi}(N_1+1)} + N_1\left(\gamma - \beta + \frac{\rho\gamma_0}{\frac{\xi}{1+\xi}+(N_1+1)\rho}\right) \\ -\left(N-1\right)\frac{(N_1+1)\rho\gamma+\rho\gamma_0+\frac{\gamma\xi}{1+\xi}}{\frac{\xi}{1+\xi}+(N_1+1)\rho} - M\alpha_L \end{array} \right). \quad \text{(S5)}$$

Imposing symmetric trading strategies on equations (S1)–(S5), i.e., $\alpha_{v,k} = \alpha_v, \alpha_{L,k} = \alpha_L, \beta_i = \beta, \gamma_{0,i} = \gamma_0, \gamma_i = \gamma$, we can obtain the equilibrium trading strategies $\{\alpha_v, \alpha_L, \beta, \gamma_0, \gamma\}$ as functions of $(N_1, \lambda; M, N)$.

The pricing rule is $\tilde{p} = \lambda\tilde{\omega}$, where

$$\lambda = \frac{M\alpha_v + MN_1\alpha_L + N_1^2\beta + \gamma\left(N-N_1\right)N_1 + \gamma_0\left(N-N_1\right)}{\left(\begin{array}{c} \left(M\alpha_v + MN_1\alpha_L + N_1^2\beta + \gamma\left(N-N_1\right)N_1 + \gamma_0\left(N-N_1\right)\right)^2 \\ +\left(MN_1\alpha_L + N_1^2\beta + \gamma\left(N-N_1\right)N_1 + \gamma_0\left(N-N_1\right)\right)^2\frac{1}{\xi} \\ +\left(M\alpha_L + N_1\beta + \left(N-N_1\right)\gamma\right)^2\frac{N_1}{\rho} + \gamma_0^2\frac{N-N_1}{\rho} + \sigma_u^2 \end{array} \right)}.$$

Together with the equilibrium trading strategies, we can solve for the equilibrium pricing rule as a function of $(N_1; M, N)$.

Next, given $(N_1; M, N)$, we can compute the expected profits of Ls that share their private information, denoted by $\pi_{L,s}\left(N_1; M, N\right)$ and those of Ls that do not share their private information, $\pi_{L,ns}(N_1; M, N)$:

$$\begin{aligned}\pi_{L,s}(N_1; M, N) &= E\left[E\left[\tilde{x}_{L,i}(\tilde{v}-\tilde{p})\,|\tilde{y}_1, ..., \tilde{y}_{N_1}\right]\right], \\ \pi_{L,ns}(N_1; M, N) &= E\left[E\left[\tilde{x}_{L,j}(\tilde{v}-\tilde{p})\,|\tilde{y}_j, \tilde{y}_1, ..., \tilde{y}_{N_1}\right]\right].\end{aligned}$$

Therefore, the equilibrium number of Ls that share their private information is solved as follows.

- If $\pi_{L,s}(N; M, N) \geq \pi_{L,ns}(N-1; M, N)$, it can be sustained in equilibrium that all Ls share their information, i.e., $N_1^* = N$;
- If $\pi_{L,s}(N-1; M, N) \geq \pi_{L,ns}(N-2; M, N)$ and $\pi_{L,ns}(N-1; M, N) \geq \pi_{L,s}(N; M, N)$, then $N_1^* = N-1$ can be sustained in equilibrium; ...
- If $\pi_{L,s}(n; M, N) \geq \pi_{L,ns}(n-1; M, N)$ and $\pi_{L,ns}(n; M, N) \geq \pi_{L,s}(n+1; M, N)$, then $N_1^* = n$ can be sustained in equilibrium; ...
- If $\pi_{L,ns}(0; M, N) \geq \pi_{L,s}(1; M, N)$, then it can be sustained in equilibrium that none of L shares her information, i.e., $N_1^* = 0$.

S3 Equilibrium Characterization in Section 4.5

Recall that we assume that (i) Hs do not share their information (Assumption 1) and (ii) each L shares either all or none of her information (Assumption 2).In this extended economy, we are interested in if L still shares her private information once the shared information can be observable to market makers.

When L does not share her information, the economy becomes the benchmark studied in Corollary 2. We thus only focus on the economy in which L shares information $\tilde{y}$. Investor H receives signal $\tilde{q}_H = \tilde{y} + \tilde{\zeta}_H$ and we conjecture his linear trading strategy as $\tilde{x}_H = \alpha_v \tilde{v} + \alpha_L \tilde{q}_H$. We conjecture L's trading strategy as $\tilde{x}_L = \beta_y \tilde{y}$. In addition to total order flow $\tilde{\omega}$, market makers observe $\tilde{q}_M = \tilde{y} + \tilde{\zeta}_M$ and they adopt a linear pricing rule $\tilde{p} = \lambda_\omega \tilde{\omega} + \lambda_L \tilde{q}_M$. Note that L could observe $\tilde{q}_H$ or $\tilde{q}_M$ because the noise terms ζ_H and ζ_M are receiver noises.

For investor H, his conditional trading profits on $t = 1$ are:

$$E\left[\tilde{x}_H(\tilde{v}-\tilde{p})|\tilde{v},\tilde{q}_H\right] = \tilde{x}_H\left(\tilde{v}-\lambda_\omega\left(\tilde{x}_H+\beta_y\frac{\rho\tilde{v}+\chi_H\tilde{q}_H}{\rho+\chi_H}\right)-\lambda_L\frac{\rho\tilde{v}+\chi_H\tilde{q}_H}{\rho+\chi_H}\right),$$

Maximizing the profits yields investor H's optimal trading strategy: $\tilde{x}_H = \alpha_v\tilde{v}+\alpha_L\tilde{q}_H$ with

$$\alpha_v = \frac{\chi_H+\rho\left(-\lambda_L\right)+\rho-\rho\lambda_w\beta_y}{2\lambda_w\left(\chi_H+\rho\right)}, \tag{S6}$$

$$\alpha_L = -\frac{\chi_H\left(\lambda_L+\lambda_w\beta_y\right)}{2\lambda_w\left(\chi_H+\rho\right)}. \tag{S7}$$

Similarly, investor L's conditional trading profits are

$$E\left[\tilde{x}_L(\tilde{v}-\tilde{p})|\tilde{y}\right] = \tilde{x}_L\left(\frac{\rho}{1+\rho}\tilde{y}-\lambda_\omega\left(\tilde{x}_L+\alpha_v\frac{\rho}{1+\rho}\tilde{y}+\alpha_L\tilde{y}\right)-\lambda_L\tilde{y}\right), \tag{S8}$$

Maximizing the profits yields investor L's optimal trading strategy: $\tilde{x}_L = \beta_y\tilde{y}$ with

$$\beta_y = \frac{\rho\left(1-\alpha_v\lambda_w\right)-\left(\rho+1\right)\left(\lambda_L+\alpha_L\lambda_w\right)}{2(\rho+1)\lambda_w}. \tag{S9}$$

Then, market makers' pricing rule is $\tilde{p} = E[\tilde{v}|\tilde{\omega},\tilde{q}_M] = \lambda_\omega\tilde{\omega}+\lambda_L\tilde{q}_M$, with λ_ω and λ_L computed as follows:

$$\lambda_\omega = \chi_H\left(\rho\alpha_L+\alpha_v\left(\chi_M+\rho\right)+\rho\beta_y\right)\Gamma^{-1}, \tag{S10}$$

$$\lambda_L = \chi_M\left(-\chi_H\alpha_L\alpha_v+\chi_H\left(\rho\sigma_u^2-\alpha_v\beta_y\right)+\rho\alpha_L^2\right)\Gamma^{-1}, \tag{S11}$$

where

$$\Gamma = \alpha_L^2\left((\rho+1)\chi_H+(\rho+1)\chi_M+\rho\right)+2\chi_H\alpha_L\left(\rho\alpha_v+(\rho+1)\beta_y\right)$$
$$+\chi_H\left(\sigma_u^2\left((\rho+1)\chi_M+\rho\right)+\alpha_v^2\left(\chi_M+\rho\right)+2\rho\alpha_v\beta_y+(\rho+1)\beta_y^2\right).$$

Based on the equations (S6), (S7), (S9), (S10), and (S11), we are able to solve for optimal trading strategies $\{\alpha_v,\alpha_L,\beta_y\}$ and optimal pricing rules $\{\lambda_\omega,\lambda_L\}$. We then replace them into L's profit functions (S8) and compute her unconditional trading profits as $\pi_L \equiv$

$E\left[E\left[\tilde{x}_L(\tilde{v}-\tilde{p})|\tilde{y}\right]\right]$.

Finally, investor L is willing to share her private information if $\pi_L > \pi_L^0$, where π_L^0 is given by Corollary 2.

S4 Extension: Information Acquisition by L

In the baseline model, the two investors are endowed with their respective private information and we find that the less informed investor L would like to fully share her information with the well informed investor H. One natural question arises: will investor L still share her information if the information needs to be acquired at a non-negligible cost?

In this section, we endogenize investor L's information acquisition. Assume that before the two investors' information sharing on $t = 0$, to acquire information of precision ρ investor L needs to incur a cost according to a linear cost function, $c \cdot \rho$, where c is a positive constant.[7] Investor L chooses the precision ρ to maximize her expected trading profits net of the information-acquisition cost. We then study the effect of information sharing on investor L's information-acquisition incentives by comparing this extended model to its benchmark economy in which there is endogenous information production but no information sharing (i.e., investor L can produce information and $\tau_H = \tau_L = 0$).

In the benchmark economy in which there is no information sharing and investor L can decide how much information to produce, based on equation (18), investor L's expected trading profits net of the information-acquisition cost can be expressed as follows:

$$\pi_L^0 - c \cdot \rho = \frac{\rho(1+\rho)\sigma_u}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}} - c \cdot \rho.$$

Maximizing the net profits yields investor L's optimal information-acquisition decision, ρ^0,

[7]The linear information-acquisition cost is assumed for tractability. Linearity in precision can be an analog of the case with discrete sampling with a constant cost per independent sample. Such an assumption is commonly made in the literature (e.g., Verrecchia, 1982; Kim and Verrecchia, 1991; Myatt and Wallace, 2002).

which is uniquely determined by the following equation:

$$\frac{c}{\sigma_u} = \frac{32 + 84\rho + 69\rho^2 + 19\rho^3}{2(4 + 3\rho)^2(4 + 5\rho + 2\rho^2)^{3/2}}. \tag{S12}$$

When information sharing is permitted, according to Section 3 we know that in equilibrium investor L shares her information "as is" whereas investor H does not share any information. Based on equation (15), investor L's expected profits net of the information-acquisition cost can be calculated as follows:

$$\pi_L^* - c \cdot \rho = \frac{2\rho\sigma_u}{3\sqrt{(1+\rho)(9+8\rho)}} - c \cdot \rho.$$

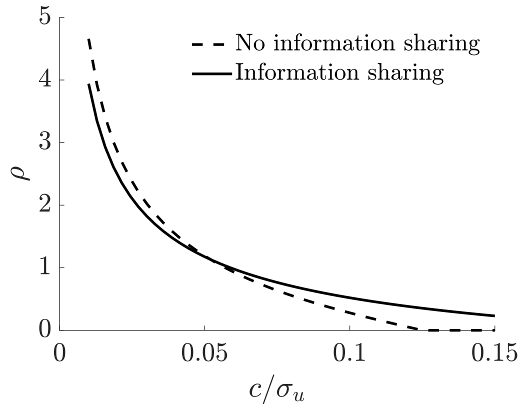
Again, maximizing the net profits yields the optimal information-acquisition decision, ρ^*, which is uniquely determined by the following equation:

$$\frac{c}{\sigma_u} = \frac{18 + 17\rho}{3(9 + 17\rho + 8\rho^2)^{3/2}}. \tag{S13}$$

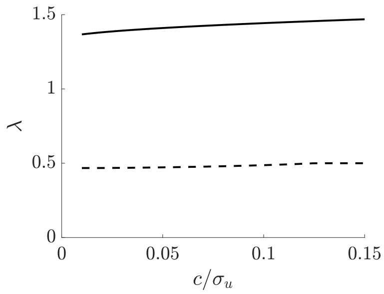
Then, a comparison of ρ^* with ρ^0 yields the following proposition.

Proposition S1. *Assume that information acquisition is costly for investor L. There exists a constant $\hat{c}$, where $\hat{c} \approx 0.0520$, such that relative to the economy without information sharing, when information sharing is permitted, if $c/\sigma_u > (<)\, \hat{c}$, investor L acquires more (less) information; that is, $\rho^* > (<)\, \rho^0$.*

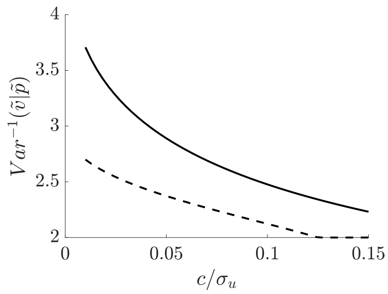
Relative to the benchmark economy without information sharing, when information sharing is permitted, investor L's information acquisition is determined by the following trade off. On the one hand, as discussed above, by sharing information with investor H, investor L can better hide her informed order flows, thereby trading more aggressively even though her information remains as noisy as before. This trading-against-error effect depresses L's incentives of acquiring information. On the other hand, with higher trading profits after sharing her information, investor L can afford to acquire more information about the fundamental and make more informed trading decisions. This in turn encourages investor L to acquire more information.



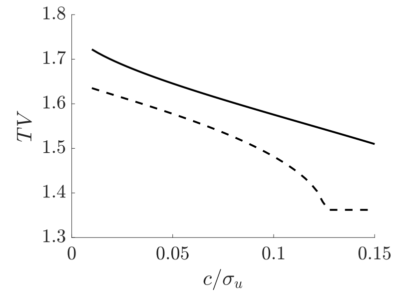
(a) Information precision



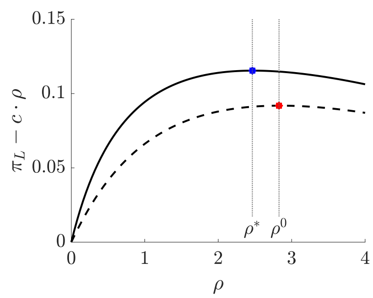
(b1) Market liquidity



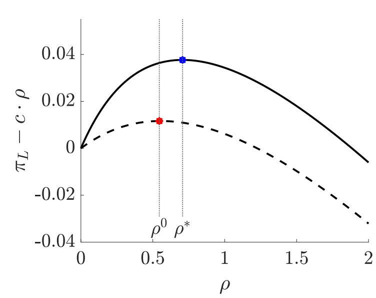
(b2) Market efficiency



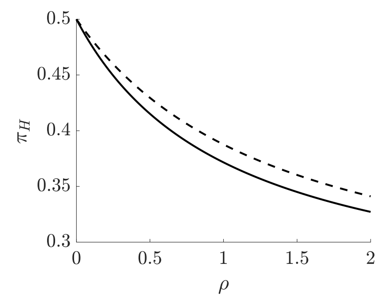
(b3) Trading volume



(c1) L's net profits ($c = 0.02$)



(c2) L's net profits ($c = 0.08$)



(c3) H's total profits

Figure S1: Costly information acquisition for investor L

Proposition S1 formalizes the above trade off faced by investor L when making information-acquisition decisions. The relative strength of the two forces depends on the primitives of the model, namely, the cost of acquiring information and the noise trading volatility. Ceteris paribus, investor L's net gains of information acquisition decrease with the information-acquisition cost c and increases with the noise trading volatility σ_u, so c and σ_u have opposite effects on investor L's incentives to acquire information. Nevertheless, the optimal information acquisition only depends on c/σ_u.

If c/σ_u is low, investor L has acquired a great deal of information. She is less concerned about improving the forecasting ability of the fundamental, but cares more about hiding her informed order flows. When information sharing is permitted, investor L would like to induce investor H to trade against her information and help her offset the order flow. Therefore, investor L is less incentivized to acquire information. However, if c/σ_u is high, investor L only acquires a limited amount of information. When information sharing is permitted, with the higher profits, investor L would like to produce more information to further enhance her trading decision-making. Panel (a) of Figure S1 graphically illustrates the information-acquisition result.

Then, how will investor L's information-acquisition behavior affect the market quality? We plot market liquidity, market efficiency, and trading volume across the extended economy and its benchmark economy in panels (b1)–(b3) of Figure S1, respectively. We find that regardless of investor L's information acquisition, the economy with information sharing is always featured with lower market liquidity, higher market efficiency, and higher trading volume. Again, with investor L sharing her information, investor H tends to trade against it, which reduces the noise in the total order flow and induces market makers to raise the price impact. Therefore, market liquidity decreases. Further, despite that investor L may acquire less information when information sharing is permitted, the intensive trading by the two investors render the total order flow more correlated with the fundamental, which always improves market efficiency. Finally, L's aggressive trading after information sharing raises total trading volume.

Next, to take a further look at investor L's optimal information-acquisition decisions, we plot how L's information production ρ affects her expected trading profits net of the information-acquisition cost $\pi_L - c \cdot \rho$ in panels (c1) and (c2) of Figure S1 under the cost $c = 0.02$ and $c = 0.08$, respectively. The other parameter is $\sigma_u = 1$. The solid (dashed) line denotes the economy with (without) information sharing. Consistent with Proposition S1, with a fixed σ_u, if c is low (high), investor L acquires less (more) information when information sharing is permitted, namely, $\rho^* < (>)\, \rho^0$. More importantly, Panels (c1) and (c2) show that allowing investor L's endogenous information acquisition can only reinforce her gains from the information sharing. In other words, when information sharing is permitted, by choosing ρ^* investor L makes higher profits than those in the situation where the information precision is exogenously given. Finally, regardless of L's information-acquisition cost c or her information production ρ, relative to the benchmark economy without information sharing, investor H always makes lower profits, as shown in Panel (c3) of Figure S1.

Proof of Proposition S1

Denote the right-hand-side of equations (S12) and (S13) as $f_{NS}(\rho)$ and $f_S(\rho)$, respectively. It is easy to see that $f'_{NS}(\rho) < 0$ and $f'_S(\rho) < 0$. Therefore, as c/σ_u increases, investor L acquires less information in both cases with and without information sharing; that is, both ρ^* and ρ^0 decrease in c/σ_u. Further, setting $f_{NS}(\rho) = f_S(\rho)$ yields $\rho = \hat{\rho} \approx 1.1307$ and we know that $f_{NS}(\hat{\rho}) = f_S(\hat{\rho}) \equiv \hat{c} \approx 0.0520$. Therefore, the function $f_S(\rho) - f_{NS}(\rho) = 0$ has a unique root at $\rho = \hat{\rho}$.

Since $\lim_{\rho \to 0} f_S(\rho) - f_{NS}(\rho) = 7/72 > 0$, if $c/\sigma_u > \hat{c}$ so that both ρ^0 and ρ^* are small, we know that $\rho^* > \rho^0$. Similarly, since $\lim_{\rho \to \infty} f_S(\rho) - f_{NS}(\rho) = 0$, when $c/\sigma_u < \hat{c}$, both ρ^0 and ρ^* are large and we know that $\rho^* < \rho^0$. QED.

S5 Extension: Heterogeneous Information Precision

While in the extension in Section 4.4 we have considered the economy with multiple coarsely informed investors, their private information is of the same precision. In this section, we further study the case in which these multiple coarsely informed investors own information of different precision. We focus on the simplest case in which there are one perfectly informed investor H and two coarsely informed investors L_1 and L_2. Moreover, the two Assumptions 1 (i.e., H does not share his information) and 2 (i.e., Ls share either all or none of their information) kick in.

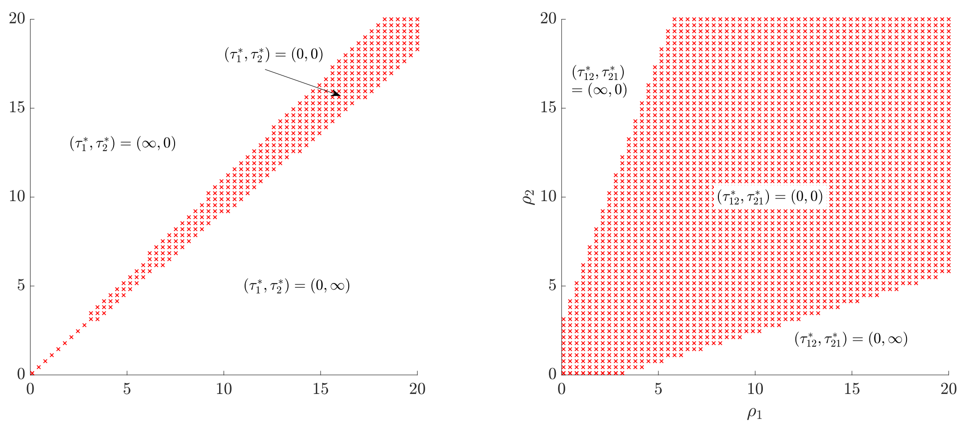
In this extended economy, H learns the fundamental value $\tilde{v}$ and investor L_i, where $i \in \{1, 2\}$, only observes a noisy signal about the fundamental:

$$\tilde{y}_i = \tilde{v} + \tilde{e}_i, \text{ with } \tilde{e}_i \sim N(0, \rho_i^{-1}).$$

We consider the following two cases. First, as in Sections 4.3 and 4.4, we assume that the information, once shared, is observable to all rational investors. Therefore, investor L_i decides whether or not to share the following signal to all rational investors including H and L_j, where $j \neq i$:

$$\tilde{s}_i = \tilde{y}_i + \tilde{\varepsilon}_i, \text{ with } \tilde{\varepsilon}_i \sim N(0, \tau_i^{-1}), \tau_i \in \{0, \infty\}. \tag{S14}$$

Panel (a) of Figure S2 plots the information-sharing behavior of the two coarsely informed investors L_1 and L_2 in this case. We find that no Ls shares information when their information is of similar precision, and only the L with relatively coarser information is willing to share her private information. The intuition is the same as we analyzed in Section 4.2.



(a) Public sharing (b) Private sharing

Figure S2: Information sharing by L_1 and L_2

Second, we allow Ls to make selective sharing; that is, if L_1 shares her private information with H, the third party including the other investor L_2 or market makers is unable to observe it. In this case, investor L_i shares the following two signals to investor H and the other investor L_j respectively:

$$\tilde{s}_{iH} = \tilde{y}_i + \tilde{\varepsilon}_{iH}, \tag{S15}$$

$$\tilde{s}_{ij} = \tilde{y}_i + \tilde{\varepsilon}_{ij}, \tag{S16}$$

where $\tilde{\varepsilon}_{iH} \sim N(0, \tau_{iH}^{-1})$, $\tilde{\varepsilon}_{ij} \sim N(0, \tau_{ij}^{-1})$, and $\tau_{iH}, \tau_{ij} \in \{0, \infty\}$. We find that both Ls are willing to fully share their private information with H and the intuition directly follows the trading-against-error effect, i.e., $\tau_{1H}^* = \tau_{2H}^* = \infty$. We then use Panel (b) of Figure S2 to examine the information sharing between the two coarsely informed investors. Again, there is no information sharing between Ls when their information is of similar precision, and information only flows from the investor with lower information precision to the one with higher information precision. Moreover, compared with Panel (a), the regime of no information flows between the coarsely informed investors is much larger. This is because sharing private information with peer investor L_j greatly dissipates L_i's informational advantage; now with selective information sharing possible, L_i will be able to alleviate this concern by withholding her information from the peer coarsely informed investor.