FDI and firm productivity in host countries: The role of financial constraints*

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Abstract

This paper studies the effect of FDI firms' financial advantages on firm productivity in host countries and examines the related policy implications. If FDI firms face lower financing costs but have higher fixed production costs than local firms, a simple Melitz-type model predicts that because of their financial advantages, FDI firms could have even lower cutoff productivity than local firms, especially in financially vulnerable sectors. The same mechanism will also lower the average productivity of FDI firms especially in financially vulnerable sectors, although FDI firms on average are still more productive than local firms. These predictions are supported by the Chinese firm-level data. Then, we study policy implications in a two-country model that resembles these empirical patterns. The counterfactual policy analysis shows that offering tax benefits to FDI firms could be counterproductive because it attracts FDI firms that are even less productive than local firms. The policy in the host country to improve its financial market efficiency could also hurt the country's welfare because of the interaction between financial market reforms and the distortionary taxes imposed on local firms to finance FDI subsidies.

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1 Introduction

Multinational companies (MNCs) are usually less financially constrained than local firms in emerging markets and recent studies have examined this financial advantage and how it drives foreign direct investment (FDI) flows and alleviates financial constraints in host countries (e.g., Wang and Wang (2015), Manova et al. (2015), Desbordes and Wei (2017), Bilir et al. (2019), and Alquist et al. (2019), among others). These studies complement previous studies on FDI firms' productivity advantages in benefiting host countries (e.g., Javorcik (2004), Yasar and Paul (2007), Keller and Yeaple (2009), and Alfaro et al. (2013), among others).[1] In this paper, we show that the proper recognition of MNCs' financial advantages has profound implications on our understanding of FDI's effects on the firm productivity in host countries and the effectiveness of FDI policies.

Many emerging markets provide tax and other incentives to attract FDI; the policy is motivated by the belief that FDI can benefit host countries' economic growth by directly introducing new technology and/or managerial skills from MNCs.[2] FDI firms are on average more productive than local firms in host countries. This empirical pattern is consistent with the Melitz-type model with heterogeneous firms (e.g., Helpman et al. (2004)) in which MNCs face a higher fixed cost than local firms to operate in a foreign country. Keeping everything else constant in the model, the higher fixed production cost induces a higher cutoff productivity (and average productivity) for FDI firms relative to local firms. Empirical studies document convincing evidence of technology transfers from FDI firms to local

[1]See Grossman and Helpman (1995) for a review on FDI and technology spillovers. In contrast, Aitken and Harrison (1999) document that FDI has a negative spillover effect on the productivity of domestic firms, though it has a positive effect on FDI firms in firm-level data of Venezuela. The two effects are almost canceled out, leaving a very small overall effect. Using macro-level data, Borensztein et al. (1998) and Carkovic and Levine (2005) find little evidence that FDI has a positive effect on host country's economic growth.

[2]In addition, FDI is also less prone to short-term runs by international investors, which became a very desirable feature following the emerging market financial crises in the 1980s and 1990s. For instance, see Krugman (2000), Aguiar and Gopinath (2005), and Alquist et al. (2016), among others.

firms through technology diffusion, labor turnover, and many other channels. In this case, policies to attract FDI can improve host countries' welfare if MNCs do not fully consider the productivity spillover effects of FDI.

However, the financial advantages of MNCs can cause a tradeoff to the cutoff productivity. MNCs are usually less financially constrained than local firms because of their easy access to international financial markets. Wang and Wang (2015) and Alquist et al. (2019) show empirically that the financial advantage of FDI is an important factor in driving foreign mergers and acquisitions in emerging markets. Although high fixed production costs allow only the very productive FDI firms to compete with local firms, the financial advantages of MNCs work in the opposite direction. In this case, the relative cutoff productivity of FDI firms to local firms will depend on which of the above effects dominates.

We first show this tradeoff in a simple Melitz-type model with financial frictions for local firms, which guides our empirical work with two testable predictions. For simplicity, we assume that FDI firms are financially unconstrained.[3] In contrast, local firms face the financial frictions proposed in Manova (2012). Therefore, financing costs are higher for local firms than FDI firms, although local firms face lower fixed production costs in our model. If the financial disadvantage of local firms is larger than its advantage in the fixed production cost, FDI firms can have lower cutoff productivity than local firms, and this condition is more pronounced in the financially more vulnerable sectors (e.g., sectors that are more dependent on external finance). In addition, the model predicts that the same mechanism will also affect the average productivity of FDI and local firms. Although FDI firms may still have higher average productivity than local firms because MNCs have a flatter tail in their productivity

[3] Relaxing this assumption will not qualitatively affect our results so long as FDI firms are assumed to be financially less constrained than local firms. See Bilir et al. (2019) and Desbordes and Wei (2017) for studies on the effects of host country's financial markets on financially constrained FDI firms. If foreign firms also finance funds in the host country's financial markets, it may exacerbate local firms' financial constraints because of the intensified competition for funds. Harrison and McMillan (2003) find such empirical evidence in firm-level data in the Ivory Coast.

distribution than local firms, the difference in average productivity between these firms will be much smaller in the sectors of high financial vulnerability than the sectors of low financial vulnerability due to low cutoff productivity of FDI firms.

The above theoretical predictions are supported in the Chinese firm-level data from the Annual Surveys of Industrial Production by the National Bureau of Statistics of China. Firm productivity is calculated following Ackerberg et al. (2015) and sector-level financial vulnerability is measured following Manova et al. (2015) and Alquist et al. (2019).[4] To test our first prediction on firms' cutoff productivity, we employ quantile regressions and focus on the results of bottom quantiles of productivity (e.g., 20% or less). We find that in the financially vulnerable sectors (top 25 percentile under the financial vulnerability measures), FDI firms have even lower cutoff productivity than local Chinese firms. For instance, at the 15th percentile, the productivity of FDI firms is usually 6 percent lower than that of local firms under various measures of financial vulnerability. However, no such evidence is detected for the sectors of low financial vulnerability. The OLS regressions are employed to test the prediction on firms' average productivity. We find that FDI firms on average are more productive than local firms in both high and low financial vulnerability sectors. However, the difference is much smaller in sectors of high financial vulnerability than sectors of low financial vulnerability. For instance, under most measures of financial vulnerability, the difference in the high financial vulnerability sectors is less than half of that in the low financially vulnerability sectors in our data. These empirical findings are consistent with our model's theoretical predictions and suggest that the financial advantages of MNCs are an important factor affecting firms' productivity in host countries.

Next, we study related policy implications in a two-country model where FDI firms have financial advantages relative to local ones. Many emerging markets provide tax benefits

[4] The dataset and measures of firm productivity and sector-level financial vulnerability are widely used in the literature.

to attract foreign investment based on MNCs' productivity advantages. However, financial advantage is also an important factor in driving FDI flows. For instance, Alquist et al. (2019) document that easing the target's credit constraints is an important reason for foreign acquisitions in emerging markets. We construct a two-country model to study the effects of FDI tax policies and financial market reform on the aggregate productivity and welfare in the host country when FDI firms have financial advantages over local ones. The model is modified from those in Manova (2012) and Bilir et al. (2019), which were built on the works of Melitz (2003) and Helpman et al. (2004). In our model, FDI firms are on average more productive than local firms in the host country, but have lower cutoff productivity than the local firms due to financial frictions in the host country. We show that the host country's welfare is determined by the aggregate productivity of FDI and local firms, the product varieties available to the households and the transfers from the government to households in the host country.

We first calibrate the tax rates to match the FDI tax policies in China, under which FDI firms pay lower corporate and value added taxes than local firms. Then, we remove the tax benefits of FDI firms while the total government revenues (and transfers to the household) are kept constant. In our benchmark model, the average productivity of local firms and FDI firms in the host country displays a humped shape: it first increases when we raise the tax rate of FDI firms as the cutoff productivity of FDI firms rises with the tax rate. This finding suggests that tax benefits offered to FDI firms actually reduce the average firm productivity in the host country because such policy attracts FDI firms who have even lower productivity than local firms, which is exactly the opposite to the purpose of such policies. Of course, the average productivity and welfare decrease when the host country taxes too heavily on FDI firms because high-productivity FDI firms may exit the host country if the tax rate is too high.

If we interpret taxes broadly as barriers to FDI, our findings highlight the tradeoff in determining restrictions/subsidies for FDI firms in countries with underdeveloped financial markets. On the one hand, emerging markets should remove barriers and even provide additional incentives for high-productivity FDI firms to enter. On the other hand, low-productivity firms may take advantage of the universal subsidies offered to FDI firms, which even decreases the aggregate productivity and welfare of host countries. This is particularly problematic when FDI firms have even lower cutoff productivity than local firms because of their financial advantages. Faced with this problem, some emerging markets such as China have adopted policies to ensure that FDI firms introduce advanced technology and management skills to their countries. For instance, China employed performance requirements to control for the quality of FDI firms and some tax exemptions were only provided to FDI firms that met certain performance requirements before China's accession to the WTO in 2001.[5]

In our second policy analysis, we investigate whether emerging markets can increase welfare by simply improving the quality of their financial markets. The financial market improvement is proxied by a decrease in the default rate in our model, which tends to capture a better quality of financial institutions in screening and monitoring borrowers. We find that the financial market improvement may have to be combined with tax reforms to increase the host country's welfare. The improvement of financial market efficiency reduces the disadvantages of local firms relative to FDI firms. As a result, local firms with lower productivity can enter the market when the host country's financial market becomes more efficient. It reduces the average productivity in the host country, but increases its product varieties. The overall welfare effect depends on which effect dominates. We show that distortionary taxes imposed to finance FDI subsidies in the host country can repress the increase in product varieties such that its welfare even decreases when the financial market efficiency improves.

[5] China had to abandon the policy of performance requirements after 2001 to meet WTO regulations.

Our paper contributes to several strands of literature. First, it is related to the literature that studies the role of FDI in alleviating the effects of domestic financial market imperfections because of MNCs' easy access to foreign capital markets. For instance, Manova et al. (2015) find that foreign affiliates and joint adventures in China perform better than local firm in export, especially in financially vulnerable sectors. Lin and Ye (2018) document that FDI firms in China provide more trade credits than local firms during tight domestic credit periods. They also find that a favorable global liquidity shock can amplify FDI's trade credit provision. We contribute to this literature by examining the effect of FDI firms' financial advantages on firm productivity in host countries. Second, our paper expands the empirical studies on MNCs' financial advantages in driving FDI flows such as Wang and Wang (2015) and Alquist et al. (2019). We provide additional empirical evidence for FDI firms' financial advantages and conduct counterfactual policy analysis in a two-country model. Finally, our study is related to the research on optimal FDI subsidy policies such as Chor (2009) and Han et al. (2020). We contribute to the literature by highlighting the possibility that such policies might attract low-productivity FDI firms, although they are designed to obtain the ones of high productivity.

The remainder of the paper is organized as follows. Section 2 presents a partial equilibrium model and reports the empirical support to the model's prediction. Section 3 describes a general equilibrium model and conducts counterfactual policy analysis. Section 4 concludes.

2 Model Predictions and Empirical Evidence

This section presents a partial equilibrium model of small open economy (SOE) to highlight the role of financial constraints in determining the cutoff and average firm productivity in the host country. We derive two testable predictions and provide empirical evidence from the firm-level data of China to support these model predictions.

2.1 Predictions from an SOE model

In this model, the representative household maximizes a Cobb-Douglas aggregate:

$$Y = \frac{\left(C^H\right)^\nu \left(C^F\right)^{1-\nu}}{(\nu)^\nu (1-\nu)^{1-\nu}},$$

where aggregate consumption, Y, serves as a numeráire. C^H is a CES aggregate of differentiated goods produced by local and FDI firms, and C^F is an aggregate of imported goods, which is exogenously given in this partial equilibrium model. ν captures the consumption home bias toward domestically produced goods relative to imported goods. Each sector has two types of Home firms: domestic firms of mass M and FDI firms of mass M^I.[6] These firms produce differentiated goods in the Home country and are indexed by ω and ω^*. Let Ω and Ω^I be the set of domestic and FDI firms, respectively. The CES aggregate, C^H, takes the form of

$$C^H = \left(\int_{\omega\in\Omega} \left[y^D(\omega)\right]^\rho d\omega + \int_{\omega^*\in\Omega^I} \left[y^I(\omega^*)\right]^\rho d\omega^*\right)^{\frac{1}{\rho}},$$

where $y^D(\omega)$ is output from a Home local firm ω and $y^I(\omega^*)$ is output from an FDI firm ω^*. From the above CES aggregate, we obtain the demand for local and FDI firms

$$y^D(\omega) = \left(\frac{p^D(\omega)}{P}\right)^{-\sigma} \left[\left(\frac{P^H}{P}\right)^{\sigma-1} \nu Y + Y^*\right], \quad y^I(\omega^*) = \left(\frac{p^I(\omega^*)}{P}\right)^{-\sigma} \left[\left(\frac{P^H}{P}\right)^{\sigma-1} \nu Y + Y^*\right],$$

and the aggregate price index

$$1 = P = \left(P^H\right)^\nu \left(P^F\right)^{1-\nu}, \quad P^H = \left[\int_{\omega\in\Omega} \left(p^D(\omega)\right)^{1-\sigma} d\omega + \int_{\omega^*\in\Omega^I} \left(p^I(\omega^*)\right)^{1-\sigma} d\omega^*\right]^{\frac{1}{1-\sigma}},$$

[6] The mass of firms is exogenous in this simple model. We relax this restriction later in our two-country model when we conduct counterfactual policy analysis.

where the price index P is given as unity and $\sigma \equiv \frac{1}{1-\rho}$. Y^* is the exogenous foreign demand and P^F is the exogenous price index for imported goods. In the following, we replace the firm index ω and ω^* with its productivity level z and z^*.

Each local firm has to pay a fixed entry cost F^D in labor units before it can draw its productivity, z, from a distribution $G(z)$. Likewise, each FDI firm pays F^I units of labor upon entry before it draws its productivity, z^*, from a distribution $G^*(z^*)$. The fixed entry cost has to be self-financed by firms. After drawing its productivity, each firm can decide whether to produce. If it chooses to produce, it has to pay additional fixed production costs in each period:

$$f = \begin{cases} f^D & \text{for domestic firms,} \\ f^I & \text{for FDI firms.} \end{cases}$$

It is assumed that $f^I > f^D$ holds, reflecting higher operational costs by FDI firms than local firms as discussed in Alquist et al. (2019). If the firm decides not to produce, it exits the market.

We introduce financial constraints to local firms following Manova (2012). A fraction of the fixed production cost, $\zeta f^D W$, has to be financed externally, where $\zeta \in [0, 1]$ is a constant and W is wage. A fraction of the entry cost, $\chi F^D W$, needs to be provided as a collateral for the external finance with $\chi \in [0, 1]$. Parameters ζ and χ indicate the sectoral level financial vulnerability. Firms are more financially vulnerable if they are in a sector that depends more on external finance (larger ζ) or has less tangible assets for collateral (smaller χ). Each local firm pays back its loan with an amount of $x(z)$ by a probability of $\lambda \in (0, 1)$. If a firm defaults (with a probability of $1 - \lambda$), its collateral goes to the lender. The parameter λ usually reflects the quality of financial institutions: a higher λ indicates less frictional financial markets.

Given the demand function, each domestic firm maximizes its expected profit:

$$\pi^D(z) = p^D(z)y^D(z) - \frac{W}{z}y^D(z) - (1-\zeta)f^D W - \lambda x(z) - (1-\lambda)\chi F^D W, \tag{1}$$

where a superscript D denotes domestic firms in Home. The profit maximization of local firms is subject to two conditions. First, the operational profit must be larger than the promised loan payment:

$$p^D(z)y^D(z) - \frac{W}{z}y^D(z) - (1-\zeta)f^D W \geq x(z). \tag{2}$$

Otherwise, the firm will choose not to produce. The second condition states that the bank's expected income is greater than or equal to its costs:

$$\lambda x(z) + (1-\lambda)\chi F^D W \geq \zeta f^D W, \tag{3}$$

where the condition is binding in the equilibrium because the banking sector is assumed to be competitive.

For simplicity, we assume that no financial constraint exists for FDI firms because they can gain access to credits from international financial markets and parent companies. Bilir et al. (2019) assumes that FDI firms also face financial constraints and Desbordes and Wei (2017) documents empirical evidence that FDI firms might require local credit. Our results can be interpreted in a relative term, rather than in an absolute term. We interpret the financial constraint of domestic firms as the wedge between domestic and FDI firms' borrowing constraints.

Substituting out demands and pricing rules in firms' variable profits, we obtain

$$\overline{\pi}^D(z) \equiv p^D(z)y^D(z) - \frac{W}{z}y^D(z) = \frac{1}{\sigma-1}\left(\frac{W}{z}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\left[\left(P^H\right)^{\sigma-1}\nu Y + Y^*\right] \tag{4}$$

for local firms and

$$\overline{\pi}^I(z^*) \equiv p^I(z^*)y^I(z^*) - \frac{W}{z^*}y^I(z^*) = \frac{1}{\sigma-1}\left(\frac{W}{z^*}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\left[\left(P^H\right)^{\sigma-1}\nu Y + Y^*\right] \quad (5)$$

for FDI firms. FDI firms' periodic profits are given by $\pi^I(z^*) = \overline{\pi}^I(z^*) - f^I W$.

Condition for cutoff productivity: Firms choose to produce only when their productivity is above a threshold. Let Z^D be the productivity cutoff of domestic firms. From equations (2) and (3), we can derive the entry condition for a marginal domestic firm:

$$\begin{aligned} \tfrac{1}{\sigma-1}\left(\tfrac{W}{Z^D}\right)^{1-\sigma}\left(\tfrac{\sigma}{\sigma-1}\right)^{-\sigma}\left[\left(P^H\right)^{\sigma-1}\nu Y + Y^*\right] &= (1-\zeta)f^D W + x\left(Z^D\right) \\ &= f^D W + \left(\tfrac{1}{\lambda}-1\right)W\left(\zeta f^D - \chi F^D\right) \\ &> f^D W. \end{aligned} \quad (6)$$

The last inequality is from the assumption that the loan amount is larger than the collateral: $\zeta f^D W > \chi F^D W$.

Let Z^I be the productivity cutoff for FDI firms. The zero-profit condition for a cutoff FDI firm implies that the variable profit of the firm equals the fixed production cost: $\overline{\pi}^I\left(Z^I\right) = f^I W$. Substituting equation (5) to this entry condition, we have:

$$\frac{1}{\sigma-1}\left(\frac{W}{Z^I}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\left[\left(P^H\right)^{\sigma-1}\nu Y + Y^*\right] = f^I W. \quad (7)$$

Dividing equation (6) by (7), we have:

$$\left(\frac{Z^D}{Z^I}\right)^{\sigma-1} = \frac{f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)}{f^I}, \quad (8)$$

where $\sigma > 1$.

It is straightforward from equation (8) that the productivity cutoff of FDI firms is higher than that of domestic firms ($\frac{Z^D}{Z^I} < 1$), if domestic firms do not face financial constraints ($\lambda = 1$). In contrast, if FDI and domestic firms have the same fixed production cost ($f^I = f^D$), financial frictions in the host country will raise the productivity cutoff of local firms above that of FDI firms. The above results indicate that our model has two offsetting effects on productivity cutoffs of domestic and FDI firms. The higher fixed production cost faced by FDI firms ($f^I > f^D$) increases the cutoff for FDI firms, while the financial friction imposed on the local firms ($0 < \lambda < 1$) raises the cutoff for local firms. If the latter effect is stronger than the former, FDI firms can have lower cutoff productivity than local firms ($\frac{Z^D}{Z^I} > 1$).

Proposition 1. *If* $\frac{f^D}{f^I} + \frac{1-\lambda}{\lambda f^I}\left(\zeta f^D - \chi F^D\right) > 1$ *holds, the productivity cutoff of FDI firms is lower than that of domestic firms. The condition is more likely to hold in the sectors that are more financially vulnerable (larger* ζ *or smaller* χ*).*

Proposition 1 is directly from equation (8). Note that $\frac{1-\lambda}{\lambda f^I}\left(\zeta f^D - \chi F^D\right)$ increases with ζ and decreases with χ. Therefore, the sectors that are more financially vulnerable are more likely to meet the condition in Proposition 1. This is the first prediction that we will empirically test in Section 2.2.

Condition for average productivity: Assume that the productivity of domestic firms and FDI firms follow Pareto distributions, given by:

$$G(z) = 1 - (z_{min})^{\eta} z^{-\eta}, \qquad G^*(z) = 1 - (z^*_{min})^{\eta^*} z^{-\eta^*},$$

where $\eta > \sigma$ and $\eta^* > \sigma$. In addition, we assume that $\eta^* < \eta$, which implies a fatter tail for the productivity distribution of FDI firms than domestic firms. This assumption is used to capture the empirical patterns that most FDI is from advanced economies to emerging markets and FDI firms have higher average productivity than local firms.

Given the above productivity distributions, we can derive average productivity for domestic firms ($\widetilde{Z}^D$) and FDI firms ($\widetilde{Z}^I$):

$$\begin{aligned}
\widetilde{Z}^D &\equiv \left[\int_{Z^D}^{\infty} z^{\sigma-1} \frac{dG(z)}{1-G(Z^D)}\right]^{\frac{1}{\sigma-1}} = \left[\frac{J(Z^D)}{1-G(Z^D)}\right]^{\frac{1}{\sigma-1}} = \left[\frac{\eta}{\eta-\sigma+1}\right]^{\frac{1}{\sigma-1}} Z^D, \\
\widetilde{Z}^I &\equiv \left[\int_{Z^I}^{\infty} z^{\sigma-1} \frac{dG^*(z)}{1-G^*(Z^I)}\right]^{\frac{1}{\sigma-1}} = \left[\frac{J^*(Z^I)}{1-G^*(Z^I)}\right]^{\frac{1}{\sigma-1}} = \left[\frac{\eta^*}{\eta^*-\sigma+1}\right]^{\frac{1}{\sigma-1}} Z^I.
\end{aligned}$$

From the above equations, we have:

$$\frac{\widetilde{Z}^D}{\widetilde{Z}^I} = \left[\frac{\eta(\eta^* - \sigma + 1)}{\eta^*(\eta - \sigma + 1)}\right]^{\frac{1}{\sigma-1}} \frac{Z^D}{Z^I}, \tag{9}$$

which shows that the ratio of average productivity of domestic and FDI firms equals the corresponding ratio of productivity cutoffs multiplied ($\frac{Z^D}{Z^I}$) by a constant, $\left[\frac{\eta(\eta^*-\sigma+1)}{\eta^*(\eta-\sigma+1)}\right]^{\frac{1}{\sigma-1}}$. Given the assumption that $\sigma < \eta^* < \eta$, we have $\left[\frac{\eta(\eta^*-\sigma+1)}{\eta^*(\eta-\sigma+1)}\right]^{\frac{1}{\sigma-1}} < 1$ and $\frac{\widetilde{Z}^D}{\widetilde{Z}^I} < \frac{Z^D}{Z^I}$. As a result, FDI firms can have higher ***average*** productivity than domestic firms ($\frac{\widetilde{Z}^D}{\widetilde{Z}^I} < 1$), even though they have lower ***cutoff*** productivity ($\frac{Z^D}{Z^I} > 1$). Because the average productivity is proportional to the cutoff productivity, similar to Proposition 1, the average productivity advantages of FDI firms relative to domestic firms are smaller in financially more vulnerable sectors.

Proposition 2. *The average productivity of FDI firms is higher than that of domestic firms if* $\frac{\widetilde{Z}^D}{\widetilde{Z}^I} = \left[\frac{\eta(\eta^*-\sigma+1)}{\eta^*(\eta-\sigma+1)}\right]^{\frac{1}{\sigma-1}} \frac{Z^D}{Z^I} < 1$. *This condition can hold even when FDI firms have lower cutoff productivity than domestic firms* ($\frac{Z^D}{Z^I} > 1$) *as in Proposition 1, because* $0 < \left[\frac{\eta(\eta^*-\sigma+1)}{\eta^*(\eta-\sigma+1)}\right]^{\frac{1}{\sigma-1}} < 1$ *under the assumption that* $1 < \sigma < \eta^* < \eta$. *The average productivity advantages of FDI firms relative to domestic firms are smaller in more financially vulnerable sectors as* $\frac{Z^D}{Z^I}$ *increases with financial vulnerability.*

2.2 Empirical evidence from Chinese firm data

In this subsection, we provide empirical support for the predictions in Propositions 1 and 2. Our main dataset contains firm-level data from the Annual Surveys of Industrial Production (ASIP) by the National Bureau of Statistics of China, which is widely used in the literature.[7] We will briefly describe the data and leave the details in Appendix A.1.

[7] Examples of studies using the dataset include Bai et al. (2017), Wang and Wang (2015), Kee and Tang (2015), Manova et al. (2015), Ma et al. (2014), and Lu (2010), among others.

2.2.1 Data

The ASIP dataset covers all state-owned and private manufacturing firms with sales greater than 5 million RMB (approximately 600,000 dollars at the exchange rate of 2000) between 2000 and 2007. On average, there are 120,000 firm-level observations each year. The dataset contains detailed information about each firm's balance sheet and income statement, from which we can calculate its FDI share and productivity.

The balance sheet data includes disaggregate-level information on the ownership of capital (e.g., government collective, corporate, special districts, foreign, etc.). Thus, we can calculate the FDI share of each firm, which is measured by the share of capital from Hong Kong, Macau, Taiwan, and foreign countries. Firm productivity is calculated following Ackerberg et al. (2015) and re-scaled around industry productivity mean and divided by industry productivity standard deviation.[8] The dataset also includes basic information on firms, such as registration type, start year, location, operating status, and total employment that can be used to control for firm and location specific characteristics. For instance, the location information of the firm enables us to find out whether it is in a special economic development zone.

In our empirical exercises, we only include firms that enter the market after 2002 because China had strict restrictions on FDI firms before it joined the WTO in 2001. China initially imposed performance requirements on all foreign firms entering the country. Foreign firms were required to meet certain specified goals to qualify for investing in China, such as promoting the country's technologies and exports. Before China joined the WTO, it only opened to foreign investors a limited number of industries that were considered strategically important for China such that the country could benefit strongly from foreign technologies (e.g., automobiles). Therefore, it was difficult for foreign firms with only financial advantages

[8]Examples of using this method to calculate the firm productivity include Wang and Wang (2015), Alfaro et al. (2013) and De Loecker and Warzynski (2012), among others.

to invest in China because of the performance requirements. The performance restrictions were removed when China joined the WTO in 2001 and more industries were opened to foreign investors. As a result, FDI inflow to China increased dramatically after 2002 (Figure 1) and the newly entered FDI firms may match our model better than those that entered China before 2001.

Five sector-level measures for the financial vulnerability are employed in our exercises following Manova et al. (2015) and Alquist et al. (2019): (i) external finance dependence, (ii) inventory ratio, (iii) R&D ratio, (iv) asset tangibility, and (v) trade credit. Financial vulnerability is positively correlated with external finance dependence, inventory ratio, and R&D ratios but negatively correlated with asset tangibility and trade credit. These five measures are described in Table 1 and calculated from the data of all publicly traded firms based in the U.S.[9] Using the U.S. data ensures that the financial vulnerability measures are not endogenously determined by China's level of financial development. Indeed, these measures are intended to capture the features inherent to the nature of the manufacturing process, which is supposed to be the same across countries.[10] Each financial vulnerability variable is measured by the median among all firms in the sector and are available for 3-digit ISIC sectors. We match them with the 4-digit Chinese industry code in our dataset. To match firms with financial vulnerability variables, we first map the 4-digit Chinese industry code of each firm to the 3-digit ISIC-Rev.3 industry code according to the contrast table provided by the Ministry of Commerce of China.[11] Next, the 3-digit ISIC-Rev.3 industry code is mapped to ISIC-Rev.2 industry code according to the concordance table of the United Nations.[12] Then, we can match the sector-level financial vulnerability variables with

[9] The raw data of U.S. firms are obtained from Compustat's annual industrial files. More details about these measures are included in Appendix A.1.2.

[10] Consistent with this argument, those measures display more cross-sectoral variations than cross-firm variations within a sector.

[11] Matching details can be found at http://www.fdi.com.cn/industry/IndustryEn.html.

[12] Details can be found at: http://unstats.un.org/unsd/cr/registry/regso.asp?Ci=1&Lg=1.

the firms.

The five measures are not highly correlated, which indicates that they capture different dimensions of financial vulnerability. Following Manova et al. (2015), we calculate the first principal component (FPC) of the five indicators and use it as our preferred proxy for each sector's financial vulnerability. Manova et al. (2015) argue that FPC provides a cleaner index of financial vulnerability than each individual measure because the individual measures might be correlated with industrial characteristics unrelated to financial frictions. The FPC index has a positive loading on external finance dependence, inventory ratio, and R&D ratio, and a negative loading on asset tangibility and trade credit. The sign of the loading on each measure is consistent with the implication of the measure of financial vulnerability. In the end, FPC accounts for 45.9% of the variance for all five measures.

2.2.2 Empirical results

We compare the productivity of FDI and domestic firms using the following benchmark model to test the entry condition in our theoretical model:

$$Productivity_{fipt} = \alpha + \beta FDI_{fipt} + \gamma Firmcontrol_{ft} + \gamma_i + \gamma_p + \gamma_t + \varepsilon_{fipt}, \tag{10}$$

where the subscripts f, i, p, and t refer to firm, industry, location, and year, respectively. The dummy variable FDI_{fipt} equals one for firms whose FDI share is equal to or greater than 10%.[13] The firm controls ($Firmcontrol_{ft}$) include firm size, the export share in total sales and an economic zone dummy indicating whether the firm is located in an economic zone. The fixed effects at the industry, location (province), and year levels are also included, which are represented by γ_i, γ_p, and γ_t respectively.

The prediction in Proposition 1 pertains to the cutoff productivity of FDI and local firm-

[13] We try different definitions of FDI firms, such as changing the cutoff value of the FDI share and using the registration type of the firms. The results remain qualitatively unchanged. Our results also hold up if we use the FDI share in the regression directly.

s. Therefore, we need to compare the productivity of FDI and local firms at the bottom of the productivity ranking in each type of firms. To achieve this goal, we employ quantile regressions to show that in financially more vulnerable sectors, FDI firms at the bottom of productivity ranking (around the cutoff productivity) are more likely to have lower productivity than their domestic counterparts.

Panel A of Table 2 reports the results when the financial vulnerability is measured by FPC. In the high financial vulnerability sector (top 25% of FPC), we find that the coefficient estimate of FDI dummy is significantly negative (at the 5% or 1% significance level) for firms at the bottom 20% of productivity, indicating that FDI firms have even lower productivity than local firms. In contrast, the evidence for the low financial vulnerability sector (bottom 25% of FPC) is much weaker, which is consistent with our Proposition 1. In both the low and high financial vulnerability sectors, the coefficient estimate of FDI dummy turned significantly positive for 50% and higher quantiles, thereby suggesting that productivity distribution of FDI firms may have a fatter tail than that of local firms.

The above findings also hold for most other measures of financial vulnerability as shown in Panel B of Table 2. In this panel, we report the results of the 15th percentile for the other five measures of financial vulnerability. In four of these five measures, the coefficient estimate of FDI dummy is significantly negative in the high financial vulnerability sector, but not for the low financial vulnerability sector. These findings strongly support the predictions in Proposition 1.

We employ the OLS regressions to test the predictions on the average productivity in Proposition 2. Table 3 shows the coefficient estimates of the FDI dummy in the sectors of low and high financial vulnerability under different measures of financial vulnerability. The coefficient estimate is significantly positive in the sectors of low financial vulnerability under all six measures of financial vulnerability and in the sector of high financial vulnerability under five out of six measure of financial vulnerability. These results suggest that FDI firms are on average more productive than local firms, even though they have lower cutoff productivity than local firms in financially vulnerable sectors. In addition, the coefficient estimate of FDI dummy is much smaller under four out of six measures of financial vulnerability for the sector of high financial vulnerability than the sector of low financial vulnerability, which is consistent with the predictions of Proposition 2. In particular, under our favorite measure of FPC, the coefficient estimate for the sector of high financial vulnerability is less than half of that for the sector of low financial vulnerability, and the difference is statistically significant at the 1% level.

3 Policy Analysis in a Two-country Model

In this section, we study the policy implications when FDI firms have funding advantages relative to local firms in a two-country model.

3.1 Model description

Figure 2 shows the model structure. There are two countries, Home and Foreign (H and F), and each country has two sectors. Sector 0 provides a constant returns-to-scale homogeneous good, which serves as a numeráire in each country, while the other sector produces differentiated goods. The two countries are almost symmetric, and we focus on the Home country to describe the model when it causes no confusion. The only asymmetry is that Foreign firms can open subsidiaries in the Home country, while Home firms only operate in their own country. This setup is used to capture the fact that most FDI is from advanced economies to emerging markets. The detailed description of the Foreign country can be found in appendix A.2.

The representative consumer in the Home country derives utilities from the homogeneous good and the aggregate composite of differentiated goods:

$$V \equiv \Phi {C_0}^{\theta_0} {C_1}^{\theta}, \tag{11}$$

where V is the utility function of the Home consumer, C_0 is the consumption of the homogeneous (numeráire) good, and C_1 is the composite consumption of differentiated goods with $\theta_0 + \theta = 1$ and $\Phi \equiv \theta_0^{-\theta_0}\theta^{-\theta}$.

C_1 is a CES aggregator of the composites of domestic and imported goods:

$$C_1 = \frac{\left(C^H\right)^{\nu}\left(C^F\right)^{1-\nu}}{(\nu)^{\nu}(1-\nu)^{1-\nu}}, \tag{12}$$

where C^H and C^F are the composites of Home goods and imported Foreign goods, respectively. Parameter $\nu > 0.5$ captures the degree of consumption bias toward local goods.[14] C^H and C^F are, respectively, the CES aggregators of locally-produced products and imported goods:

$$C^H = \left(\int_{\omega\in\Omega}\left[y^D(\omega)\right]^{\rho} d\omega + \int_{\omega^*\in\Omega^I}\left[y^I(\omega^*)\right]^{\rho} d\omega^*\right)^{\frac{1}{\rho}}$$
$$C^F = \left(\int_{\omega^*\in\Omega^*}\left[y^{D,X*}(\omega^*)\right]^{\rho} d\omega^*\right)^{\frac{1}{\rho}},$$

where $y^D(\omega)$ is a good produced by a Home domestic firm ω and $y^I(\omega^*)$ is a good produced by a Home FDI firm ω^*. Note that the composite of locally-produced goods includes products of both local domestic firms and FDI firms in Home. We refer to local domestic firms as local firms in the rest of the paper when it causes no confusion. $y^{D,X*}(\omega^*)$ is an imported good produced by a Foreign firm ω^*. Ω and Ω^I denote the set of goods produced by the local and FDI firms, respectively, in the Home country. Ω^* is the set of goods produced by the firms in the Foreign country.

Labor is the sole factor of production and is immobile across countries. Given wage

[14] The presence of consumption home bias is consistent with the data and also essential to ensure the existence of the unique equilibrium in our model.

W and the transfer from government T, the representative household maximizes its utility subject to the budget constraint:

$$PC = WL + T, \tag{13}$$

where C is aggregate consumption given by $C = \Phi {C_0}^{\theta_0} {C_1}^{\theta}$, which is identical to the utility V. P is the consumer price index and L denotes the labor supply.

Financially constrained Home local firms: There is a continuum of Home local firms, each producing a different variety of Home goods with the production function $y(\omega) = zl(\omega)$, where each firm is indexed by $\omega \in \Omega$, z is the productivity of firm ω, and $l(\omega)$ is the firm's labor input for production.

A local entrepreneur pays F^D units of labor as a fixed cost to draw her productivity z from a distribution of $G(z)$. As is standard in the literature, z follows a Pareto distribution, given by $G(z) = 1 - (z_{min})^{\eta} z^{-\eta}$ for $z \geq z_{min}$, with $\eta > \sigma - 1$ and $\sigma \equiv \frac{1}{1-\rho}$. After observing her productivity, the entrepreneur can decide whether to produce. If she chooses to produce, the firm needs to pay an additional fixed cost for production, f^D units of labor. Otherwise, she exits.

We assume that Home domestic firms are financially constrained as in Manova (2012) and Bilir et al. (2019). A fraction $\zeta \in [0, 1]$ of the fixed production cost, f^D, has to be funded externally, although the firm can finance the variable production cost internally. To borrow the loan of $\zeta f^D W$, Home local firms must pledge a collateral equal to a portion $\chi \in [0, 1]$ of the fixed entry cost F^D (a proxy for tangible assets) and promise to pay back an amount of $x(z)$ in home currency when the loan matures. After observing their revenues, Home domestic firms redeem their loans with a probability $\lambda \in [0, 1]$. The non-defaulting probability λ reflects the quality of financial institutions, such as the effectiveness of contract enforcement or bank's ability of screening and monitoring of borrowers: a higher λ indicates better quality of financial institutions. In the case of default, the bank seizes the collateral $\chi F^D W$ and the defaulting firm must replace it before it can obtain external financing in the future.[15] Firms face an exogenous probability (δ) of exiting the market involuntarily.

The government imposes taxes on firms' profits and revenues, which is captured by the wedges τ_C^D and τ_V^D in the model.[16] Given the above setup, a Home local firm with productivity z chooses price $p^D(z)$, quantity for local sales $y^D(z)$, quantity for exports $y^{D,X}(z)$, labor input $l^D(z)$, and loan payment $x(z)$ to maximize its profit

$$\pi^D(z) = \max \ \ \tau_C^D \begin{bmatrix} \tau_V^D p^D(z) \left(y^D(z) + y^{D,X}(z)\right) & -Wl^D(z) - f^D W + \zeta f^D W \\ & -\lambda x(z) - (1-\lambda)\chi F^D W \end{bmatrix}, \tag{14}$$

subject to

$$\overline{\pi}^D(z) \equiv \tau_V^D p^D(z) \left(y^D(z) + y^{D,X}(z)\right) - Wl^D(z) - f^D W + \zeta f^D W \geq x(z), \tag{14a}$$

$$\lambda x(z) + (1-\lambda)\chi F^D W \geq \zeta f^D W. \tag{14b}$$

There are two constraints in local firms' profit maximization. First, the firm's operational profit, $\overline{\pi}^D(z)$, must be larger than or equal to the loan payment $x(z)$ as in equation (14a). Second, the constraint in equation (14b) requires the bank's expected proceeds from the debt contract to be greater than or equal to its costs. We assume that the banks operate in a perfectly competitive market. As a result, the constraint in equation (14b) is always binding.

Note that the ex-ante profit before the realization of the defaulting event, $\pi^D(z)$, is different from the ex-post profit of the firm that repays its debt, $\overline{\pi}^D(z)$. The additional cost from the financial constraints drives out low-productivity firms that would produce without

[15] Note that the defaulting firms do not exit the market. They stay in operation by replacing the collateral seized by the bank.

[16] Specifically, taxes on the firms' profits and revenues equal to $\left(1-\tau_C^D\right)$ and $\left(1-\tau_V^D\right)$, respectively.

such a cost. Financial frictions affect neither the optimal pricing of a firm nor its ex-ante profit, which are given by $\frac{p^D(z)}{W} = \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)\frac{1}{z}$ and $\frac{\pi^D(z)}{W} = \tau_C^D\left[z^{\sigma-1}\left(\frac{\tau_V^D}{\sigma}\right)\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} A - f^D\right]$. In the last equations, A denotes the market demand for the products produced by Home domestic firms:

$$A \equiv \left[\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right].$$

Because financial frictions affect the ex-post profit of a non-defaulting firm, the cutoff productivity for Home local firms, Z^D, depends on the financing cost through

$$\frac{\pi^D(Z^D)}{W} \;=\; \tau_C^D\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right). \tag{15}$$

As the degree of financial market imperfection increases, i.e., the non-defaulting probability λ decreases, the cutoff productivity, Z^D, increases as the external financing cost rises. The cutoff local firms have to be more productive to cover the financing cost.

An unbounded pool of prospective domestic entrants into the production exist, and the incumbents exit with an exogenous probability of δ. Due to the free entry condition, the expected life-time operating profit of a potential entrant should equal the entry cost: $\frac{\pi^D(\widetilde{Z}^D)}{W} = \frac{\delta F^D}{1-G(Z^D)}$, where the average productivity of Home local firms is denoted by $\widetilde{Z}^D \equiv \left[\int_{Z^D}^{\infty} z^{\sigma-1}\frac{dG(z)}{1-G(Z^D)}\right]^{\frac{1}{\sigma-1}}$. The mass of Home local firms evolves according to the law of motion: $M = \left(1-G(Z^D)\right)M^E + (1-\delta)M$. There are M^E mass of entrants who pay fixed entry cost to draw their productivity but only $\left(1-G(Z^D)\right)$ fraction of them successfully enter the market after their productivity is revealed.

Financially unconstrained Foreign firms: Foreign firms also bear fixed entry and production costs in the Foreign country, which are denoted by F^{D*} and f^{D*} in terms of Foreign labor. In addition, Foreign firms can choose to establish subsidiaries in the Home country via FDI after paying a fixed production cost f^I in terms of Home labor. FDI firms face a higher fixed production cost every period than Home domestic firms: $f^I \geq f^D$.

The aggregate profit of all Foreign firms in terms of Foreign currency is given by the following:

$$\int_{\omega^* \in \Omega^*} \pi^{D*}(z) d\omega^* + \int_{\omega^* \in \Omega^I} \frac{1}{\epsilon} \pi^I(z) d\omega^*,$$

where $\pi^{D*}(z)$ represents the profit of a Foreign local firm with the productivity z and $\pi^I(z)$ denotes the profit of a FDI firm. ϵ denotes the nominal exchange rate, which is unity in our model.

Foreign and FDI firms are financially less constrained than Home local firms because firms in advanced economies usually have access to the credits from international financial markets and FDI firms can also finance from their parent companies.[17] For simplicity, we assume Foreign and FDI firms face no financial constraint. These firms face a standard profit maximization problem, which we leave to appendix A.2.

Equilibrium conditions and the cutoff productivity tradeoff: The model equilibrium is defined by 18 equations for 18 endogenous variables. To save space, we leave the equilibrium conditions to appendix A.2.2 and only discuss the main results in the paper.

In the general equilibrium model, FDI firms face a similar tradeoff between the financial advantage and high fixed production cost similar to the partial equilibrium model in Section 2. The cutoff productivity of FDI firms can be lower than that of Home local firms if $\frac{f^D + \left(\frac{1}{\lambda} - 1\right)\left(\zeta f^D - \chi F^D\right)}{f^I} \left(\frac{\tau_V^I}{\tau_V^D}\right)^\sigma > 1$. In case $\tau_V^I = \tau_V^D$, the above condition is reduced to the one in Proposition 1. Because $\sigma > 1$, the condition is more likely to hold when the wedges satisfy the inequality: $\tau_V^I > \tau_V^D$. In other words, FDI firms are more likely to have lower cutoff productivity than Home local firms if their revenue tax rate is lower than that of the local firms. It suggests that tax benefits to FDI firms in many emerging markets may attract

[17] See Bilir et al. (2019), Desai et al. (2007) and Manova et al. (2015) for discussions on FDI firm's financial advantages.

FDI firms whose productivity is even lower than local firms, which is exactly the opposite of what these tax policies are designed to achieve, that is, to attract high-tech and productive MNCs.

3.2 Calibration

We calibrate our model by taking China as the Home country and the advanced economies as the Foreign country. Table 4 presents the parameter values in our calibration and the reasons for choosing these values.

Most parameter values are standard in the literature. Following Bernard et al. (2003), the elasticity of substitution among differentiated goods is set to $\sigma = 3.8$, which implies a price markup of 36%. The exogenous exit probability δ is 10%. The home-bias parameter ν is calibrated to match the empirical ratio of total Home import sales to total Home product sales, 0.163, which is obtained from the Chinese Customs and firm data in 2000. We assume that the Foreign country exhibits the same level of home bias in consumption ($\nu^* = \nu$). The Home labor endowment is normalized to be one and the Foreign labor supply is chosen to match the empirical ratio of total Home export sales to total Home import sales, 0.184.

Following Alquist et al. (2019), we impose additional restriction on FDI. When Foreign multinationals open subsidiaries in the host country, their productivity is reduced by a factor α.[18] The parameter α is calibrated to match the empirical ratio of total Home FDI product sales to total Home product sales, 0.388. We apply the same lower bound for the Pareto distribution of firm productivity in Home and Foreign, which is normalized to be $z_{min} = z^*_{min} = 0.20$ as in Bernard et al. (2007). The dispersion parameter in Home, η, is set to 3.3 as in Bernard et al. (2003) and Bernard et al. (2007). The Foreign dispersion parameter, η^*, is calibrated to 3.15, such that the average productivity of FDI firms is higher

[18] If a Foreign firm draws its productivity z in the Foreign country, then its productivity is reduced to αz with $\alpha < 1$ when its subsidiaries run in the host country. The parameter α captures operational frictions that MNCs face in the FDI host country.

than that of the Home firms, though the cutoff productivity of FDI firms is lower than that of the Home local firms. Figure 3 shows the firm distributions and cutoff productivities in Home and Foreign.

Insert Figure 3 Here

For parameters of financial frictions, we assume that all fixed production costs have to be externally financed ($\zeta = 1$). χ is chosen such that banks can recover 30% of the collateral when firms default and the value of the collateral is 1.2 times that of the total borrowing. In each period, 30% of Home firms default on their debt ($\lambda = 0.70$). The financial market reform is captured by an increase in λ. Even though the literal interpretation of λ is the non-defaulting probability of domestic firms, we take λ as a measure for the imperfectness of Home financial markets in a broad sense. When λ equals one, the local financial market becomes a frictionless one as in the standard models. We choose $\lambda = 0.70$, which is low enough to ensure that the cutoff productivity of FDI firms is lower than that of local firms as we find in the Chinese firm-level data.

The sunk entry cost is the same for Home and Foreign firms, which is equal to two units of labor following Bernard et al. (2007). The fixed production cost of domestic firms in Home and Foreign is set to 5% of entry costs. The fixed production cost of FDI firms is assumed to be higher than that of local firms due to additional transaction costs. It is calibrated to match the empirical ratio of tangible assets between FDI firms and local firms, 1.115, which is presented in Table A.3 in the appendix. The fixed production cost mainly consists of the depreciation of tangible assets, interest expense, and utility costs. Because asset depreciation accounts for most of the fixed production costs, we use the tangible asset ratio as a proxy to pin down the ratio of fixed production costs between FDI firms and local firms.[19]

[19]We assume the depreciation rate is the same across firms.

Lastly, the wedges in the firms' profit maximization problem are calibrated to match the practice of corporate taxation in China. For Home local firms, we set 33% corporate tax on profits and 17% value-added tax. Since intermediate inputs are absent from firms' optimization problem in our model, the value-added tax is equivalent to the tax on revenues. For FDI firms, we assume 15% corporate tax and 15% value-added tax due to tax incentives to attract FDI. The empirical ratios and corresponding values from our calibration are shown in the Table 5 while the other details are relegated to appendix A.3.[20]

3.3 Counterfactual policy analysis

This section investigates the welfare effect of two policy changes. First, we raise the tax rate of FDI firms in our benchmark model, while keeping the total government revenue constant. In this case, the tax rate of local firms will also change endogenously when we adjust the tax rate of FDI firms. This exercise examines the effect of removing tax benefits offered to FDI firms (or even taxing FDI firms more than local firms) on the host country's welfare. The second exercise investigates the welfare effect of raising λ, which simulates a financial market reform that reduces the financial disadvantages of local firms. In these two exercises, we only compare the equilibrium results of the benchmark model at different parameter values, which represent the targeted policy changes. No dynamic path is considered in our analysis.

Welfare is measured by the representative household's utility and the appendix (A.2.1)

[20] The consumption weight on the homogeneous-good sector, θ_0, is fixed at 0.07. We choose this number to maximize the weight on the heterogeneous-good sector, which is the main focus of our counterfactuals, while holding consumption and labor in the homogeneous-good sector positive in the equilibrium: $C_0, L_0, C_0^*, L_0^* > 0$. If the weight, θ_0, is too small or labor endowments, L and L^*, are not large enough, then consumption or labor allocation in the homogeneous-good sector, C_0, L_0, C_0^*, and L_0^*, can be negative, which implies allocations are in disequilibrium.

shows that the utility in equation (11) can be written out as

$$
\begin{aligned}
V &= \Phi {C_0}^{\theta_0} {C_1}^{\theta}, \text{and} \\
C_1 &= \underbrace{\left(\rho \widetilde{Z}^{HF}\right)}_{\text{Productivity Effect}} \underbrace{\left(M^{HF}\right)^{\frac{1}{\sigma-1}}}_{\text{Variety Effect}} \underbrace{\theta\left(L + \frac{T}{W}\right)}_{\text{Income Effect}} . \qquad (16)
\end{aligned}
$$

Since the only purpose of homogeneous-good consumption C_0 in the model is to have a common numeráire across countries and C_0 has a small weight in total consumption, ignoring C_0 in our analysis will not affect our main results. Therefore, C_1 and welfare are determined by the three effects in equation (16). First, welfare is positively affected by the composite productivity (the productivity effect), $\widetilde{Z}^{HF}$, since higher productivity leads to more output for consumption. The second effect (the variety effect) is from the composite mass of firms, M^{HF}, because the consumer loves the variety. The last one is from the income effect of the household, $L + \frac{T}{W}$. The household becomes better off if it receives more transfers from the government, $\frac{T}{W}$.

We study the equilibrium outcome of the three effects to a policy change of our interest and the policy's overall welfare effect in our model. In addition, we explore the factors driving these three effects to develop intuitions behind our results.

3.3.1 Removing tax benefits of FDI firms

The tax benefits offered by emerging markets to FDI firms could be counter productive. The tax benefits are usually offered to attract FDI inflows under the conventional wisdom that FDI firms can promote the host country's productivity. Under certain circumstances, such a policy could be optimal because it boosts a country's productivity by attracting highly productive FDI firms to invest in the country. In addition, previous studies document convincing microeconomic evidence of positive technology spillovers from FDI firms to local firms. We fully acknowledge these positive effects of FDI firms. However, whether the tax benefits offered to FDI firms can achieve their intended goal if FDI firms have other advantages such as low financing costs, rather than advantages in productivity, is less clear.

Figure 4 presents the results when the revenue tax rate of FDI firms is set at different levels (from 15% to 55%). For each level of the FDI tax rate, we endogenously choose the corresponding revenue tax rate of local firms such that government revenues (and the transfers to household) remain the same as in our benchmark model. In each chart, the horizonal axis displays the revenue tax rate of FDI firms and the vertical axis presents the corresponding equilibrium outcome for the variables of interest. For instance, the top-left subfigure shows the corresponding revenue tax rate of local firms when the revenue tax rate of FDI firms is set at a level from 15% to 55%. The tax rate of local firms first decreases because the tax revenue from the FDI firms increases with the tax rate hike on the FDI firms. The government can lower the tax rate of local firms while keeping its total tax revenue constant. However, the higher tax rate will also make more FDI firms exit from the Home country, which reduces the tax base and the revenue. This effect will eventually dominate and the Home government will have to increase the tax rate of local firms once the tax rate of FDI firms exceeds a threshold (33%). Note that the change in the tax rate is more moderate for local firms (between 12.6% and 17.0%), relative to FDI firms (from 15% to 55%) because local firms account for a bigger fraction of total output in the Home country than FDI firms. Keeping the total tax revenue constant, a small change in the tax rate of local firms can offset the effect from a much larger tax rate change for FDI firms.

We find that the tax benefits offered to FDI firms actually reduce the total productivity of the host country ($\widetilde{Z}^{HF}$) in our model. The productivity effect displays a hump-shaped pattern in the left subfigure of the second row in Figure 4: $\widetilde{Z}^{HF}$ first increases if we remove the tax benefits offered to FDI firms and remains true even when we start to tax FDI firms more than local firms. However, $\widetilde{Z}^{HF}$ eventually declines with the tax rate of FDI firms when the tax rate is higher than 33%.

The hump-shaped pattern of the productivity effect is due to two reasons: the increase in FDI firms' cutoff productivity and the change in the market share of FDI firms. Recall that the total productivity of the consumption composite in Home is defined as

$$\log\left(\widetilde{Z}^{HF}\right) \equiv \nu \log\left(\widetilde{Z}^{DI}\right) + (1-\nu)\log\left(\widetilde{Z}^{D*}\right), \tag{17}$$

which states that the total productivity of home consumption bundle is the average productivity of Home firms (both local firms and FDI firms, $\widetilde{Z}^{DI}$) and foreign firms ($\widetilde{Z}^{D*}$). This result is intuitive because the home consumption bundle includes both products produced by Home firms and products imported from Foreign. The tax rate change in Home has a very small spillover effect on the foreign productivity, $\widetilde{Z}^{D*}$, and the consumption bundle is biased towards Home produced goods ($\nu = 0.83$).[21] Thus, $\widetilde{Z}^{D*}$ has a negligible effect on $\widetilde{Z}^{HF}$. We can safely ignore $\widetilde{Z}^{D*}$ and focus our analysis of $\widetilde{Z}^{HF}$ on $\widetilde{Z}^{DI}$.

$\widetilde{Z}^{DI}$ is determined by the tax rate, cutoff productivity, and mass of local and FDI firms in Home. Note that $\widetilde{Z}^{DI}$ is defined by

$$\widetilde{Z}^{DI} \equiv \left[\frac{1}{M+M^I}\left(M\left(\tau_V^D \widetilde{Z}^D\right)^{\sigma-1} + M^I\left(\tau_V^I \widetilde{Z}^I\right)^{\sigma-1}\right)\right]^{\frac{1}{\sigma-1}}, \tag{18}$$

where $\widetilde{Z}^D$ and $\widetilde{Z}^I$ are the average productivity of Home local firms and FDI firms, respectively. The average productivity of Home firms ($\widetilde{Z}^{DI}$) is the weighted average of local-firm productivity and FDI-firm productivity with the relative mass of these two types of firms being the weight. We show in the online appendix (Section A.2) that the average productivity of local and FDI firms are linear functions of the corresponding cutoff productivity: $\widetilde{Z}^D = \left[\frac{\eta}{\eta-\sigma+1}\right]^{\frac{1}{\sigma-1}} Z^D$, and $\widetilde{Z}^I = \left[\frac{\eta^*}{\eta^*-\sigma+1}\right]^{\frac{1}{\sigma-1}} \alpha Z^I$, where Z^D and αZ^I are the cutoff produc-

[21] The productivity in Foreign actually decreases in the model when the tax rate of FDI firms increases. Please see appendix A.2.2.3 for details.

tivity for local and FDI firms, respectively. As discussed in Section 3.1, the tax benefits of FDI firms reduce their cutoff productivity relative to that of local firms. When we increase FDI firms' tax rate, their cutoff productivity rises substantially as shown in the right subfigure of the second row in Figure 4. In contrast, the cutoff productivity of local firms does not change. As a result, the average productivity of FDI firms increases significantly while the average productivity of local firms only changes slightly, mimicking the patterns of the cutoff productivity of FDI and local firms.

Both the cutoff productivity and the after-tax wedges (τ_V^D and τ_V^I) affect $\widetilde{Z}^{DI}$ in equation (18). For instance, the cutoff productivity of FDI firms increases but the after-tax wedge decreases when the tax rate of FDI firms increases. To understand the overall effect, the right subfigure of the third row in Figure 4 shows the effective productivity measures that augment the average productivity by the after-tax wedges: $\tau_V^D \widetilde{Z}^D$ for Home local firms and $\tau_V^I \widetilde{Z}^I$ for FDI firms. The effective productivity of FDI firms continues to increase with the tax rate even after we take into account the after-tax wedge. The effective productivity of local firms changes only moderately: it mainly traces the after-tax wedge ($\tau_V^D \widetilde{Z}^D$) as local firms' cutoff productivity stays almost constant.

Another important change in our counterfactual analysis is that the market share of FDI firms declines because fewer FDI firms operate in the Home country if a higher revenue tax rate is imposed on these firms. As shown in left panel of the third row in Figure 4. When the market share of FDI firms shrinks, it pulls down the average productivity in the Home country because the effective productivity of FDI firms is higher than that of local firms as shown in the right subfigure of the third row in Figure 4. Therefore, we have two offsetting effects from raising the tax rate of FDI firms. Given the market shares of local and FDI firms, raising the tax rate of FDI firms will increase the aggregate productivity in the Home country as the cutoff and effective productivity of FDI firms rise substantially. However, the decrease in the market share of FDI firms will reduce the average productivity of the Home country. As a result, the aggregate productivity in the Home country shows a humped shape when we adjust the tax rate of FDI firms.

Welfare consists mainly of three components: the productivity effect, the variety effect, and the income effect. The income effect is constant in this exercise. The productivity effect displays a humped shape and the variety effect is concavely increasing. Overall, the welfare also has a humped shape similar to the productivity effect as shown in the last subfigure of Figure 4.

The above analysis highlights a trade off faced by the FDI policy. On the one hand, the policymakers would like to provide tax and other benefits to attract high-productivity foreign firms in order to boost the domestic average productivity. On the other hand, the low-productivity foreign firms will also take advantage of these policies. As we show, the FDI firms could even have lower productivity than local firms if the host country's financial markets are underdeveloped. In this case, some FDI firms with low-productivity are subsidized by taxing high-productivity local firms, which defies the purpose of such policies.

Several previous studies also emphasize the quality of domestic financial sector as a crucial factor for a country to derive the benefits of international capital flows.[22] In particular, Alfaro et al. (2004) document that economies with better-developed financial markets are able to benefit more from FDI to promote their economic growth. They argue that technology spillovers from FDI firms to local firms are financially costly and well-functioning domestic financial markets help local firms with their financing to adopt new technology from FDI firms. Similar empirical findings are also documented in Prasad et al. (2005) and Kose et al. (2009). In this paper, we explore a different channel through which underdeveloped local financial markets may undermine FDI's benefits to the productivity of host countries. We

[22]More generally, the quality of the domestic financial markets plays a critical role for a country to benefit from financial market globalization. See Kose et al. (2010) for a review of these studies.

emphasize that FDI decision depends endogenously on local financial markets: inefficient local financial markets may attract low-productivity FDI firms.[23]

China adopted policies in the 1990s to alleviate the problem of attracting low-productive FDI firms. In the beginning of the 1990s, China had very strong capital controls and several FDI policy measures were adopted to ensure that FDI firms introduced new technology and management skills to China. For instance, China used performance requirements to control for the quality of FDI firms before its accession to the WTO in 2001. Policies, such as tax exemption were also adopted to encourage FDI firms to transfer advanced technology to China before 2001.[24] Thus, FDI firms are more likely to have higher productivity than local firms during this period. However, China went through capital account liberalization in the 2000s by removing restrictions on what sectors foreign firms can invest and also called off the performance requirements.

3.3.2 Financial Market Reform under Tax Distortions

The financial disadvantage of local firms can be attributed to the financial market friction in the Home country of our model. If the financial market is reformed to improve its efficiency, will the reform definitely improve the Home country's welfare? We examine this question by changing the non-defaulting probability of local firms in our model, λ. Financial market efficiency improves when λ increases.

Under the benchmark setup, the Home welfare displays a humped shape when λ increases from 0.5 to 1. Figure 5 presents our results, and the welfare of the Home country measured by the composite consumption is displayed in the top left chart. The composite consumption exhibits a humped shape, peaking when λ is about 78%. Beyond this point, a further improvement of the Home financial market efficiency can even reduce the country's welfare.

[23]Bilir et al. (2019) find that financially advanced economies attract more affiliates of US multinationals, which are usually more productive than FDI from other countries.

[24]See Long (2005) for more details.

This counter-intuitive result can be attributed to the interaction between the financial friction and tax distortions. If we decrease the profit tax of local firms, the humped shape of Home country welfare flattens and the welfare increases with λ. The top-right chart of Figure 5 shows the percent change of consumption relative to the benchmark model ($\lambda = 0.70$) in cases with different profit tax rates of $1 - \tau_C^D$.

The second row of Figure 5 and the left chart in the third row present the three effects that determine the Home welfare: the aggregate productivity, $\widetilde{Z}^{HF}$, the aggregate measure of product varieties, M^{HF}, and the lump-sum transfers to households, $\frac{T}{W}$. When the financial constraints of local firms improve, more local firms with low productivity can now enter the market. As shown in the right panel of row three in Figure 5, the cutoff productivity of local firms decreases with λ, dragging down the aggregate productivity, $\widetilde{Z}^{HF}$. At the same time, the entry of new local firms raises the product varieties, M^{HF}, which increases almost linearly with λ in Figure 5. The lump-sum transfers barely change and have a negligible effect on the welfare. We can ignore this income effect in the analysis below.

Equation (16) reveals that the welfare is a linear function of the aggregate productivity and is a concave function of the aggregate measure of product varieties. As a result, the welfare is a concave function of λ. In particular, it shows a humped shape in our benchmark model because the decrease in the aggregate productivity dominates the increase in product varieties when λ is above 78%.

The aggregate measure of product varieties increases more strongly with λ when the profit tax rate of local firms is set to a lower level. The increase in the product varieties is strong enough to offset the decreases in the aggregate productivity for all values of λ when the profit tax rate is low enough. In contrast, under our benchmark parameterizations, the effects from the decrease in aggregate productivity dominate the increase in product varieties for large λ such that the welfare becomes a hump-shaped function of λ.

To understand the role of the profit tax rate in driving our results, consider the labor market clearing condition in Home:

$$L - L_0 \;=\; M\left(\frac{\delta F^D}{1-G(Z^D)}\right) + Mf^D + M^I f^I + Ml^D\left(\widetilde{Z}^D\right) + M^I l^I\left(\widetilde{Z}^I\right),$$

which states the labor in the heterogeneous goods sector is used for the fixed entry cost of local firms ($M\left(\frac{\delta F^D}{1-G(Z^D)}\right)$), the fixed production costs ($Mf^D + M^I f^I$) and the production of goods ($Ml^D\left(\widetilde{Z}^D\right) + M^I l^I\left(\widetilde{Z}^I\right)$). From the free entry condition in the Home country, we have $\frac{\delta F^D}{1-G(Z^D)} = \frac{\pi^D(\widetilde{Z}^D)}{W} = \tau_C^D\left[\frac{l^D(\widetilde{Z}^D)}{\sigma-1} - f^D\right]$. For a given fixed entry cost, a decrease in the tax rate (an increase in τ_C^D) will reduce the average profit of each local firm, and therefore, the average size of local firms. In other words, the decrease in the Home profit tax rate increases the competition in the Home market. When the average size of Home local firms is smaller, a given increase in λ allows more home firms (or product varieties) to enter the market.

Our results suggest that the financial market reform may have to be combined with tax reform, that is, a reduction in the profit tax, to benefit the economy. Under a high profit tax rate, the Home market is populated by relatively large firms. The financial market reform may not benefit enough firms to guarantee an increase in social welfare.

4 Conclusion

Recent studies have explored the role of FDI's financial advantages in driving FDI flows and alleviating firms' financial constraints in host countries. Convincing evidence that FDI firms can improve export performance of host countries, provide funding (e.g., through trade credit) to financially constraint local firms and relax financial constraints of foreign acquired firms, is documented in the literature. These financial benefits complement the conventional technology benefits that FDI firms bring to host countries, which have been widely studied in the literature.

In this paper, we emphasize that the financial advantages and productivity advantages of FDI firms are interdependent and this interdependence has strong policy implications. The financial advantages may allow FDI firms of low productivity to enter host countries. We show both theoretically and empirically that the financial advantages of MNCs will affect the relative cutoff and average productivity of FDI and local firms. Indeed, the cutoff productivity of marginal FDI firms can even be lower than that of marginal local firms as we show in the Chinese firm-level data. In this case, the policy of subsidizing FDI firms by taxing local firms is highly counterproductive.

We acknowledge that our two-country model of policy analysis abstracts from many well-documented benefits of FDI to host countries such as technology spillovers. A comprehensive welfare evaluation of FDI policies requires a model that incorporates all benefits and costs of FDI flows and the related policies, which we leave for future work.

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Table 1: Measures of Financial Vulnerability

Variable	Definition†	Higher financial vulnerability if	25th percentile in the data	75th percentile in the data
External finance dependence	U.S. industry median of (capital expenditure-cash flow)/capital expenditure for the period 1980-1999	Higher external finance dependence	-0.27	0.06
Inventory ratio	U.S. industry median of inventory/sales for the period 1980-1999	Higher inventory ratio	0.13	0.18
R&D ratio	U.S. industry median of R&D expenditure/sales for the period 1980-1999	Higher R&D ratio	0.01	0.02
Tangibility	U.S. industry median of fixed asset/total asset for the period 1980-1999	Smaller tangibility	0.20	0.40
Trade credit	U.S. industry median of account payable/total asset for the period 1980-1989	Smaller trade credit	0.05	0.08
First principal component (FPC)	Linear combination of the above five measures	Larger FPC	-0.79	0.79

Note: †–See Kroszner et al. (2007) and Fisman and Love (2003) for details.

Table 2: Results of Quantile Regressions

Panel A: Results for First Principal Component						
	Low financial vulnerability			High financial vulnerability		
Quantile (%)	Coef.	s.e.	No. obs.	Coef.	s.e.	No. obs.
5	−0·106***	0·034	48136	−0·153***	0·026	58895
10	−0·039*	0·023	48136	−0·094***	0·018	58895
15	−0·006	0·018	48136	−0·055***	0·015	58895
20	0·020	0·017	48136	−0·033**	0·013	58895
25	0·033**	0·016	48136	−0·007	0·012	58895
50	0·113***	0·014	48136	0·060***	0·012	58895
75	0·164***	0·016	48136	0·094***	0·013	58895
Panel B: Results of the 15th Percentile for other FV measures						
	Low financial vulnerability			High financial vulnerability		
FV measure	Coef.	s.e.	No. obs.	Coef.	s.e.	No. obs.
R&D ratio	0·006	0·011	102433	−0·075***	0·019	43868
Trade Credit	−0·018	0·014	56139	−0·002	0·015	61354
External Fiance	−0·013	0·018	38673	−0·066***	0·017	48656
Inventory ratio	−0·030	0·019	41304	−0·052***	0·013	66087
Tangibility	0·039**	0·019	46775	−0·061***	0·015	48451

Note: The financial vulnerability in Panel A is measured by the first principle component (FPC). Panel B shows the results of the 15th percentile for other measures of financial vulnerability. The low and high financial vulnerability refers to the bottom and top 25% of each financial vulnerability measure, respectively. The sample includes all firms that entered the market between 2002 and 2007, after China's accession to the WTO. The reported coefficient estimate is for the independent variable of FDI firm dummy. Control variables include firm size, export ratio, economic zone dummy, and industry, province, and year fixed effects. *, ** and *** denote the statistical significance at the 10%, 5% and 1% levels respectively.

Table 3: Coefficient of firm productivity on FDI firm dummy in different sectors for new entrants

	Low financial vulnerability			High financial vulnerability			χ^2
	Coef.	s.e.	No. Obs.	Coef.	s.e.	No. Obs.	
R&D ratio	0·047***	0·008	102443	0·059***	0·012	43868	0·73
Trade Credit	0·037***	0·009	56139	0·063***	0·010	61354	0·05**
External Fiance	0·027**	0·012	38673	0·011	0·011	48651	1·47
Inventory ratio	0·091***	0·012	41304	0·044***	0·009	66083	9·52***
Tangibility	0·068***	0·012	46775	0·018*	0·011	48442	15·93**
First Principal Component	0·083***	0·011	48136	0·038***	0·010	58895	8·66***

Note: By definition, the low financial vulnerability refers to the bottom 25% of external finance, inventory ratio, R&D ratio and first principle component, and the top 25% of asset tangibility and trade credit. The high financial vulnerability follows the opposite: the top 25% of the first three and the bottom 25% of the last two. The sample includes all firms that entered the market between 2002 and 2007, after China's accession to the WTO. The reported coefficient estimate is for the independent variable of FDI firm dummy. Control variables include firm size, export ratio, economic zone dummy, and industry, province, and year fixed effects. *, ** and *** denote the statistical significance at the 10%, 5% and 1% levels respectively.

Table 4: Calibrated Parameters for the two-country model with financial frictions

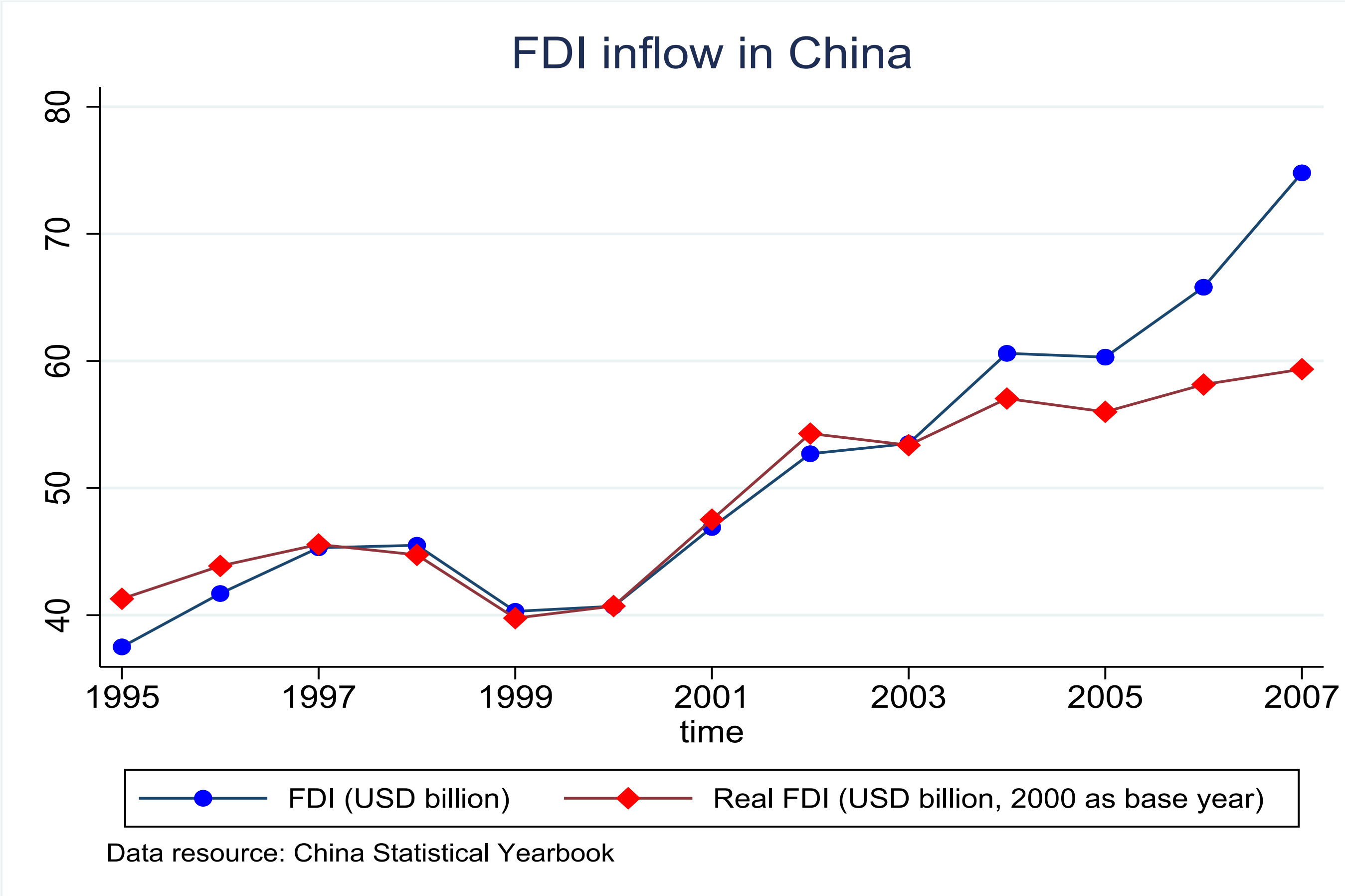
Parameter	Description	Value	Source
$\sigma = \frac{1}{1-\rho}$	Substitutability between differentiated goods	3.80	Bernard et al. (2003): Monopoly markup $\frac{\sigma}{\sigma-1} = 1.36$, i.e. 36.0%
δ	Exogenous exit probability	0.10	10% job destruction per year
ν, ν^*	Home and Foreign consumption bias	0.83	Target $\frac{\text{Total Home Imports}}{\text{Total Home Production}} = 0.163$
L	Labor endowment of Home country (FDI receiver)	1.00	Normalization
L^*	Labor endowment of Foreign country (FDI sender)	1.49	Target $\frac{\text{Total Home Exports}}{\text{Total Home Imports}} = 1.184$
α	Productivity loss of MNCs in the FDI-host country	0.48	Target $\frac{\text{Total Home FDI Production}}{\text{Total Home Production}} = 0.388$, 52% loss in productivity of MNCs in the host country
z_{min}, z^*_{min}	Lower bound in Pareto distribution for Home and Foreign firms	0.20	Normalization: Bernard et al. (2007)
η	Dispersion in Pareto distribution for Home firms	3.30	Bernard et al. (2003)
η^*	Dispersion in Pareto distribution for Foreign firms	3.15	Higher average productivity of FDI firms than local firms
λ	Probability at which domestic firms do not default	0.70	Lower cutoff productivity of FDI firms than local firms, 30% of default probability
ζ	Working capital fraction, i.e. the portion of fixed production costs that the local firm must finance externally	1.00	All fixed production costs must be externally financed.
χ	30% recovery rate times 1.2 leverage ratio	0.02	$30\% \times 1.2 \times \frac{\zeta f^D}{F^D}$: ζf^D is total borrowing and $\frac{1}{F^D}$ for normalization.
F^D	Fixed entry costs for Home firms	2.00	Normalization: follow Bernard et al. (2007) and choose 2 units of labor.
f^D	Fixed production costs for Home firms	0.10	Bernard et al. (2007) : 5% of sunk entry costs
f^I	Fixed production costs for FDI firms	0.11	$\frac{f^I}{f^D} = 1.1150$, the ratio of tangible assets between FDI and local firms
F^{D*}	Fixed entry costs for Foreign firms	2.00	The same value with F^D is assumed.
f^{D*}	Fixed production costs for Foreign firms	0.10	Bernard et al. (2007) : 5% of sunk entry costs
τ^D_C	Wedge on profits of Home local firms	0.67	33% corporate tax on profits of local firms
τ^D_V	Wedge on revenues of Home local firms	0.83	17% value added tax on local firms
τ^I_C	Wedge on profits of Home FDI firms	0.85	15% corporate tax on profits of FDI firms
τ^I_V	Wedge on revenues of Home FDI firms	0.85	15% value added tax on FDI firms

Table 5: Empirical Target Moments for Calibration

Year	$\frac{\text{Total Exports}}{\text{Total Imports}}$	$\frac{\text{Total Exports}}{\text{Total Domestic Sales}}$	$\frac{\text{Total Imports}}{\text{Total Production}}$	$\frac{\text{Total FDI Production}}{\text{Total Production}}$ (Foreign Capital $\geq$ 10%)	$\frac{\text{Total FDI Production}}{\text{Total Production}}$ (Foreign Capital $\geq$ 25%)
2000	1.184	0.2333	0.163	0.388	0.343
2001	1.171	0.2229	0.144	0.400	0.364
2002	1.187	0.2422	0.141	0.411	0.376
2003	1.070	0.2527	0.156	0.432	0.401
2004	1.007	0.2911	0.160	0.468	0.441
2005	1.066	0.2809	0.154	0.461	0.430
2006	1.224	0.3010	0.321	0.471	0.443
2007	1.275	0.2857	0.288	0.457	0.430
Model	1.1840	0.2391	0.1629	0.3881	

All data are from manufacturing sectors. Exports and imports are from Chinese Customs data. Total domestic sales, total production, and FDI production are from Chinese firm data.

Figure 1: Sharp Increase of FDI Flows in China after 2001



Note: Real FDI is deflated by the price index of capital formation.

Figure 2: Model Structure

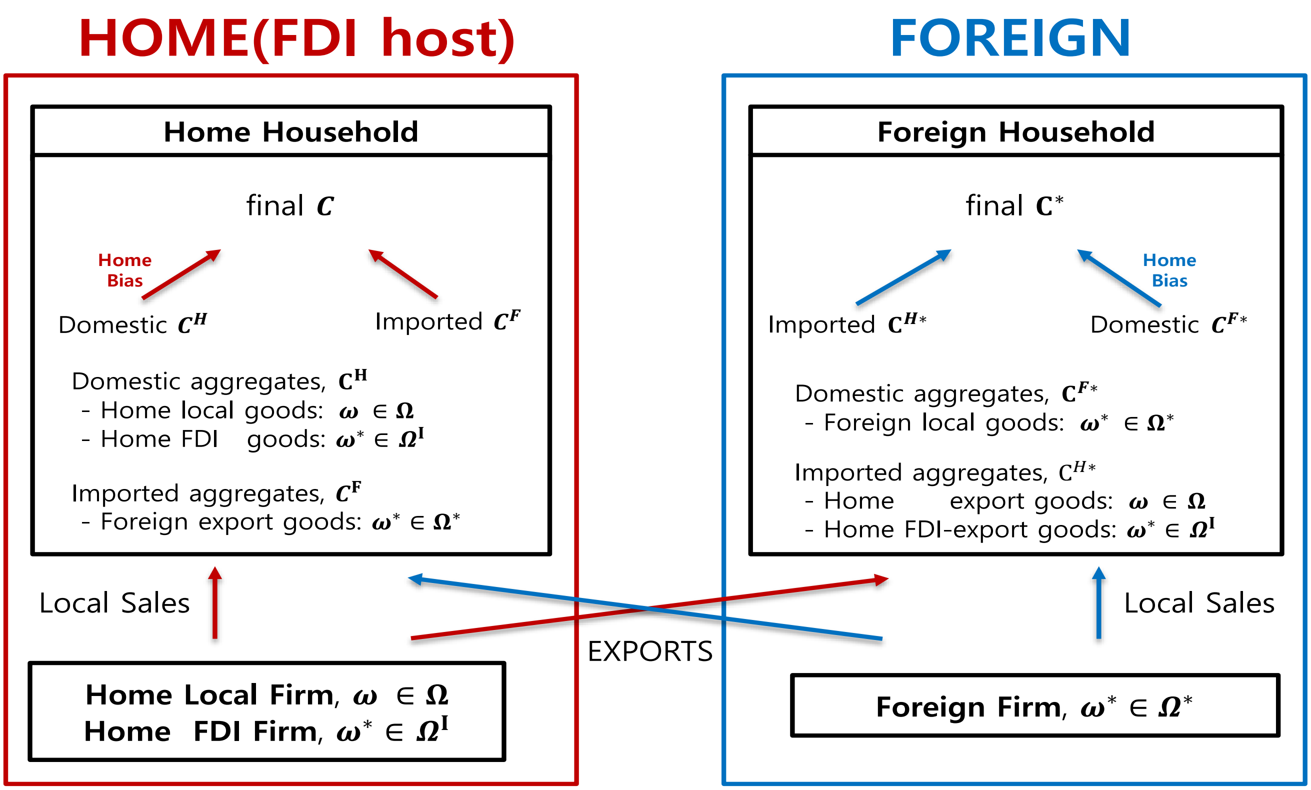


Figure 3: Firm Distribution and Cutoff Productivity

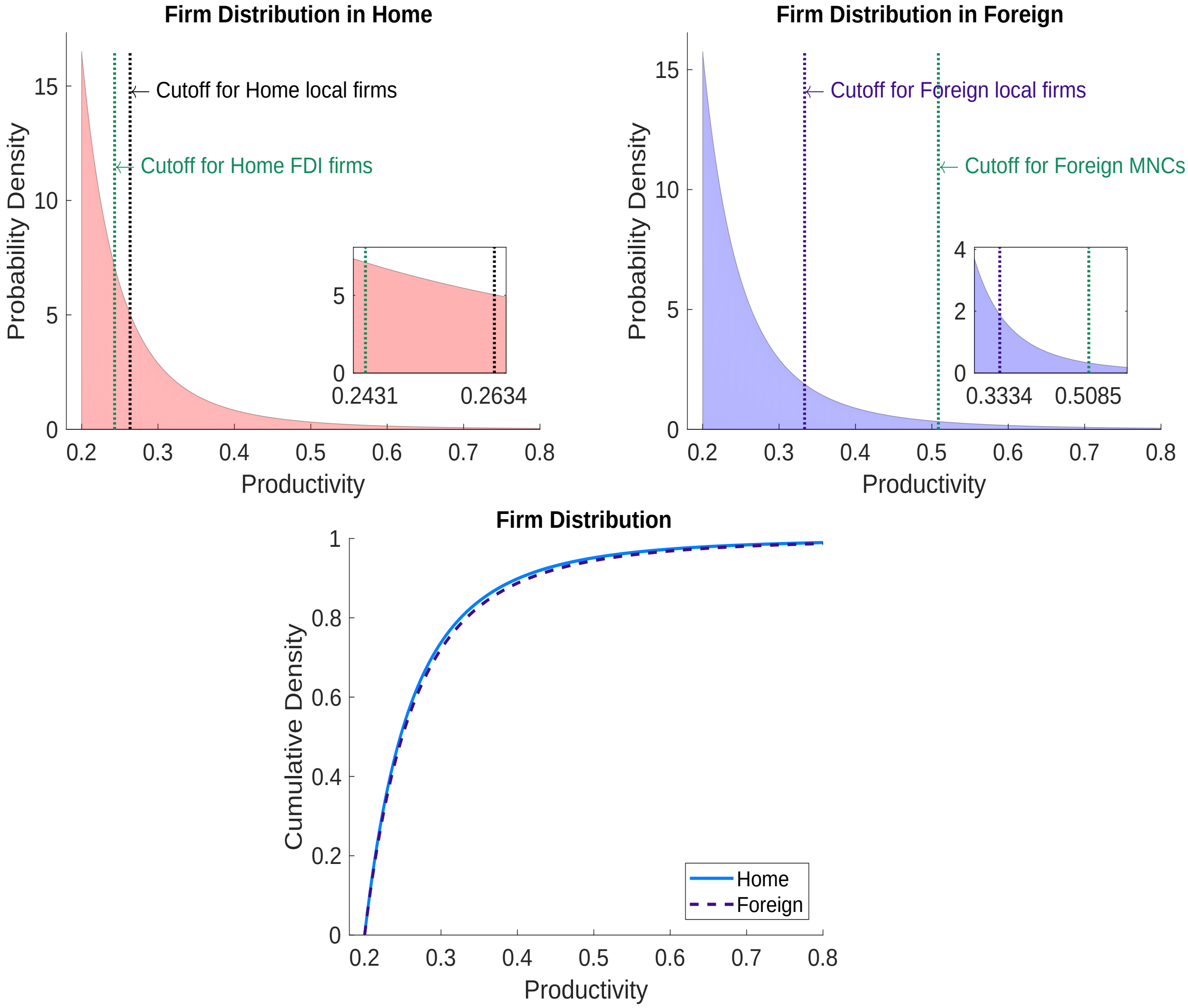


Figure 4: Value-Added Tax Reform with varying τ_V^D under $\lambda = 0.70$, $\tau_C^D = 0.67$, and $\tau_C^I = 0.85$

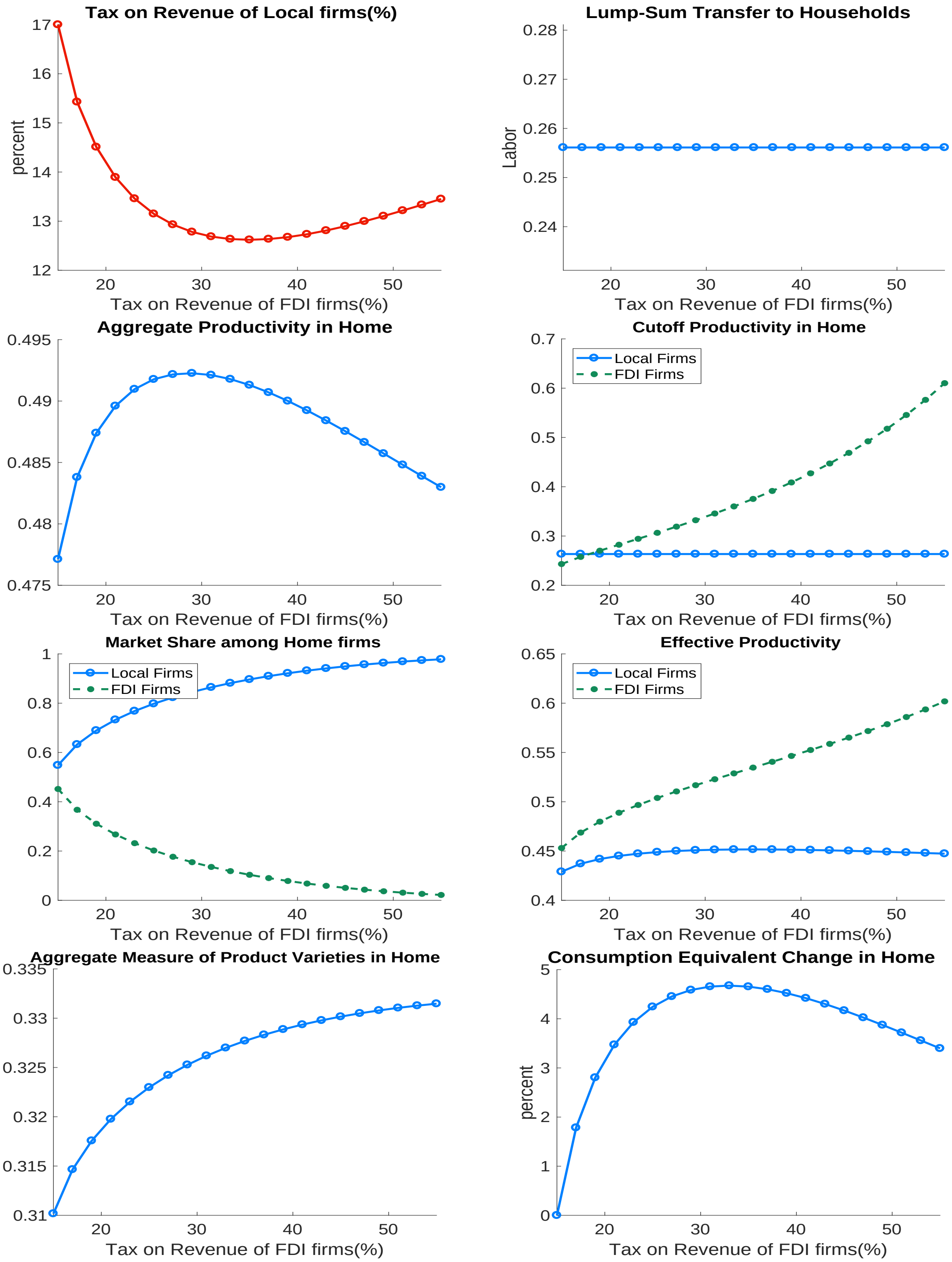
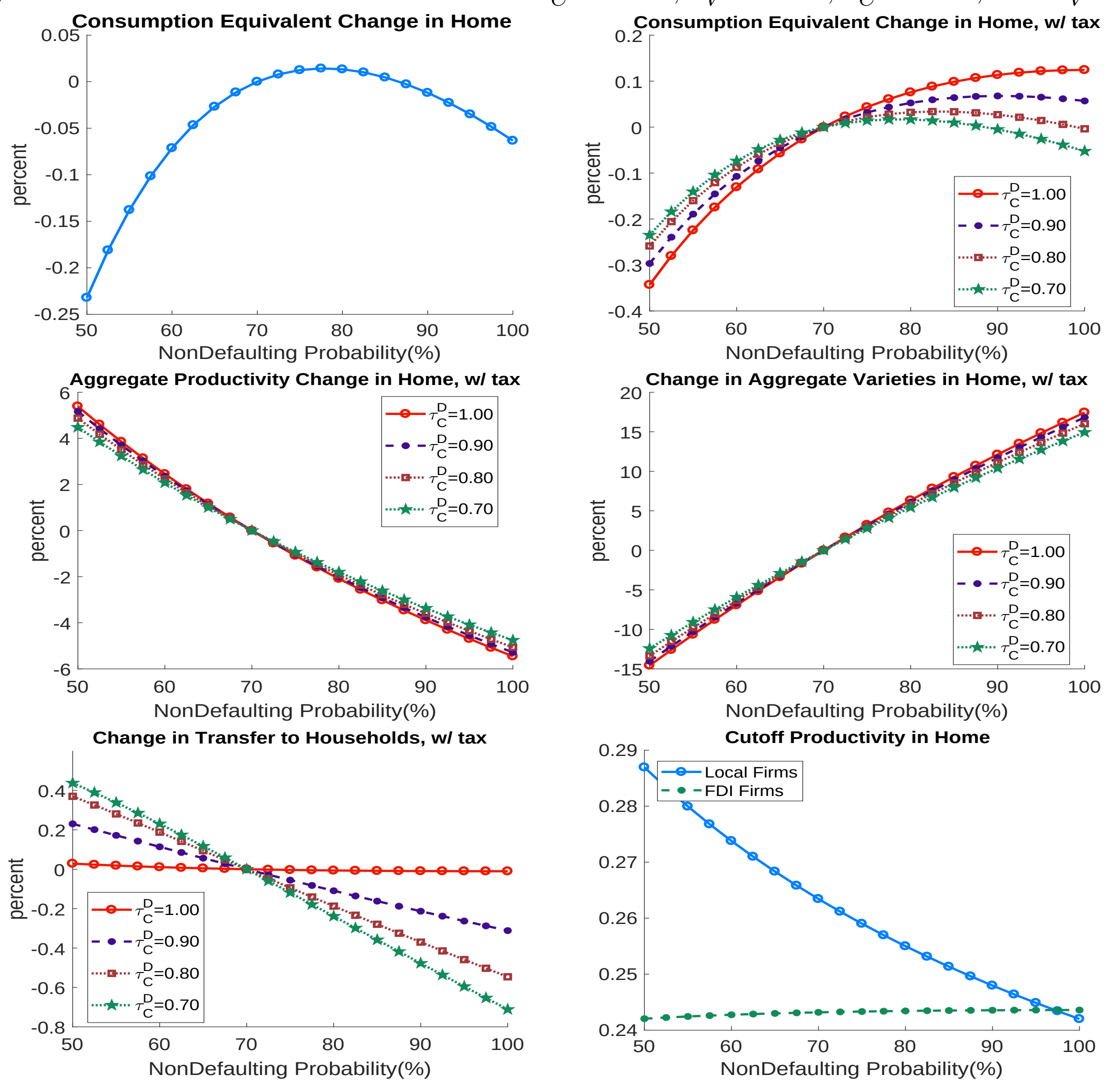


Figure 5: Financial Market Reform under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$



Appendix (not for publication)

A.1 Data

The ASIP dataset covers all state-owned manufacturing firms and private manufacturing firms with sales greater than 5 million RMB (approximately 600,000 dollars at the exchange rate of 2000) between 2000 and 2007. On average, there are 120,000 firm-level observations each year. The firm-level data include some basic firm information such as firm identification number, registration type, start year, operating status and total employment. In addition, the dataset contains detailed information about each firm's balance sheet and income statement. The balance sheet data report detailed information about assets and liabilities such as total assets, fixed assets, current assets, long-run investment, total liabilities, total equities and capital. Capital information include disaggregate-level information about the ownership of capital (e.g., government collective, corporate, special districts, foreign). So we can use such information to calculate the FDI share of each firm, which is measured by the share of capital from Hong Kong, Macau, Taiwan and foreign countries.

The data of income statement include each firm's total sales, total industry production, value added, export volume, income from main product, cost from main product, financing cost, interest cost, tax, wage, employee benefit, total intermediate input, total profit, etc. The above data are used to calculate the productivity of each firm. We will describe the method of calculating firm productivity shortly.

The dataset contains the location information of the firm that enables us to find out if it is in a special economic development zone. A 4-digit Chinese industry code is also provided for each firm, which is used to match firm with sector-level financial vulnerability measures.

We obtain the following industry-level and province-level data from China Statistic Yearbook: : industry PPI and province-level variables (GDP, GDP per capital, retail sale, transportation, investment, R&D, import and export).

A.1.1 Firm Productivity

Firm productivity is calculated following Ackerberg et al. (2015) and re-scaled around industry productivity mean and divided by industry productivity standard deviation. The method of Ackerberg et al. (2015) uses the ideas in Olley and Pakes (1996) and Levinsohn and Petrin (2003) to identify firm's productivity, but does not suffer from the collinearity problems in the literature. Examples of using this method include Alfaro et al. (2013) and De Loecker and Warzynski (2012).

Consider the following production function for firm i in a given industry:

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \varepsilon_{it}, \tag{A.1.1}$$

where y_{it} is the log of output, k_{it} is the log of capital input and l_{it} is the log of labor input. These variables are observable to the econometrician. ω_{it} is the productivity shock that is observable to the firm, but unobservable to the econometrician. ε_{it} is the error term that is not predictable to the firm. OLS cannot be used to estimate equation (A.1.1) if the choice of k_{it} or l_{it} is a function of ω_{it}, which is likely to be true in reality. We follow Ackerberg et al. (2015) to solve this endogeneity issue.

First assume ω_{it} follow an exogenous first-order Markov process:

$$p(\omega_{it+1}|I_t) = p(\omega_{it+1}|\omega_t), \tag{A.1.2}$$

where I_t is firm i's information set at time t. It is further assumed that firm's intermediate input is determined after its choices of labor and capital input and the realization of ω_{it}. Suppose the demand for intermediate input takes the form of:

$$m_{it} = f_t(\omega_{it}, k_{it}, l_{it}). \tag{A.1.3}$$

It is assumed that f_t is monotonic in ω_{it}. Therefore, we can invert the input demand function to get ω_{it}:

$$\omega_{it} = f_t^{-1}(m_{it}, k_{it}, l_{it}). \tag{A.1.4}$$

Substitute equation (A.1.4) to (A.1.1), we have:

$$\begin{aligned} y_{it} &= \beta_k k_{it} + \beta_l l_{it} + f_t^{-1}(m_{it}, k_{it}, l_{it}) + \varepsilon_{it} \\ &= \Phi_t(m_{it}, k_{it}, l_{it}) + \varepsilon_{it}, \end{aligned}$$

where $\Phi_t(m_{it}, k_{it}, l_{it}) \equiv \beta_k k_{it} + \beta_l l_{it} + f_t^{-1}(m_{it}, k_{it}, l_{it})$. We employ a second-order approximation for $f_t^{-1}(m_{it}, k_{it}, l_{it})$. So the estimate of $\Phi_t(m_{it}, k_{it}, l_{it})$, $\widehat{\Phi}_t(m_{it}, k_{it}, l_{it})$, is obtained by regressing y_{it} on m_{it}, k_{it}, l_{it} and their second-order terms.[1]

Next, two moment conditions are employed to estimate β_k and β_l:

$$E\left[\xi_{it}\begin{pmatrix} k_{it} \\ l_{it} \end{pmatrix}\right] = 0, \tag{A.1.5}$$

where $\xi_{it} = \omega_t - E[\omega_t|\omega_{t-1}]$ is the innovation in ω_t. These two moment conditions are from the assumption that capital and labor inputs are chosen before the realization of ω_t.

To be specific, for given $\widehat{\beta}_k$ and $\widehat{\beta}_l$, we have:

$$\widehat{\omega}_{it} = \widehat{\Phi}_t(m_{it}, k_{it}, l_{it}) - \widehat{\beta}_k k_{it} - \widehat{\beta}_l l_{it}. \tag{A.1.6}$$

Then $\widehat{\xi}_{it}$ is obtained with an third-order approximation by regressing $\widehat{\omega}_{it}$ on $\widehat{\omega}_{it-1}$, $\widehat{\omega}_{it-1}^2$ and $\widehat{\omega}_{it-1}^3$. In the estimation, $\widehat{\beta}_k$ and $\widehat{\beta}_l$ are selected to minimize the sample analogue to the moment conditions in equation (A.1.5):

$$\min_{\widehat{\beta}_k, \widehat{\beta}_l} \Lambda = \frac{1}{T}\frac{1}{N}\sum_{t=1}^{T}\sum_{i=1}^{N} \widehat{\xi}_{it}(\widehat{\beta}_k, \widehat{\beta}_l)\begin{pmatrix} k_{it} \\ l_{it} \end{pmatrix}, \tag{A.1.7}$$

where T is the number of sample periods and N is the number of firms in the industry.

[1] Cross terms of these variables are also included in the regression.

In our exercise, we first group firms according to China's 2-digit industry code. For each industry, we follow the above procedure to estimate firm's productivity during the period 2000-2007 ($T = 8$). In this way, we allow β_k and β_l to vary across different industries, but remain constant over time.

In our estimation, k_{it} is measured by fixed capital reported in firm's balance sheet, l_{it} is measure by the total number of employees and m_{it} is measured by intermediate input reported in firm's income statement. Both fixed capital and intermediate input are deflated by industry-level PPI obtained from China Statistic Yearbook.

Given the estimated $\widehat{\beta}_k$ and $\widehat{\beta}_l$ from equation (A.1.7), we can calculate firm i's productivity in year t, $\widehat{\omega}_{it}$, from equation (A.1.6). Then $\widehat{\omega}_{it}$ is normalized around the industrial mean:

$$\widetilde{\omega}_{it} = \frac{\widehat{\omega}_{it} - \mu_t}{\sigma_t}, \tag{A.1.8}$$

where μ_t is the industrial mean of $\widehat{\omega}_{it}$ and σ_t is the standard deviation of $\widehat{\omega}_{it}$. $\widetilde{\omega}_{it}$ is our final measure of firm i's productivity in all our empirical exercises.

A.1.2 Financial vulnerability

We employ five measures for financial vulnerability at the sector level, following Manova et al. (2015). These five measures are described in Table 1 and are calculated from data on all publicly traded U.S.-based firms.[2] The use of the U.S. data ensures that the financial vulnerability measures are not endogenously determined by China's level of financial development. Indeed, these measures are intended to capture features inherent to the nature of the manufacturing process, which remain the same across countries and are beyond the control of individual firms. Consistent with this argument, the measures display more cross-sector variations than cross-firm variations within a sector. Each financial vulnerability variable

[2]The raw data on U.S. firms are obtained from Compustat's annual industrial files.

is measured by the median among all firms in the sector and are available for 3-digit ISIC sectors. We will describe later how to match these 3-digit ISIC data with the 4-digit Chinese industry code in our dataset.

The first three measures use firm's dependence on external finance in a sector as proxy for the sector's liquidity constraint. The first measure is the share of capital expenditure that is not financed by operation cash flow, which we refer to as external finance dependence. The other two measures are the share of R&D in total sales and the share of inventory to sales, which we refer to as the inventory ratio and R&D ratio. Capital expenditure, R&D investment and inventory are important up-front costs and may reflect a firm's liquidity constraint. While companies in all industries may have to pay fixed costs and face liquidity constraints, the relative importance of such costs varies systematically across sectors. The above three measure can hopefully captures the systematical differences across sectors.

The fourth measure considers other sources of external finance that are in the form of trade credit. If a firm has access to buyer or seller trade credit, it is less dependent on the formal financial market and hence less financially constrained. This financial vulnerability variable is measured by the ratio of the change in account payable to the change in total asset.

The last measure of financial vulnerability, asset tangibility, captures firm's ability to raise external finance. Tangible assets can usually serve as collateral for external finance. Therefore, firms with a higher share of tangible assets (defined as the ratio of net plant, property and equipment to total book value assets) are less financially constrained.

Following Manova et al. (2015) and other studies in the literature, we obtain the external finance, inventory ratio, R&D ratio and asset tangibility from Kroszner et al. (2007), who follow the methodology of Rajan and Zingales (1998) and Claessens and Laeven (2003). They are averages over the 1980-1999 period for the median U.S. firms in each sector. Trade credit measure is obtained from Fisman and Love (2003), who calculate from the same data for 1980-1989.

The five measures are not highly correlated indicating that they capture conceptionally different dimensions of financial vulnerability. Following Manova et al. (2015), we calculate the first principal component (FPC) of the five indicators and use it as our preferred proxy for sector's financial vulnerability. Manova et al. (2015) argue that FPC provides a cleaner index of financial vulnerability than each individual measure because the individual measures might be correlated with industrial characteristics unrelated to financial frictions. The FPC index has a positive loading on external finance, the inventory ratio, and the R&D ratio, but a negative loading on asset tangibility and trade credit. This is the consistent with the intuitions we discussed above. In the end, FPC accounts for 45.9% of variance for all five measures.

Table A.1: The Elasticity of Productivity w.r.t.FDI for new firms (age$\leq$ 2) in quantile regressions

	Low financial vulnerability			High financial vulnerability		
Quantile (%)	Coef.	s.e.	Psudo-R^2	Coef.	s.e.	Psudo-R^2
5	$-0{\cdot}043$	0·040	0·185	$-0{\cdot}181^{***}$	0·023	0·100
10	$-0{\cdot}007$	26·000	0·197	$-0{\cdot}122^{***}$	0·023	0·100
15	0·029	0·022	0·199	$-0{\cdot}076^{***}$	0·026	0·099
20	0·015	0·018	0·195	$-0{\cdot}043^{**}$	0·017	0·098
25	0·005	0·021	0·190	$-0{\cdot}012$	0·018	0·098
50	$0{\cdot}083^{***}$	0·022	0·168	$0{\cdot}086^{***}$	0·009	0·093
75	$0{\cdot}149^{***}$	0·020	0·149	0·110	0·011	0·090

Note: The financial vulnerability is measured by the first principle component (FPC). The low and high financial vulnerability refers to the bottom and top 25% of FPC, respectively. New firms are defined as the firms whose age equals two years or less. *, ** and *** denote the statistical significance at the 10%, 5% and 1% levels respectively.

Table A.2: The Elasticity of Productivity w.r.t.FDI for new firms (age≤ 4) in quantile regressions

	Low financial vulnerability			High financial vulnerability		
Quantile (%)	Coef.	s.e.	Psudo-R^2	Coef.	s.e.	Psudo-R^2
5	$-0{\cdot}002$	0·031	0·185	$-0{\cdot}130^{***}$	0·031	0·098
10	$0{\cdot}050^{**}$	0·024	0·198	$-0{\cdot}062^{***}$	0·019	0·102
15	$0{\cdot}051^{**}$	0·021	0·201	$-0{\cdot}028^{**}$	0·014	0·103
20	$0{\cdot}071^{***}$	0·018	0·199	$-0{\cdot}008$	0·011	0·103
25	$0{\cdot}071^{***}$	0·016	0·195	0·014	0·009	0·103
50	$0{\cdot}120^{***}$	0·013	0·177	$0{\cdot}092^{***}$	0·011	0·099
75	$0{\cdot}157^{***}$	0·017	0·157	$0{\cdot}124^{***}$	0·010	0·096

Note: The financial vulnerability is measured by the first principle component (FPC). The low and high financial vulnerability refers to the bottom and top 25% of FPC, respectively. New firms are defined as the firms whose age equals two years or less. *, ** and *** denote the statistical significance at the 10%, 5% and 1% levels respectively.

A.2 The Two-Country Model of FDI under Financial Frictions

Households:

$$\max \quad V = C = \Phi C_0^{\theta_0} C_1{}^{\theta}$$
$$s.t. \quad PC = WL + T$$

where $\Phi = \theta_0^{-\theta_0}\theta^{-\theta}$ and $\theta_0 + \theta = 1$. C_0 denotes the consumption on the homogeneous good which is produced in the perfectly competitive industry with constant return to scale technology. When θ_0 is zero, then our framework will be a general equilibrium, otherwise, it is a partial equilibrium where real exchange rate is unity. Home efficiency conditions are

$$\begin{aligned}
C_1 &= \left(\frac{P_1}{P}\right)^{-1} \theta C \\
P &= (P_0)^{\theta_0} {P_1}^{\theta} \\
PC &= P_0 C_0 + P_1 C_1 = WL + T
\end{aligned}$$

Symmetrically, Foreign efficiency conditions are:

$$\begin{aligned}
C_1^* &= \left(\frac{P_1^*}{P^*}\right)^{-1} \theta C^* \\
P^* &= (P_0^*)^{\theta_0} {P_1^*}^{\theta} \\
P^* C^* &= P_0^* C_0^* + P_1^* C_1^* = W^* L^* + T^*
\end{aligned}$$

The Industry for the Homogeneous Good: The industry for the homogenous good is perfectly competitive and uses CRTS technology: it requires one unit of labor to produce one unit of the good. Producers do local currency pricing.

Home country:

$$\begin{aligned}
\max \quad & P_0 Y_0 + \epsilon P_0^* Y_0^X - W L_0 \\
s.t. \quad & Y_0 + Y_0^X = L_0
\end{aligned}$$

where ϵ is the nominal exchange rate: units of Home currency per one unit of Foreign currency.

Foreign country:

$$\begin{aligned}
\max \quad & P_0^* Y_0^* + \frac{1}{\epsilon} P_0 Y_0^{X*} - W^* L_0^* \\
s.t. \quad & Y_0^* + Y_0^{X*} = L_0^*
\end{aligned}$$

Demand and equilibrium conditions are given by:

$$
\begin{aligned}
C_0 &= Y_0 + Y_0^{X*} \\
C_0^* &= Y_0^* + Y_0^X \\
C_0 + C_0^* &= L_0 + L_0^* \\
W &= P_0 = \epsilon P_0^* \\
W^* &= P_0^* = \frac{1}{\epsilon} P_0
\end{aligned}
$$

We take the homogeneous good as a numéraire, and $W = \epsilon W^* = P_0 = \epsilon P_0^*$ holds if the weight on the homogeneous good is not zero: $\theta_0 > 0$. This implies Home(Foreign) labor is also the numéraire. If the world were a currency union, then the nominal exchange rate would be unity: $\epsilon = 1$. We define two kinds of real exchange rate: one is the labor real exchange rate Q_L as units of home labor in exchange for one unit of foreign labor $Q_L \equiv \epsilon \frac{W^*}{W}$ and the other is the consumption real exchange rate Q as units of home consumption basket in exchange for one unit of foreign consumption basket $Q \equiv \epsilon \frac{P^*}{P}$. Therefore, $Q = Q_L \frac{\frac{W}{P}}{\frac{W^*}{P^*}}$ or $Q_L = Q \frac{\frac{P}{W}}{\frac{P^*}{W^*}}$ hold.

Demand on differentiated goods in Home country:

The model features consumption home bias. Households in Home prefer domestically-produced goods C^H to imported goods C^F, which is captured by the parameter ν.

$$
C_1 = \frac{\left(C^H\right)^{\nu} \left(C^F\right)^{1-\nu}}{(\nu)^{\nu} (1-\nu)^{1-\nu}}
$$

Then the efficiency conditions are

$$\begin{aligned}
C^H &= \left(\frac{P^H}{P_1}\right)^{-1} \nu C_1 = \left(\frac{P^H}{P}\right)^{-1} \nu\theta C \\
C^F &= \left(\frac{P^F}{P_1}\right)^{-1} (1-\nu) C_1 = \left(\frac{P^F}{P}\right)^{-1} (1-\nu)\theta C \\
P_1 &= \left(P^H\right)^{\nu} \left(P^F\right)^{1-\nu} \\
P_1 C_1 &= P^H C^H + P^F C^F
\end{aligned}$$

where we have $\frac{P_1}{P} C_1 = \frac{P^H}{P} C^H + \frac{P^F}{P} C^F = \nu\theta C + (1-\nu)\theta C = \theta C$.

The composite consumption of domestic products in Home, C^H, is comprised of those products made by domestic firms and by FDI firms. We can find Hicksian demand by solving

$$\begin{aligned}
\min \quad & \int_{\omega\in\Omega} p^D(\omega) y^D(\omega) d\omega + \int_{\omega^*\in\Omega^I} p^I(\omega^*) y^I(\omega^*) d\omega^* \\
s.t. \quad & \left(\int_{\omega\in\Omega} \left[y^D(\omega)\right]^{\rho} d\omega + \int_{\omega^*\in\Omega^I} \left[y^I(\omega^*)\right]^{\rho} d\omega^*\right)^{\frac{1}{\rho}} \geq C^H
\end{aligned}$$

Then efficiency conditions are

$$\begin{aligned}
\left\{P^H\right\}^{1-\sigma} &= \int_{\omega\in\Omega} \left(p^D(\omega)\right)^{1-\sigma} d\omega + \int_{\omega^*\in\Omega^I} \left(p^I(\omega^*)\right)^{1-\sigma} d\omega^* \\
y^D(\omega) &= \left(\frac{p^D(\omega)}{P^H}\right)^{-\sigma} C^H = \left(\frac{p^D(\omega)}{P}\right)^{-\sigma} \left(\frac{P^H}{P}\right)^{\sigma-1} \nu\theta C \\
y^I(\omega^*) &= \left(\frac{p^I(\omega^*)}{P^H}\right)^{-\sigma} C^H = \left(\frac{p^I(\omega^*)}{P}\right)^{-\sigma} \left(\frac{P^H}{P}\right)^{\sigma-1} \nu\theta C \\
P^H C^H &= \int_{\omega\in\Omega} p^D(\omega) y^D(\omega) d\omega + \int_{\omega^*\in\Omega^I} p^I(\omega^*) y^I(\omega^*) d\omega^*
\end{aligned}$$

where we define $\sigma \equiv \frac{1}{1-\rho}$, $1-\sigma = \frac{-\rho}{1-\rho}$, and $\rho = \frac{\sigma-1}{\sigma}$ with $\sigma > 1$ and $0 < \rho < 1$. The composite consumption of products imported from Foreign under producer-currency pricing is given by

$$\begin{aligned}
\min \quad & \int_{\omega^*\in\Omega^*} \epsilon p^{D*}(\omega^*) y^{D,X*}(\omega^*) d\omega^* \\
s.t. \quad & \left(\int_{\omega^*\in\Omega^*} \left[y^{D,X*}(\omega^*)\right]^{\rho} d\omega^*\right)^{\frac{1}{\rho}} \geq C^F
\end{aligned}$$

Efficiency conditions are

$$\begin{aligned}
\left\{P^F\right\}^{1-\sigma} &= \int_{\omega^*\in\Omega^*} \left(\epsilon p^{D*}(\omega^*)\right)^{1-\sigma} d\omega^* \\
y^{D,X*}(\omega^*) &= \left(\frac{\epsilon p^{D*}(\omega^*)}{P^F}\right)^{-\sigma} C^F = \left(\frac{\epsilon p^{D*}(\omega^*)}{P}\right)^{-\sigma} \left(\frac{P^F}{P}\right)^{\sigma-1} (1-\nu)\theta C \\
P^F C^F &= \int_{\omega^*\in\Omega^*} \epsilon p^{D*}(\omega^*) y^{D,X*}(\omega^*) d\omega^*
\end{aligned}$$

where we define $\sigma \equiv \frac{1}{1-\rho}$, $1-\sigma = \frac{-\rho}{1-\rho}$, and $\rho = \frac{\sigma-1}{\sigma}$ with $\sigma > 1$ and $0 < \rho < 1$.

Demand on differentiated goods in Foreign country: Likewise, households in Foreign are also more inclined to consume domestic goods C^{F*} than imported goods C^{H*}, which is captured by the parameter ν^*.

$$C_1^* = \frac{\left(C^{F*}\right)^{\nu^*}\left(C^{H*}\right)^{1-\nu^*}}{\left(\nu^*\right)^{\nu^*}\left(1-\nu^*\right)^{1-\nu^*}}$$

Then efficiency conditions are:

$$\begin{aligned}
C^{F*} &= \left(\frac{P^{F*}}{P_1^*}\right)^{-1} \nu^* C_1^* = \left(\frac{P^{F*}}{P^*}\right)^{-1} \nu^* \theta C^* \\
C^{H*} &= \left(\frac{P^{H*}}{P_1^*}\right)^{-1} (1-\nu^*) C_1^* = \left(\frac{P^{H*}}{P^*}\right)^{-1} (1-\nu^*)\theta C^* \\
P_1^* &= \left(P^{F*}\right)^{\nu^*}\left(P^{H*}\right)^{1-\nu^*} \\
P_1^* C_1^* &= P^{F*}C^{F*} + P^{H*}C^{H*}
\end{aligned}$$

where we have $\frac{P_1^*}{P^*}C_1^* = \frac{P^{F*}}{P^*}C^{F*} + \frac{P^{H*}}{P^*}C^{H*} = \nu^*\theta C^* + (1-\nu^*)\theta C^* = \theta C^*$.

The composite consumption of domestic products in Foreign, C^{F*}, is comprised of those products made by Foreign domestic firms. We can find Hicksian demand by solving

$$\begin{aligned}
\min \quad & \int_{\omega^*\in\Omega^*} p^{D*}(\omega^*) y^{D*}(\omega^*) d\omega^* \\
s.t. \quad & \left(\int_{\omega^*\in\Omega^*} \left[y^{D*}(\omega^*)\right]^{\rho} d\omega^*\right)^{\frac{1}{\rho}} \geq C^{F*}
\end{aligned}$$

Efficiency conditions are:

$$
\begin{aligned}
{P^{F*}}^{1-\sigma} &= \int_{\omega^* \in \Omega^*} \left(p^{D*}(\omega^*)\right)^{1-\sigma} d\omega^* \\
y^{D*}(\omega^*) &= \left(\frac{p^{D*}(\omega^*)}{P^{F*}}\right)^{-\sigma} C^{F*} = \left(\frac{p^{D*}(\omega^*)}{P^*}\right)^{-\sigma} \left(\frac{P^{F*}}{P^*}\right)^{\sigma-1} \nu^* \theta C^* \\
P^{F*} C^{F*} &= \int_{\omega^* \in \Omega^*} p^{D*}(\omega^*) y^{D*}(\omega^*) d\omega^*
\end{aligned}
$$

where we define $\sigma \equiv \frac{1}{1-\rho}$, $1-\sigma = \frac{-\rho}{1-\rho}$, and $\rho = \frac{\sigma-1}{\sigma}$ with $\sigma > 1$ and $0 < \rho < 1$.

The composite consumption of products imported from Home under producer-currency pricing is also defined by the standard CES aggregator and Hicksian demand can be derived by solving

$$
\begin{aligned}
\min \quad & \int_{\omega \in \Omega} \frac{1}{\epsilon} p^D(\omega) y^{D,X}(\omega) d\omega + \int_{\omega^* \in \Omega^I} \frac{1}{\epsilon} p^I(\omega^*) y^{I,X}(\omega^*) d\omega^* \\
s.t. \quad & \left(\int_{\omega \in \Omega} \left[y^{D,X}(\omega)\right]^\rho d\omega + \int_{\omega^* \in \Omega^I} \left[y^{I,X}(\omega^*)\right]^\rho d\omega^*\right)^{\frac{1}{\rho}} \geq C^{H*}
\end{aligned}
$$

Therefore, efficiency conditions are

$$
\begin{aligned}
{P^{H*}}^{1-\sigma} &= \int_{\omega \in \Omega} \left(\frac{p^D(\omega)}{\epsilon}\right)^{1-\sigma} d\omega + \int_{\omega^* \in \Omega^I} \left(\frac{p^I(\omega^*)}{\epsilon}\right)^{1-\sigma} d\omega^* \\
y^{D,X}(\omega) &= \left(\frac{p^D(\omega)}{\epsilon P^{H*}}\right)^{-\sigma} C^{H*} = \left(\frac{p^D(\omega)}{\epsilon P^*}\right)^{-\sigma} \left(\frac{P^{H*}}{P^*}\right)^{\sigma-1} (1-\nu^*)\theta C^* \\
y^{I,X}(\omega^*) &= \left(\frac{p^I(\omega^*)}{\epsilon P^{H*}}\right)^{-\sigma} C^{H*} = \left(\frac{p^I(\omega^*)}{\epsilon P^*}\right)^{-\sigma} \left(\frac{P^{H*}}{P^*}\right)^{\sigma-1} (1-\nu^*)\theta C^* \\
P^{H*} C^{H*} &= \int_{\omega \in \Omega} \frac{p^D(\omega)}{\epsilon} y^{D,X}(\omega) d\omega + \int_{\omega^* \in \Omega^I} \frac{p^I(\omega^*)}{\epsilon} y^{I,X}(\omega^*) d\omega^*
\end{aligned}
$$

where we define $\sigma \equiv \frac{1}{1-\rho}$, $1-\sigma = \frac{-\rho}{1-\rho}$, and $\rho = \frac{\sigma-1}{\sigma}$ with $\sigma > 1$ and $0 < \rho < 1$.

Domestic Firms under imperfect financial markets in Home:

$$
\begin{aligned}
\pi^D(z) &= \max\ \tau_C^D \left[\begin{array}{ll} \tau_V^D p^D(z)\left(y^D(z)+y^{D,X}(z)\right) & -\tau_L^D W l^D(z) - f^D W + \zeta f^D W \\ & -\lambda x(z) - (1-\lambda)\chi F^D W \end{array} \right] \\
s.t. \quad & \tau_V^D p^D(z)\left(y^D(z)+y^{D,X}(z)\right) - \tau_L^D W l^D(z) - f^D W + \zeta f^D W \geq x(z) \\
& \lambda x(z) + (1-\lambda)\chi F^D W \geq \zeta f^D W \\
& l^D(z) = \frac{y^D(z)+y^{D,X}(z)}{z} \\
& y^D(z) = \left(\frac{p^D(z)}{W}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} \\
& y^{D,X}(z) = \left(\frac{p^D(z)}{W}\right)^{-\sigma} {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}
\end{aligned}
$$

Solving the maximization problem delivers equilibrium conditions:

$$
\begin{aligned}
\frac{p^D(z)}{W} &= \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)\frac{1}{z} \\
l^D(z) &= z^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \frac{\frac{r^D(z)}{W}}{\tau_V^D\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)} = \frac{\rho}{\tau_L^D}\frac{r^D(z)}{W} \\
\frac{r^D(z)}{W} &\equiv \tau_V^D\frac{p^D(z)}{W}\left(y^D(z)+y^{D,X}(z)\right) = z^{\sigma-1}\tau_V^D\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \sigma\left[\frac{1}{\tau_C^D}\frac{\pi^D(z)}{W} + f^D\right] \\
\frac{\pi^D(z)}{W} &= \tau_C^D\left[\tau_V^D\frac{p^D(z)}{W}\left(y^D(z)+y^{D,X}(z)\right) - \tau_L^D l^D(z) - f^D\right] = \tau_C^D\left[\frac{\tau_V^D}{\sigma}\frac{p^D(z)}{W}\left(y^D(z)+y^{D,X}(z)\right) - f^D\right] \\
&= \tau_C^D\left[\frac{1}{\sigma}\frac{r^D(z)}{W} - f^D\right] = \tau_C^D\left[\frac{\tau_L^D}{\sigma-1}l^D(z) - f^D\right] \\
\frac{\xi^D(z)}{W} &= \tau_C^D\frac{r^D(z)}{W} - \frac{\pi^D(z)}{W} = \tau_C^D\left[\frac{\sigma-1}{\sigma}\frac{r^D(z)}{W} + f^D\right]
\end{aligned}
$$

where $\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] = \left(\frac{W}{P}\right)^{-\sigma}\left[\left(\frac{P^H}{P}\right)^{\sigma-1}\nu\theta C + Q^{\sigma}\left(\frac{P^{H*}}{P^*}\right)^{\sigma-1}(1-\nu^*)\theta C^*\right]$. From the firm's participation constraint, we can get the cutoff productivity Z^D:

$$\begin{aligned}
\frac{r^D(Z^D)}{W} - \tau_L^D l^D(Z^D) &= \frac{1}{\sigma}\frac{r^D(Z^D)}{W} \\
&= (Z^D)^{\sigma-1}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \frac{x(z)}{W} + f^D - \zeta f^D \\
&= \left[f^D + \left(\frac{1}{\lambda}-1\right)(\zeta f^D - \chi F^D)\right]
\end{aligned}$$

That is,

$$\begin{aligned}
\frac{\pi^D(Z^D)}{W} &= \tau_C^D\left[\frac{1}{\sigma}\frac{r^D(Z^D)}{W} - f^D\right] \\
&= \tau_C^D\left(\frac{1}{\lambda}-1\right)(\zeta f^D - \chi F^D)
\end{aligned}$$

Note that we have nonzero ex-ante profit for the marginal firm of productivity Z^D: $\frac{\pi^D(Z^D)}{W} > 0$. Instead, after the defaulting shock with probability $(1-\lambda)$ is realized, the ex-post profit of the firm becomes zero if the firm turns out not to default.

Foreign firms under no financial frictions in Foreign:

When Foreign firms establish FDI subsidiaries, their productivity in the host country is exogenously reduced by a factor of $\alpha \in (0,1]$: $g = \alpha z$, where z is drawn from the distribution $G^*(z)$. Aggregate profit across all Foreign firms in terms of Foreign currency is given by

$$\int_{\omega^*\in\Omega^*}\pi^{D*}(z)d\omega^* + \int_{\omega^*\in\Omega^I}\frac{1}{\epsilon}\pi^I(\alpha z)d\omega^*$$

(1) Domestic Sales and Exporting of Foreign firms

$$\begin{aligned}
\pi^{D*}(z) &= \max \ \tau_C^{D*}\left[\tau_V^{D*}p^{D*}(z)\left(y^{D*}(z)+y^{D,X*}(z)\right)-\tau_L^{D*}W^* l^{D*}(z)-f^{D*}W^*\right] \\
s.t. \quad & l^{D*}(z)=\frac{y^{D*}(z)+y^{D,X*}(z)}{z} \\
& y^{D*}(z)=\left(\frac{p^{D*}(z)}{W^*}\right)^{-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*} \\
& y^{D,X*}(z)=\left(\frac{p^{D*}(z)}{W^*}\right)^{-\sigma}Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}
\end{aligned}$$

Note that $\sigma>1$ implies the good z is a substitute among intra-industry goods. Therefore, if the overall price level $\frac{P^{F*}}{P^*}$ increases, this means other intra-industry differentiated goods become more expensive and so the demand on the good z increases. Then equilibrium conditions are:

$$\begin{aligned}
\frac{p^{D*}(z)}{W^*} &= \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)\frac{1}{z} \\
l^{D*}(z) &= z^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma}\left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}+Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right] \\
&= \frac{\frac{r^{D*}(z)}{W^*}}{\tau_V^{D*}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)}=\frac{\rho}{\tau_L^{D*}}\frac{r^{D*}(z)}{W^*} \\
\frac{r^{D*}(z)}{W^*} &\equiv \tau_V^{D*}\frac{p^{D*}(z)}{W^*}\left(y^{D*}(z)+y^{D,X*}(z)\right) \\
&= z^{\sigma-1}\tau_V^{D*}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}+Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right] \\
&= \sigma\left[\frac{1}{\tau_C^{D*}}\frac{\pi^{D*}(z)}{W^*}+f^{D*}\right] \\
\frac{\pi^{D*}(z)}{W^*} &\equiv \tau_C^{D*}\left[\tau_V^{D*}\frac{p^{D*}(z)}{W^*}\left(y^{D*}(z)+y^{D,X*}(z)\right)-\tau_L^{D*}l^{D*}(z)-f^{D*}\right] \\
&= \tau_C^{D*}\left[\frac{\tau_V^{D*}}{\sigma}\frac{p^{D*}(z)}{W^*}\left(y^{D*}(z)+y^{D,X*}(z)\right)-f^{D*}\right] \\
&= \tau_C^{D*}\left[\frac{1}{\sigma}\frac{r^{D*}(z)}{W^*}-f^{D*}\right]=\tau_C^{D*}\left[\frac{\tau_L^{D*}}{\sigma-1}l^{D*}(z)-f^{D*}\right] \\
\frac{\xi^{D*}(z)}{W^*} &= \tau_C^{D*}\frac{r^{D*}(z)}{W^*}-\frac{\pi^{D*}(z)}{W^*}=\tau_C^{D*}\left[\frac{\sigma-1}{\sigma}\frac{r^{D*}(z)}{W^*}+f^{D*}\right]
\end{aligned}$$

where

$$
\begin{aligned}
& \left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1} \nu^* \theta \frac{P^* C^*}{W^*} + Q_L{}^{-\sigma} \left(\frac{P^F}{W}\right)^{\sigma-1} (1-\nu)\theta \frac{PC}{W}\right] \\
= & \left(\frac{W^*}{P^*}\right)^{-\sigma} \left[\left(\frac{P^{F*}}{P^*}\right)^{\sigma-1} \nu^* \theta C^* + Q^{-\sigma} \left(\frac{P^F}{P}\right)^{\sigma-1} (1-\nu)\theta C\right].
\end{aligned}
$$

From the firm's zero profit condition, we can get the cutoff productivity Z^{D*}:

$$
\begin{aligned}
\frac{r^{D*}(Z^{D*})}{W^*} - \tau_L^{D*} l^{D*}(Z^{D*}) &= \frac{1}{\sigma} \frac{r^{D*}(Z^{D*})}{W^*} \\
&= (Z^{D*})^{\sigma-1} \frac{\tau_V^{D*}}{\sigma} \left(\frac{1}{\rho} \frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} \left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1} \nu^* \theta \frac{P^* C^*}{W^*} + Q_L{}^{-\sigma} \left(\frac{P^F}{W}\right)^{\sigma-1} (1-\nu)\theta \frac{PC}{W}\right] \\
&= f^{D*}
\end{aligned}
$$

That is,

$$
\frac{\pi^{D*}(Z^{D*})}{W^*} = \tau_C^{D*} \left[\frac{1}{\sigma} \frac{r^{D*}(Z^{D*})}{W^*} - f^{D*}\right] = 0
$$

(2) FDI Subsidiaries of Foreign firms

When Foreign firms establish FDI subsidiaries, their productivity is exogenously reduced by a factor of $\alpha \in (0, 1]$: $g = \alpha z$, where z is drawn from the distribution $G^*(z)$.

$$
\begin{aligned}
\pi^I(g) \quad = \quad & \max \ \tau_C^I \left[\tau_V^I p^I(g) \left(y^I(g) + y^{I,X}(g)\right) - \tau_L^I W l^I(g) - f^I W\right] \\
s.t. \quad & l^I(g) = \frac{y^I(g) + y^{I,X}(g)}{g} \\
& y^I(g) = \left(\frac{p^I(g)}{W}\right)^{-\sigma} \left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta \frac{PC}{W} \\
& y^{I,X}(g) = \left(\frac{p^I(g)}{W}\right)^{-\sigma} Q_L{}^{\sigma} \left(\frac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \frac{P^* C^*}{W^*}
\end{aligned}
$$

Solving the maximization problem leads to equilibrium conditions:

$$\begin{aligned}
\frac{p^I(g)}{W} &= \left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)\frac{1}{g} \\
l^I(g) &= g^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \frac{\frac{r^I(g)}{W}}{\tau_V^I\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)} = \frac{\rho}{\tau_L^I}\frac{r^I(g)}{W} \\
\frac{r^I(g)}{W} &\equiv \tau_V^I\frac{p^I(g)}{W}\left(y^I(g)+y^{I,X}(g)\right) = g^{\sigma-1}\tau_V^I\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \sigma\left[\frac{1}{\tau_C^I}\frac{\pi^I(g)}{W}+f^I\right] \\
\frac{\pi^I(g)}{W} &\equiv \tau_C^I\left[\tau_V^I\frac{p^I(g)}{W}\left(y^I(g)+y^{I,X}(g)\right)-\tau_L^I l^I(g)-f^I\right] = \tau_C^I\left[\frac{\tau_V^I}{\sigma}\frac{p^I(g)}{W}\left(y^I(g)+y^{I,X}(g)\right)-f^I\right] \\
&= \tau_C^I\left[\frac{1}{\sigma}\frac{r^I(g)}{W}-f^I\right] = \tau_C^I\left[\frac{\tau_L^I}{\sigma-1}l^I(g)-f^I\right] \\
\frac{\xi^I(g)}{W} &= \tau_C^I\frac{r^I(g)}{W}-\frac{\pi^I(g)}{W} = \tau_C^I\left[\frac{\sigma-1}{\sigma}\frac{r^I(g)}{W}+f^I\right]
\end{aligned}$$

where

$$\begin{aligned}
&\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
=\ &\left(\frac{W}{P}\right)^{-\sigma}\left[\left(\frac{P^H}{P}\right)^{\sigma-1}\nu\theta C+Q^{\sigma}\left(\frac{P^{H*}}{P^*}\right)^{\sigma-1}(1-\nu^*)\theta C^*\right].
\end{aligned}$$

From the firm's zero profit condition, we can obtain the cutoff productivity Z^I:

$$\begin{aligned}
\frac{r^I(\alpha Z^I)}{W}-\tau_L^I l^I(\alpha Z^I) &= \frac{1}{\sigma}\frac{r^I(\alpha Z^I)}{W} \\
&= \left(\alpha Z^I\right)^{\sigma-1}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= f^I
\end{aligned}$$

That is,

$$\frac{\pi^I\left(\alpha Z^I\right)}{W} = \tau_C^I\left[\frac{1}{\sigma}\frac{r^I\left(\alpha Z^I\right)}{W}-f^I\right] = 0$$

Productivity Distribution:

Assume productivity of Home firms and Foreign firms follow Pareto distribution, given by:

$$G(z) = 1 - (z_{min})^{\eta} z^{-\eta}, \qquad G^*(z) = 1 - (z^*_{min})^{\eta^*} z^{-\eta^*}.$$

Define $J(z)$ and $J^*(z)$ as:

$$J(z) \equiv \int_z^\infty a^{\sigma-1} dG(a) = \underbrace{\frac{\eta (z_{min})^{\eta}}{\eta - \sigma + 1}}_{\equiv \widetilde{\eta}} \{z\}^{-\eta+\sigma-1}, \qquad J^*(z) \equiv \int_z^\infty a^{\sigma-1} dG^*(a) = \underbrace{\frac{\eta^* (z^*_{min})^{\eta^*}}{\eta^* - \sigma + 1}}_{\equiv \widetilde{\eta}^*} \{z\}^{-\eta^*+\sigma-1},$$

where $\eta > \sigma - 1$ and $\eta^* > \sigma - 1$ are required. In addition, we can compute:

$$\int_z^\infty a^{-1} dG(a) = \tfrac{\eta (z_{min})^{\eta}}{\eta+1} \{z\}^{-\eta-1}, \qquad \int_z^\infty a^{\sigma} dG(a) = \tfrac{\eta (z_{min})^{\eta}}{\eta-\sigma} \{z\}^{-\eta+\sigma},$$
$$\int_z^\infty a^{-1} dG^*(a) = \tfrac{\eta^* (z^*_{min})^{\eta^*}}{\eta^*+1} \{z\}^{-\eta^*-1}, \qquad \int_z^\infty a^{\sigma} dG^*(a) = \tfrac{\eta^* (z^*_{min})^{\eta^*}}{\eta^*-\sigma} \{z\}^{-\eta^*+\sigma},$$

where $\eta > \sigma$ and $\eta^* > \sigma$ are required. Define average productivity $\widetilde{Z}^D$, $\widetilde{Z}^I$, and $\widetilde{Z}^{D*}$ using cutoff productivity Z^D, Z^I, and Z^{D*}:

$$\begin{aligned}
\widetilde{Z}^D &\equiv \left[\int_{Z^D}^\infty z^{\sigma-1} \frac{dG(z)}{1 - G(Z^D)}\right]^{\frac{1}{\sigma-1}} = \left[\frac{J(Z^D)}{1 - G(Z^D)}\right]^{\frac{1}{\sigma-1}} = \left[\frac{\eta}{\eta - \sigma + 1}\right]^{\frac{1}{\sigma-1}} Z^D, \\
\widetilde{Z}^I &\equiv \left[\int_{Z^I}^\infty (\alpha z)^{\sigma-1} \frac{dG^*(z)}{1 - G^*(Z^I)}\right]^{\frac{1}{\sigma-1}} = \alpha \left[\frac{J^*(Z^I)}{1 - G^*(Z^I)}\right]^{\frac{1}{\sigma-1}} = \alpha \left[\frac{\eta^*}{\eta^* - \sigma + 1}\right]^{\frac{1}{\sigma-1}} Z^I, \\
\widetilde{Z}^{D*} &\equiv \left[\int_{Z^{D*}}^\infty z^{\sigma-1} \frac{dG^*(z)}{1 - G^*(Z^{D*})}\right]^{\frac{1}{\sigma-1}} = \left[\frac{J^*(Z^{D*})}{1 - G^*(Z^{D*})}\right]^{\frac{1}{\sigma-1}} = \left[\frac{\eta^*}{\eta^* - \sigma + 1}\right]^{\frac{1}{\sigma-1}} Z^{D*}.
\end{aligned}$$

That is,

$$\begin{aligned}
\frac{J(Z^D)}{1 - G(Z^D)} &= \left(\widetilde{Z}^D\right)^{\sigma-1} \text{ and } \left(\frac{\widetilde{Z}^D}{Z^D}\right)^{\sigma-1} = \int_{Z^D}^\infty \left(\frac{z}{Z^D}\right)^{\sigma-1} \frac{dG(z)}{1 - G(Z^D)} = \frac{\eta}{\eta - \sigma + 1}, \\
\frac{J^*(Z^I)}{1 - G^*(Z^I)} &= \left(\frac{\widetilde{Z}^I}{\alpha}\right)^{\sigma-1} \text{ and } \left(\frac{\widetilde{Z}^I}{Z^I}\right)^{\sigma-1} = \int_{Z^I}^\infty \left(\frac{\alpha z}{Z^I}\right)^{\sigma-1} \frac{dG^*(z)}{1 - G^*(Z^I)} = \frac{\eta^*}{\eta^* - \sigma + 1} \alpha^{\sigma-1}, \\
\frac{J^*(Z^{D*})}{1 - G^*(Z^{D*})} &= \left(\widetilde{Z}^{D*}\right)^{\sigma-1} \text{ and } \left(\frac{\widetilde{Z}^{D*}}{Z^{D*}}\right)^{\sigma-1} = \int_{Z^{D*}}^\infty \left(\frac{z}{Z^{D*}}\right)^{\sigma-1} \frac{dG^*(z)}{1 - G^*(Z^{D*})} = \frac{\eta^*}{\eta^* - \sigma + 1},
\end{aligned}$$

where

$$\int_{Z^I}^{\infty}(\alpha z)^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^I)}=\frac{\alpha^{\sigma-1}J^*(Z^I)}{1-G^*(Z^I)}=\left(\widetilde{Z}^I\right)^{\sigma-1}.$$

Therefore, even if the productivity of FDI subsidiaries is reduced by a factor α when it starts its business in the host country, it does not change the ex-post conditional probability density, $\frac{dG^*(z)}{1-G^*(Z^I)}$.

Free Entry Condition:

Incumbents might exit with exogenous probability δ. The expected life-time operating profit of a potential entrant should equal the entry costs. Home local firms equate their ex-ante expected profit to sunk entry costs.

$$\begin{aligned}\frac{W}{P}F^D &= \left(1-G(Z^D)\right)\begin{bmatrix} & \int_{Z^D}^{\infty}\frac{\pi^D(z)}{P}\frac{dG(z)}{(1-G(Z^D))} \\ +(1-\delta) & \begin{bmatrix} \int_{Z^D}^{\infty}\frac{\pi^D(z)}{P}\frac{dG(z)}{(1-G(Z^D))} \\ + \quad \cdots \end{bmatrix}\end{bmatrix} \\ &= \left(1-G(Z^D)\right)\sum_{t=0}^{\infty}(1-\delta)^t\int_{Z^D}^{\infty}\frac{\pi^D(z)}{P}\frac{dG(z)}{(1-G(Z^D))}=\left(\frac{1-G(Z^D)}{\delta}\right)\int_{Z^D}^{\infty}\frac{\pi^D(z)}{P}\frac{dG(z)}{(1-G(Z^D))}\end{aligned}$$

That is,

$$\begin{aligned}& F^D \\ =& \left(\frac{1-G(Z^D)}{\delta}\right)\int_{Z^D}^{\infty}\frac{\pi^D(z)}{W}\frac{dG(z)}{(1-G(Z^D))} \\ =& \left(\frac{1-G(Z^D)}{\delta}\right)\tau_C^D\left\{\frac{J(Z^D)}{1-G(Z^D)}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]-f^D\right\} \\ =& \left(\frac{1-G(Z^D)}{\delta}\right)\tau_C^D\left\{\left(\widetilde{Z}^D\right)^{\sigma-1}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]-f^D\right\} \\ =& \left(\frac{1-G(Z^D)}{\delta}\right)\frac{\pi^D\left(\widetilde{Z}^D\right)}{W}\end{aligned}$$

Therefore,

$$\frac{\pi^D(\widetilde{Z}^D)}{W} = \frac{\delta F^D}{1-G(Z^D)}$$

Likewise, Foreign firms equate their ex-ante expected profit to sunk entry costs.

$$\frac{W^*}{P^*}F^{D*} = \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\int_{Z^{D*}}^{\infty}\frac{\pi^{D*}(z)}{P^*}\frac{dG^*(z)}{1-G^*(Z^{D*})} + \left(\frac{1-G^*(Z^I)}{Q\cdot\delta}\right)\int_{Z^I}^{\infty}\frac{\pi^I(\alpha z)}{P}\frac{dG^*(z)}{1-G^*(Z^I)}$$

That is,

$$\begin{aligned}
& F^{D*} \\
= & \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\int_{Z^{D*}}^{\infty}\frac{\pi^{D*}(z)}{W^*}\frac{dG^*(z)}{1-G^*(Z^{D*})} + \left(\frac{1-G^*(Z^I)}{Q_L\cdot\delta}\right)\int_{Z^I}^{\infty}\frac{\pi^I(\alpha z)}{W}\frac{dG^*(z)}{1-G^*(Z^I)} \\
= & \left(\tfrac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}\left\{\tfrac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\tfrac{\tau_V^{D*}}{\sigma}\left(\tfrac{1}{\rho}\tfrac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left[\left(\tfrac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\tfrac{P^*C^*}{W^*} + {Q_L}^{-\sigma}\left(\tfrac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\tfrac{PC}{W}\right] - f^{D*}\right\} \\
& + \left(\tfrac{1-G^*(Z^I)}{Q_L\cdot\delta}\right)\tau_C^I\left\{\alpha^{\sigma-1}\tfrac{J^*(Z^I)}{1-G^*(Z^I)}\tfrac{\tau_V^I}{\sigma}\left(\tfrac{1}{\rho}\tfrac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left[\left(\tfrac{P^H}{W}\right)^{\sigma-1}\nu\theta\tfrac{PC}{W} + {Q_L}^{\sigma}\left(\tfrac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\tfrac{P^*C^*}{W^*}\right] - f^I\right\} \\
= & \left(\tfrac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}\left\{\left(\widetilde{Z}^{D*}\right)^{\sigma-1}\tfrac{\tau_V^{D*}}{\sigma}\left(\tfrac{1}{\rho}\tfrac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left[\left(\tfrac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\tfrac{P^*C^*}{W^*} + {Q_L}^{-\sigma}\left(\tfrac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\tfrac{PC}{W}\right] - f^{D*}\right\} \\
& + \left(\tfrac{1-G^*(Z^I)}{Q_L\cdot\delta}\right)\tau_C^I\left\{\left(\widetilde{Z}^I\right)^{\sigma-1}\tfrac{\tau_V^I}{\sigma}\left(\tfrac{1}{\rho}\tfrac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left[\left(\tfrac{P^H}{W}\right)^{\sigma-1}\nu\theta\tfrac{PC}{W} + {Q_L}^{\sigma}\left(\tfrac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\tfrac{P^*C^*}{W^*}\right] - f^I\right\} \\
= & \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} + \left(\frac{1-G^*(Z^I)}{Q_L\cdot\delta}\right)\frac{\pi^I(\widetilde{Z}^I)}{W}
\end{aligned}$$

Therefore,

$$\frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} + \frac{1}{Q_L}\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{\pi^I(\widetilde{Z}^I)}{W} = \frac{\delta F^{D*}}{1-G^*(Z^{D*})}$$

Free Entry Condition and Zero-Profit Cutoff Productivity:

In Home country, by combining zero-profit cutoff productivity condition and free entry condition, given by:

$$\begin{aligned}
\frac{1}{\sigma}\frac{r^D(Z^D)}{W} &= f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right), \\
F^D &= \left(\frac{1-G(Z^D)}{\delta}\right)\frac{\pi^D\left(\widetilde{Z}^D\right)}{W},
\end{aligned}$$

we can specify the cutoff productivity for Home local firms, Z^D:

$$
\begin{aligned}
& F^D \\
= & \left(\frac{1-G(Z^D)}{\delta}\right)\tau_C^D\left[f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)\right]\left[\frac{\frac{r^D(\widetilde{Z}^D)}{W}}{\frac{r^D(Z^D)}{W}}-\frac{f^D}{f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)}\right] \\
= & \left(\frac{1-G(Z^D)}{\delta}\right)\tau_C^D\left[f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)\right]\left[\left(\frac{\widetilde{Z}^D}{Z^D}\right)^{\sigma-1}-\frac{f^D}{f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)}\right] \\
= & \left(\frac{1-G(Z^D)}{\delta}\right)\tau_C^D\left[f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)\right]\left[\int_{Z^D}^{\infty}\left(\frac{z}{Z^D}\right)^{\sigma-1}\frac{dG(z)}{1-G(Z^D)}-\frac{f^D}{f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)}\right] \\
= & \left(\frac{1-G(Z^D)}{\delta}\right)\tau_C^D\left[f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)\right]\left[\frac{\eta}{\eta-\sigma+1}-\frac{f^D}{f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)}\right]
\end{aligned}
$$

$$
\therefore (Z^D)^\eta = \left(\frac{\tau_C^D}{\delta}\right)\left(\frac{f^D}{F^D}\right)\left(\frac{(\sigma-1)(z_{min})^\eta}{\eta-\sigma+1}\right)\left[1+\frac{\eta}{\sigma-1}\left(\frac{1}{\lambda}-1\right)\left(\zeta-\chi\frac{F^D}{f^D}\right)\right].
$$

In Home country, by combining two zero-profit cutoff productivity conditions, given by:

$$
\begin{aligned}
\frac{1}{\sigma}\frac{r^D(Z^D)}{W} &= \left[f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)\right] \\
\frac{1}{\sigma}\frac{r^I(\alpha Z^I)}{W} &= f^I
\end{aligned}
$$

we can solve for the cutoff productivity for FDI firms, Z^I:

$$
\begin{aligned}
\frac{\frac{r^D(Z^D)}{W}}{\frac{r^I(\alpha Z^I)}{W}} &= \frac{(Z^D)^{\sigma-1}\tau_V^D\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^\sigma\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]}{(\alpha Z^I)^{\sigma-1}\tau_V^I\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^\sigma\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]} \\
&= \frac{f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)}{f^I} \\
\therefore \alpha Z^I &= Z^D\left(\frac{f^I}{\left[f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)\right]}\right)^{\frac{1}{\sigma-1}}\left(\frac{\tau_V^D}{\tau_V^I}\right)^{\frac{\sigma}{\sigma-1}}\frac{\tau_L^I}{\tau_L^D} \\
&= z_{min}\left(\frac{\tau_C^D}{\delta}\right)^{\frac{1}{\eta}}\left(\frac{\eta}{F^D(\eta-\sigma+1)}\right)^{\frac{1}{\eta}}\frac{\left[\frac{f^D(\sigma-1)}{\eta}+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)\right]^{\frac{1}{\eta}}}{\left[f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)\right]^{\frac{1}{\sigma-1}}}(f^I)^{\frac{1}{\sigma-1}}\left(\frac{\tau_V^D}{\tau_V^I}\right)^{\frac{\sigma}{\sigma-1}}\frac{\tau_L^I}{\tau_L^D}
\end{aligned}
$$

Similarly, in Foreign country, by combining three equilibrium conditions which are given as,

$$\begin{aligned}
\frac{1}{\sigma}\frac{r^{D*}(Z^{D*})}{W^*} &= f^{D*}, \\
\frac{1}{\sigma}\frac{r^{I}(\alpha Z^{I})}{W} &= f^{I}, \\
F^{D*} &= \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} + \left(\frac{1-G^*(Z^{I})}{Q_L\cdot\delta}\right)\frac{\pi^{I}(\widetilde{Z}^{I})}{W},
\end{aligned}$$

we can derive the cutoff productivity of Foreign local firms, Z^{D*}:

$$\begin{aligned}
& F^{D*} \\
=& \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}f^{D*}\left[\frac{\frac{r^{D*}(\widetilde{Z}^{D*})}{W^*}}{\frac{r^{D*}(Z^{D*})}{W^*}}-1\right] + \left(\frac{1-G^*(Z^{I})}{Q_L\cdot\delta}\right)\tau_C^{I}f^{I}\left[\frac{\frac{r^{I}(\widetilde{Z}^{I})}{W}}{\frac{r^{I}(\alpha Z^{I})}{W}}-1\right] \\
=& \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}f^{D*}\left[\left(\frac{\widetilde{Z}^{D*}}{Z^{D*}}\right)^{\sigma-1}-1\right] + \left(\frac{1-G^*(Z^{I})}{Q_L\cdot\delta}\right)\tau_C^{I}f^{I}\left[\left(\frac{\widetilde{Z}^{I}}{\alpha Z^{I}}\right)^{\sigma-1}-1\right] \\
=& \quad \left(\tfrac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}f^{D*}\left[\int_{Z^{D*}}^{\infty}\left(\tfrac{z}{Z^{D*}}\right)^{\sigma-1}\tfrac{dG^*(z)}{1-G^*(Z^{D*})}-1\right] \\
& + \quad \left(\tfrac{1-G^*(Z^{I})}{Q_L\cdot\delta}\right)\tau_C^{I}f^{I}\left[\int_{Z^{I}}^{\infty}\left(\tfrac{z}{Z^{I}}\right)^{\sigma-1}\tfrac{dG^*(z)}{1-G^*(Z^{I})}-1\right] \\
=& \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}f^{D*}\left[\frac{\eta^*}{\eta^*-\sigma+1}-1\right] + \left(\frac{1-G^*(Z^{I})}{Q_L\cdot\delta}\right)\tau_C^{I}f^{I}\left[\frac{\eta^*}{\eta^*-\sigma+1}-1\right] \\
=& \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}f^{D*}\left[\frac{\sigma-1}{\eta^*-\sigma+1}\right] + \left(\frac{1-G^*(Z^{I})}{Q_L\cdot\delta}\right)\tau_C^{I}f^{I}\left[\frac{\sigma-1}{\eta^*-\sigma+1}\right]
\end{aligned}$$

$$\therefore \left(Z^{D*}\right)^{\eta^*} = \frac{\tau_C^{D*}f^{D*}\frac{(\sigma-1)\left(z^*_{min}\right)^{\eta^*}}{\eta^*-\sigma+1}}{\delta F^{D*}-\left(\frac{1}{Q_L}\right)\tau_C^{I}f^{I}\frac{(\sigma-1)\left(z^*_{min}\right)^{\eta^*}}{\eta^*-\sigma+1}\left(Z^{I}\right)^{-\eta^*}} = \frac{\left(\frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*}\right)\left(z^*_{min}\right)^{\eta^*}}{\delta F^{D*}-\left(\frac{1}{Q_L}\right)\left(\frac{\pi^{I}(\widetilde{Z}^{I})}{W}\right)\left(z^*_{min}\right)^{\eta^*}\left(Z^{I}\right)^{-\eta^*}}$$

Price Index: For Home country, we obtain:

$$
\begin{aligned}
\left(\frac{P_1}{W}\right) &= \left(\frac{P^H}{W}\right)^{\nu}\left(\frac{P^F}{W}\right)^{(1-\nu)} \\
\left(\frac{P^H}{W}\right)^{1-\sigma} &= \int_{\omega\in\Omega}\left(\frac{p^D(\omega)}{W}\right)^{1-\sigma} d\omega + \int_{\omega^*\in\Omega^I}\left(\frac{p^I(\omega^*)}{W}\right)^{1-\sigma} d\omega^* \\
&= M\int_{Z^D}^{\infty}\left(\frac{p^D(z)}{W}\right)^{1-\sigma}\frac{dG(z)}{1-G(Z^D)} + M^*\int_{Z^I}^{\infty}\left(\frac{p^I(\alpha z)}{W}\right)^{1-\sigma}\frac{dG^*(z)}{1-G^*(Z^{D*})} \\
&= M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} + M^*\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} \\
&= M\left(\widetilde{Z}^D\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} + \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)M^*\left(\widetilde{Z}^I\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} \\
&= M\left(\frac{p^D(\widetilde{Z}^D)}{W}\right)^{1-\sigma} + M^I\left(\frac{p^I(\widetilde{Z}^I)}{W}\right)^{1-\sigma} \\
\left(\frac{P^F}{W}\right)^{1-\sigma} &= \int_{\omega^*\in\Omega^*}\left(\frac{\epsilon p^{D*}(\omega^*)}{W}\right)^{1-\sigma} d\omega^* = M^*\int_{Z^{D*}}^{\infty}\left(\frac{Q_L p^{D*}(z)}{W^*}\right)^{1-\sigma}\frac{dG^*(z)}{1-G^*(Z^{D*})} \\
&= M^*\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{Q_L}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} = M^*\left(\widetilde{Z}^{D*}\right)^{\sigma-1}\left(\frac{Q_L}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} \\
&= M^*\left(Q_L\frac{p^{D*}(\widetilde{Z}^{D*})}{W^*}\right)^{1-\sigma}
\end{aligned}
$$

where $M^I \equiv \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) M^*$. For Foreign country, we obtain:

$$
\begin{aligned}
\left(\frac{P_1^*}{W^*}\right) &= \left(\frac{P^{F*}}{W^*}\right)^{\nu^*} \left(\frac{P^{H*}}{W^*}\right)^{(1-\nu^*)} \\
\left(\frac{P^{F*}}{W^*}\right)^{1-\sigma} &= \int_{\omega^* \in \Omega^*} \left(\frac{p^{D*}(\omega^*)}{W^*}\right)^{1-\sigma} d\omega^* = M^* \int_{Z^{D*}}^{\infty} \left(\frac{p^{D*}(z)}{W^*}\right)^{1-\sigma} \frac{dG^*(z)}{1-G^*(Z^{D*})} \\
&= M^* \frac{J^*(Z^{D*})}{1-G^*(Z^{D*})} \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} = M^* \left(\widetilde{Z}^{D*}\right)^{\sigma-1} \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} \\
&= M^* \left(\frac{p^{D*}(\widetilde{Z}^{D*})}{W^*}\right)^{1-\sigma} \\
\left(\frac{P^{H*}}{W^*}\right)^{1-\sigma} &= \int_{\omega \in \Omega} \left(\frac{p^D(\omega)}{\epsilon W^*}\right)^{1-\sigma} d\omega + \int_{\omega^* \in \Omega^I} \left(\frac{p^I(\omega^*)}{\epsilon W^*}\right)^{1-\sigma} d\omega^* \\
&= M \int_{Z^D}^{\infty} \left(\frac{p^D(z)}{WQ_L}\right)^{1-\sigma} \frac{dG(z)}{1-G(Z^D)} + M^* \int_{Z^I}^{\infty} \left(\frac{p^I(\alpha z)}{WQ_L}\right)^{1-\sigma} \frac{dG^*(z)}{1-G^*(Z^{D*})} \\
&= M \frac{J(Z^D)}{1-G(Z^D)} \left(\frac{1}{Q_L}\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} + \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) M^* \alpha^{\sigma-1} \frac{J^*(Z^I)}{1-G^*(Z^I)} \left(\frac{1}{Q_L}\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} \\
&= M \left(\widetilde{Z}^D\right)^{\sigma-1} \left(\frac{1}{Q_L}\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} + M^I \left(\widetilde{Z}^I\right)^{\sigma-1} \left(\frac{1}{Q_L}\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} \\
&= M \left(\frac{1}{Q_L}\frac{p^D(\widetilde{Z}^D)}{W}\right)^{1-\sigma} + M^I \left(\frac{1}{Q_L}\frac{p^I(\widetilde{Z}^I)}{W}\right)^{1-\sigma}
\end{aligned}
$$

Therefore, observe that

$$
\left(\frac{P^F}{W}\right) = Q_L \left(\frac{P^{F*}}{W^*}\right) \quad \text{and} \quad \left(\frac{P^{H*}}{W^*}\right) = \left(\frac{1}{Q_L}\right)\left(\frac{P^H}{W}\right)
$$

The Evolution of the Mass of Firms:

$$
\begin{aligned}
M &= \left(1-G(Z^D)\right) M^E + (1-\delta) M \\
M^* &= \left(1-G^*(Z^{D*})\right) M^{E*} + (1-\delta) M^* \\
M^I &\equiv \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) M^* = \left(\frac{Z^I}{Z^{D*}}\right)^{-\eta^*} M^*
\end{aligned}
$$

Incumbents in both countries exit with probability δ next period. Entrants exit if their productivity is so low that they cannot cover fixed overhead costs. Notice that M^E and M^{E*} represent the mass of potential entrants who pay fixed entry costs in Home and Foreign, respectively. M^I denotes the mass of Foreign firms which establish subsidiary business in the FDI host country. All Foreign firms of mass M^* serve both Home and Foreign markets through their local sales and export sales. Among them, the portion $M^I = \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) M^*$ of Foreign firms establish FDI subsidiaries in the Home country thanks to their productivity advantages. Foreign firms of mass M^I have two establishments: local headquarters in Foreign and FDI subsidiaries in Home. Both of these have the same productivity level. Their headquarters and subsidiaries serve Home and Foreign markets through local sales and export sales. That is, Foreign firms with productivity higher than Z^I manage headquarter business in Foreign and it serves Foreign and Home markets through local sales and export sales. In addition, they operate FDI affiliates in Home and it also engages in local sales and export sales. The idea here is that Foreign headquarters and Home FDI subsidiaries produce different products.

Market Demand:

Using $\left(\frac{P^F}{W}\right) = Q_L \left(\frac{P^{F*}}{W^*}\right)$ and $\left(\frac{P^{H*}}{W^*}\right) = \left(\frac{1}{Q_L}\right)\left(\frac{P^H}{W}\right)$, we derive market demand for Home firms and Foreign firms as

$$\begin{aligned}
A &\equiv \left(\frac{W}{P}\right)^{-\sigma}\left[\left(\frac{P^H}{P}\right)^{\sigma-1}\nu\theta C + Q^{\sigma}\left(\frac{P^{H*}}{P^*}\right)^{\sigma-1}(1-\nu^*)\theta C^*\right] \\
&= \left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W} + Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
A^* &\equiv \left(\frac{W^*}{P^*}\right)^{-\sigma}\left[\left(\frac{P^{F*}}{P^*}\right)^{\sigma-1}\nu^*\theta C^* + Q^{-\sigma}\left(\frac{P^F}{P}\right)^{\sigma-1}(1-\nu)\theta C\right] \\
&= \left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*} + {Q_L}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right] \\
&= \left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\left[\nu^*\theta\frac{P^*C^*}{W^*} + \frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right]
\end{aligned}$$

Labor Market Clearing Condition:

By using $M^I = \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) M^*$, we can find labor market clearing conditions for Home and Foreign:

$$
\begin{aligned}
L - L_0 &= \left(\begin{array}{ll} & M^E F^D \\ + & M \int_{Z^D}^{\infty} \left(l^D(z) + f^D \right) \frac{dG(z)}{1-G(Z^D)} \\ + & M^* \int_{Z^I}^{\infty} \left(l^I(\alpha z) + f^I \right) \frac{dG^*(z)}{1-G^*(Z^{D*})} \end{array} \right) \\
&= \left(\begin{array}{ll} & M\left(\frac{\delta F^D}{1-G(Z^D)} + f^D\right) + M^I f^I \\ + & M \frac{J(Z^D)}{1-G(Z^D)} \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma} \left[\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta \frac{PC}{W} + Q_L{}^{\sigma} \left(\frac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \frac{P^*C^*}{W^*} \right] \\ + & M^I \alpha^{\sigma-1} \frac{J^*(Z^I)}{1-G^*(Z^I)} \left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma} \left[\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta \frac{PC}{W} + Q_L{}^{\sigma} \left(\frac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \frac{P^*C^*}{W^*} \right] \end{array} \right) \\
&= M\left(\frac{\delta F^D}{1-G(Z^D)}\right) + M f^D + M l^D(\widetilde{Z}^D) + M^I f^I + M^I l^I(\widetilde{Z}^I)
\end{aligned}
$$

$$
\begin{aligned}
L^* - L_0^* &= \left(\begin{array}{ll} & M^{E*} F^{D*} \\ + & M^* \int_{Z^{D*}}^{\infty} \left(l^{D*}(z) + f^{D*} \right) \frac{dG^*(z)}{1-G^*(Z^{D*})} \end{array} \right) \\
&= \left(\begin{array}{ll} & M^*\left(\frac{\delta F^{D*}}{1-G^*(Z^{D*})} + f^{D*}\right) \\ + & M^* \frac{J^*(Z^{D*})}{1-G^*(Z^{D*})} \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} \left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1} \nu^*\theta \frac{P^*C^*}{W^*} + Q_L{}^{-\sigma} \left(\frac{P^F}{W}\right)^{\sigma-1} (1-\nu)\theta \frac{PC}{W} \right] \end{array} \right) \\
&= M^*\left(\frac{\delta F^{D*}}{1-G^*(Z^{D*})}\right) + M^* f^{D*} + M^* l^{D*}(\widetilde{Z}^{D*})
\end{aligned}
$$

Labor Market Clearing Condition and Free Entry Condition for Home: Combine Home labor market clearing condition with Home free entry condition to get:

$$
\begin{aligned}
L - L_0 &= M\left(\frac{\delta F^D}{1-G(Z^D)}\right) + M f^D + M^I f^I + M l^D\left(\widetilde{Z}^D\right) + M^I l^I\left(\widetilde{Z}^I\right) \\
&= M\frac{\pi^D(\widetilde{Z}^D)}{W} + M f^D + M^I f^I + M l^D\left(\widetilde{Z}^D\right) + M^I l^I\left(\widetilde{Z}^I\right) \\
&= M\tau_C^D\left[\frac{\tau_L^D}{\sigma-1} l^D(\widetilde{Z}^D) - f^D\right] + M f^D + M^I f^I + M l^D\left(\widetilde{Z}^D\right) + M^I l^I\left(\widetilde{Z}^I\right) \\
&= M\left[\left(\frac{\tau_C^D\tau_L^D}{\sigma-1} + 1\right) l^D(\widetilde{Z}^D) + \left(1-\tau_C^D\right) f^D\right] + M^I\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]
\end{aligned}
$$

Therefore, the mass of Home firms can be determined by:

$$M = \frac{(L-L_0) - M^I\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]}{\left(\frac{\delta F^D}{1-G(Z^D)}\right) + f^D + l^D\left(\widetilde{Z}^D\right)} = \frac{(L-L_0) - M^I\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]}{\left(\frac{\tau_C^D \tau_L^D}{\sigma-1}+1\right) l^D\left(\widetilde{Z}^D\right) + \left(1-\tau_C^D\right) f^D}$$

Labor Market Clearing Condition and Free Entry Condition for Foreign: Combine Foreign labor market clearing condition with Foreign free entry condition to get:

$$\begin{aligned}
& L^* - L_0^* \\
= \ & M^*\left(\frac{\delta F^{D*}}{1-G^*(Z^{D*})}\right) + M^* f^{D*} + M^* l^{D*}(\widetilde{Z}^{D*}) \\
= \ & \left(\frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} + \frac{1}{Q_L}\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{\pi^I(\widetilde{Z}^I)}{W}\right) + M^* f^{D*} + M^* l^{D*}(\widetilde{Z}^{D*}) \\
= \ & M^*\left\{\tau_C^{D*}\left[\frac{\tau_L^{D*}}{\sigma-1} l^{D*}(\widetilde{Z}^{D*}) - f^{D*}\right] + \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{\tau_C^I}{Q_L}\left[\frac{\tau_L^I}{\sigma-1} l^I(\widetilde{Z}^I) - f^I\right] + f^{D*} + l^{D*}(\widetilde{Z}^{D*})\right\} \\
= \ & M^*\left\{\left(\frac{\tau_C^{D*}\tau_L^{D*}}{\sigma-1}+1\right) l^{D*}(\widetilde{Z}^{D*}) + \left(1-\tau_C^{D*}\right) f^{D*} + \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{\tau_C^I}{Q_L}\left[\frac{\tau_L^I}{\sigma-1} l^I(\widetilde{Z}^I) - f^I\right]\right\}
\end{aligned}$$

Therefore, the mass of Foreign firms can be determined by:

$$\begin{aligned}
M^* &= \frac{L^* - L_0^*}{\left(\frac{\delta F^{D*}}{1-G^*(Z^{D*})}\right) + f^{D*} + l^{D*}(\widetilde{Z}^{D*})} \\
&= \frac{L^* - L_0^*}{\left\{\left(\frac{\tau_C^{D*}\tau_L^{D*}}{\sigma-1}+1\right) l^{D*}(\widetilde{Z}^{D*}) + \left(1-\tau_C^{D*}\right) f^{D*} + \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{\tau_C^I}{Q_L}\left[\frac{\tau_L^I}{\sigma-1} l^I(\widetilde{Z}^I) - f^I\right]\right\}}
\end{aligned}$$

Aggregate Prices, Outputs, and Sales of Firms:

$$\begin{aligned}
\int_{Z^D}^{\infty} \frac{p^D(z)}{W}\frac{dG(z)}{1-G(Z^D)} &= \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)\int_{Z^D}^{\infty}\frac{1}{z}\frac{dG(z)}{1-G(Z^D)} &&= \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)\frac{\eta}{\eta+1}\left\{Z^D\right\}^{-1}, \\
\int_{Z^I}^{\infty} \frac{p^I(\alpha z)}{W}\frac{dG^*(z)}{1-G^*(Z^I)} &= \left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)\int_{Z^I}^{\infty}\frac{1}{\alpha z}\frac{dG^*(z)}{1-G^*(Z^I)} &&= \left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\frac{1}{\alpha}\right)\frac{\eta^*}{\eta^*+1}\left\{Z^I\right\}^{-1}, \\
\int_{Z^{D*}}^{\infty} \frac{p^{D*}(z)}{W^*}\frac{dG^*(z)}{1-G^*(Z^{D*})} &= \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)\int_{Z^{D*}}^{\infty}\frac{1}{z}\frac{dG^*(z)}{1-G^*(Z^{D*})} &&= \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)\frac{\eta^*}{\eta^*+1}\left\{Z^{D*}\right\}^{-1}.
\end{aligned}$$

$$
\begin{aligned}
&\int_{Z^D}^{\infty} \left(y^D(z) + y^{D,X}(z)\right) \tfrac{dG(z)}{1-G(Z^D)} \\
&= \int_{Z^D}^{\infty} \left(\tfrac{p^D(z)}{W}\right)^{-\sigma} \tfrac{dG(z)}{1-G(Z^D)} \left(\left(\tfrac{P^H}{W}\right)^{\sigma-1} \nu\theta \tfrac{PC}{W} + Q_L{}^{\sigma} \left(\tfrac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \tfrac{P^*C^*}{W^*}\right), \\
&= \left(\tfrac{1}{\rho}\tfrac{\tau_L^D}{\tau_V^D}\right)^{-\sigma} \int_{Z^D}^{\infty} z^{\sigma} \tfrac{dG(z)}{1-G(Z^D)} \left(\left(\tfrac{P^H}{W}\right)^{\sigma-1} \nu\theta \tfrac{PC}{W} + Q_L{}^{\sigma} \left(\tfrac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \tfrac{P^*C^*}{W^*}\right), \\
&= \left(\tfrac{1}{\rho}\tfrac{\tau_L^D}{\tau_V^D}\right)^{-\sigma} \tfrac{\eta}{\eta-\sigma} \left\{Z^D\right\}^{\sigma} \left(\left(\tfrac{P^H}{W}\right)^{\sigma-1} \nu\theta \tfrac{PC}{W} + Q_L{}^{\sigma} \left(\tfrac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \tfrac{P^*C^*}{W^*}\right),
\end{aligned}
$$

$$
\begin{aligned}
&\int_{Z^I}^{\infty} \left(y^I(\alpha z) + y^{I,X}(\alpha z)\right) \tfrac{dG^*(z)}{1-G^*(Z^I)} \\
&= \int_{Z^I}^{\infty} \left(\tfrac{p^I(\alpha z)}{W}\right)^{-\sigma} \tfrac{dG^*(z)}{1-G^*(Z^I)} \left(\left(\tfrac{P^H}{W}\right)^{\sigma-1} \nu\theta \tfrac{PC}{W} + Q_L{}^{\sigma} \left(\tfrac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \tfrac{P^*C^*}{W^*}\right), \\
&= \left(\tfrac{1}{\rho}\tfrac{\tau_L^I}{\tau_V^I}\tfrac{1}{\alpha}\right)^{-\sigma} \int_{Z^I}^{\infty} z^{\sigma} \tfrac{dG^*(z)}{1-G^*(Z^I)} \left(\left(\tfrac{P^H}{W}\right)^{\sigma-1} \nu\theta \tfrac{PC}{W} + Q_L{}^{\sigma} \left(\tfrac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \tfrac{P^*C^*}{W^*}\right), \\
&= \left(\tfrac{1}{\rho}\tfrac{\tau_L^I}{\tau_V^I}\right)^{-\sigma} \tfrac{\eta^*}{\eta^*-\sigma} \left\{\alpha Z^I\right\}^{\sigma} \left(\left(\tfrac{P^H}{W}\right)^{\sigma-1} \nu\theta \tfrac{PC}{W} + Q_L{}^{\sigma} \left(\tfrac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \tfrac{P^*C^*}{W^*}\right),
\end{aligned}
$$

$$
\begin{aligned}
&\int_{Z^{D*}}^{\infty} \left(y^{D*}(z) + y^{D,X*}(z)\right) \tfrac{dG^*(z)}{1-G^*(Z^{D*})} \\
&= \int_{Z^{D*}}^{\infty} \left(\tfrac{p^{D*}(z)}{W^*}\right)^{-\sigma} \tfrac{dG^*(z)}{1-G^*(Z^{D*})} \left(\left(\tfrac{P^{F*}}{W^*}\right)^{\sigma-1} \nu^*\theta \tfrac{P^*C^*}{W^*} + Q_L{}^{-\sigma} \left(\tfrac{P^F}{W}\right)^{\sigma-1} (1-\nu)\theta \tfrac{PC}{W}\right), \\
&= \left(\tfrac{1}{\rho}\tfrac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} \int_{Z^{D*}}^{\infty} z^{\sigma} \tfrac{dG^*(z)}{1-G^*(Z^{D*})} \left(\left(\tfrac{P^{F*}}{W^*}\right)^{\sigma-1} \nu^*\theta \tfrac{P^*C^*}{W^*} + Q_L{}^{-\sigma} \left(\tfrac{P^F}{W}\right)^{\sigma-1} (1-\nu)\theta \tfrac{PC}{W}\right), \\
&= \left(\tfrac{1}{\rho}\tfrac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} \tfrac{\eta^*}{\eta^*-\sigma} \left\{Z^{D*}\right\}^{\sigma} \left(\left(\tfrac{P^{F*}}{W^*}\right)^{\sigma-1} \nu^*\theta \tfrac{P^*C^*}{W^*} + Q_L{}^{-\sigma} \left(\tfrac{P^F}{W}\right)^{\sigma-1} (1-\nu)\theta \tfrac{PC}{W}\right).
\end{aligned}
$$

$$\int_{Z^D}^{\infty} \frac{p^D(z)}{W}\left(y^D(z)+y^{D,X}(z)\right)\frac{dG(z)}{1-G(Z^D)}$$
$$= \int_{Z^D}^{\infty}\left(\frac{p^D(z)}{W}\right)^{1-\sigma}\frac{dG(z)}{1-G(Z^D)}\left(\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right),$$
$$= \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\int_{Z^D}^{\infty} z^{\sigma-1}\frac{dG(z)}{1-G(Z^D)}\left(\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right),$$
$$= \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left(\widetilde{Z}^D\right)^{\sigma-1}\left(\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right),$$

$$\int_{Z^I}^{\infty} \frac{p^I(\alpha z)}{W}\left(y^I(\alpha z)+y^{I,X}(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)}$$
$$= \int_{Z^I}^{\infty}\left(\frac{p^I(\alpha z)}{W}\right)^{1-\sigma}\frac{dG^*(z)}{1-G^*(Z^I)}\left(\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right),$$
$$= \left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\int_{Z^I}^{\infty}(\alpha z)^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^I)}\left(\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right),$$
$$= \left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left(\widetilde{Z}^I\right)^{\sigma-1}\left(\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right),$$

$$\int_{Z^{D*}}^{\infty} \frac{p^{D*}(z)}{W^*}\left(y^{D*}(z)+y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})}$$
$$= \int_{Z^{D*}}^{\infty}\left(\frac{p^{D*}(z)}{W^*}\right)^{1-\sigma}\frac{dG^*(z)}{1-G^*(Z^{D*})}\left(\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}+Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right),$$
$$= \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\int_{Z^{D*}}^{\infty} z^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^{D*})}\left(\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}+Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right),$$
$$= \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left(\widetilde{Z}^{D*}\right)^{\sigma-1}\left(\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}+Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right).$$

Government Budget Balance: Home government budget balance implies

$$\begin{aligned}
T &= M\int_{Z^D}^{\infty}\left(\tau_L^D-1\right)Wl^D(z)\frac{dG(z)}{1-G(Z^D)}\\
&+ M\int_{Z^D}^{\infty}\left(1-\tau_V^D\right)p^D(z)\left(y^D(z)+y^{D,X}(z)\right)\frac{dG(z)}{1-G(Z^D)}\\
&+ M\int_{Z^D}^{\infty}\left(1-\tau_C^D\right)\left\{\tau_V^D p^D(z)\left(y^D(z)+y^{D,X}(z)\right)-\tau_L^D Wl^D(z)-f^D W\right\}\frac{dG(z)}{1-G(Z^D)}\\
&+ M^I\int_{Z^I}^{\infty}\left(\tau_L^I-1\right)Wl^I(\alpha z)\frac{dG^*(z)}{1-G^*(Z^I)}\\
&+ M^I\int_{Z^I}^{\infty}\left(1-\tau_V^I\right)p^I(\alpha z)\left(y^I(\alpha z)+y^{I,X}(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)}\\
&+ M^I\int_{Z^I}^{\infty}\left(1-\tau_C^I\right)\left\{\tau_V^I p^I(\alpha z)\left(y^I(\alpha z)+y^{I,X}(\alpha z)\right)-\tau_L^I Wl^I(\alpha z)-f^I W\right\}\frac{dG^*(z)}{1-G^*(Z^I)}
\end{aligned}$$

That is,

$$
\begin{aligned}
\frac{T}{W} &= M\left(\tau_L^D - 1\right) l^D(\widetilde{Z}^D) \\
&+ M\left(1-\tau_V^D\right)\int_{Z^D}^{\infty}\frac{p^D(z)}{W}\left(y^D(z)+y^{D,X}(z)\right)\frac{dG(z)}{1-G(Z^D)} \\
&+ M\left(1-\tau_C^D\right)\left\{\tau_V^D\int_{Z^D}^{\infty}\frac{p^D(z)}{W}\left(y^D(z)+y^{D,X}(z)\right)\frac{dG(z)}{1-G(Z^D)} - \tau_L^D l^D(\widetilde{Z}^D) - f^D\right\} \\
&+ M^I\left(\tau_L^I - 1\right) l^I(\widetilde{Z}^I) \\
&+ M^I\left(1-\tau_V^I\right)\int_{Z^I}^{\infty}\frac{p^I(\alpha z)}{W}\left(y^I(\alpha z)+y^{I,X}(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)} \\
&+ M^I\left(1-\tau_C^I\right)\left\{\tau_V^I\int_{Z^I}^{\infty}\frac{p^I(\alpha z)}{W}\left(y^I(\alpha z)+y^{I,X}(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)} - \tau_L^I l^I(\widetilde{Z}^I) - f^I\right\}
\end{aligned}
$$

Foreign government budget balance implies

$$
\begin{aligned}
T^* &= M^*\int_{Z^{D*}}^{\infty}\left(\tau_L^{D*}-1\right)W^* l^{D*}(z)\frac{dG^*(z)}{1-G^*(Z^{D*})} \\
&+ M^*\int_{Z^{D*}}^{\infty}\left(1-\tau_V^{D*}\right)p^{D*}(z)\left(y^{D*}(z)+y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})} \\
&+ M^*\int_{Z^{D*}}^{\infty}\left(1-\tau_C^{D*}\right)\left\{\tau_V^{D*}p^{D*}(z)\left(y^{D*}(z)+y^{D,X*}(z)\right)-\tau_L^{D*}W^* l^{D*}(z) - f^{D*}W^*\right\}\frac{dG^*(z)}{1-G^*(Z^{D*})}
\end{aligned}
$$

That is,

$$
\begin{aligned}
\frac{T^*}{W^*} &= M^*\left(\tau_L^{D*}-1\right) l^{D*}(\widetilde{Z}^{D*}) \\
&+ M^*\left(1-\tau_V^{D*}\right)\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D*}(z)+y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})} \\
&+ M^*\left(1-\tau_C^{D*}\right)\left\{\tau_V^{D*}\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D*}(z)+y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})} - \tau_L^{D*} l^{D*}(\widetilde{Z}^{D*}) - f^{D*}\right\}
\end{aligned}
$$

The Resource Constraint:

Home household budget constraint can be written as

$$\begin{aligned}\frac{P}{W}C &= L+\frac{T}{W}=\frac{P_0}{W}C_0+\frac{P^H}{W}C^H+\frac{P^F}{W}C^F \\ &= \begin{array}{l}\frac{P_0}{W}C_0+M\int_{Z^D}^{\infty}\frac{p^D(z)}{W}\left(y^D(z)\right)\frac{dG(z)}{1-G(Z^D)} \\ +\; M^I\int_{Z^I}^{\infty}\frac{p^I(\alpha z)}{W}\left(y^I(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)}+M^*Q_L\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})}\end{array}\end{aligned}$$

Use household budget constraint, labor market clearing condition, free entry condition, and government budget balance to obtain Home resource constraint:

$$\begin{aligned}&\left(\frac{P_0}{W}C_0-L_0\right)+M^*Q_L\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})}+M^I\frac{\pi^I(\widetilde{Z}^I)}{W} \\ =\;& M\int_{Z^D}^{\infty}\frac{p^D(z)}{W}\left(y^{D,X}(z)\right)\frac{dG(z)}{1-G(Z^D)}+M^I\int_{Z^I}^{\infty}\frac{p^I(\alpha z)}{W}\left(y^{I,X}(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)}\end{aligned}$$

Similarly, Foreign household budget constraint can be written as

$$\begin{aligned}\frac{P^*}{W^*}C^* &= L^*+\frac{T^*}{W^*}=\frac{P_0^*}{W^*}C_0^*+\frac{P^{F*}}{W^*}C^{F*}+\frac{P^{H*}}{W^*}C^{H*} \\ &= \begin{array}{l}\frac{P_0^*}{W^*}C_0^*+M^*\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})} \\ +\; M\frac{1}{Q_L}\int_{Z^D}^{\infty}\frac{p^D(z)}{W}\left(y^{D,X}(z)\right)\frac{dG(z)}{1-G(Z^D)}+M^I\frac{1}{Q_L}\int_{Z^I}^{\infty}\frac{p^I(\alpha z)}{W}\left(y^{I,X}(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)}\end{array}\end{aligned}$$

Use household budget constraint, labor market clearing condition, free entry condition, and government budget balance to obtain Foreign resource constraint:

$$\begin{aligned}&\left(Q_LL_0^*-Q_L\frac{P_0^*}{W^*}C_0^*\right)+M^*Q_L\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})}+M^I\frac{\pi^I(\widetilde{Z}^I)}{W} \\ =\;& M\int_{Z^D}^{\infty}\frac{p^D(z)}{W}\left(y^{D,X}(z)\right)\frac{dG(z)}{1-G(Z^D)}+M^I\int_{Z^I}^{\infty}\frac{p^I(\alpha z)}{W}\left(y^{I,X}(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)}\end{aligned}$$

Under the partial equilibrium with $W=\epsilon W^*=P_0=\epsilon P_0^*$ and $Q_L=1$, these two Home and Foreign resource constraints are identical since $Q_LL_0^*-Q_L\frac{P_0^*}{W^*}C_0^*=\frac{P_0}{W}C_0-L_0$ holds. This is also true in the general equilibrium with zero weight on the homogeneous good: $\theta_0=0$ and $C_0=C_0^*=L_0=L_0^*=P_0=P_0^*=0$ (note that $0^0=1$).

A.2.1 Aggregate and Average Measures over Firms and Welfare Decomposition

Define

$$\begin{aligned}
M^{DI} &\equiv M + M^I \\
\widetilde{Z}^{DI} &\equiv \left[\frac{1}{M^{DI}}\left(M\left(\frac{\tau_L^D}{\tau_V^D}\frac{1}{\widetilde{Z}^D}\right)^{1-\sigma} + M^I\left(\frac{\tau_L^I}{\tau_V^I}\frac{1}{\widetilde{Z}^I}\right)^{1-\sigma}\right)\right]^{\frac{1}{\sigma-1}} \\
\log\left(M^{HF}\right) &\equiv \nu\log\left(M^{DI}\right) + (1-\nu)\log\left(M^*\right) \\
\log\left(\frac{1}{\widetilde{Z}^{HF}}\right) &\equiv \nu\log\left(\frac{1}{\widetilde{Z}^{DI}}\right) + (1-\nu)\log\left(Q_L\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right) \\
\log\left(M^{HF*}\right) &\equiv \nu^*\log\left(M^*\right) + (1-\nu^*)\log\left(M^{DI}\right) \\
\log\left(\frac{1}{\widetilde{Z}^{HF*}}\right) &\equiv \nu^*\log\left(\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right) + (1-\nu^*)\log\left(\frac{1}{Q_L}\frac{1}{\widetilde{Z}^{DI}}\right)
\end{aligned}$$

where $\rho \equiv \frac{\sigma-1}{\sigma}$.

Welfare Decomposition for Home consumption:

We can decompose Home consumption index C^H of domestically-produced goods by:

$$
\begin{aligned}
C^H &= \left(M\left[y^D\left(\widetilde{Z}^D\right)\right]^\rho + M^I\left[y^I\left(\widetilde{Z}^I\right)\right]^\rho\right)^{\frac{1}{\rho}} = \left(\int_{\omega\in\Omega}\left[y^D(\omega)\right]^\rho d\omega + \int_{\omega^*\in\Omega^I}\left[y^I(\omega^*)\right]^\rho d\omega^*\right)^{\frac{1}{\rho}} \\
&= \left(\int_{Z^D}^\infty \left(\frac{p^D(z)}{P}\right)^{1-\sigma}\frac{MdG(z)}{(1-G(Z^D))} + \int_{Z^I}^\infty\left(\frac{p^I(\alpha z)}{P}\right)^{1-\sigma}\frac{M^I dG^*(z)}{(1-G^*(Z^I))}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{P^H}{P}\right)^{\sigma-1}\nu\theta C \\
&= \left(\int_{Z^D}^\infty \left(\frac{p^D(z)}{W}\right)^{1-\sigma}\frac{MdG(z)}{(1-G(Z^D))} + \int_{Z^I}^\infty\left(\frac{p^I(\alpha z)}{W}\right)^{1-\sigma}\frac{M^I dG^*(z)}{(1-G^*(Z^I))}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{W}{P}\right)^{-\sigma}\left(\frac{P^H}{P}\right)^{\sigma-1}\nu\theta C \\
&= \left(\int_{Z^D}^\infty \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\frac{1}{z}\right)^{1-\sigma}\frac{MdG(z)}{(1-G(Z^D))} + \int_{Z^I}^\infty\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\frac{1}{\alpha z}\right)^{1-\sigma}\frac{M^I dG^*(z)}{(1-G^*(Z^I))}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} \\
&= \left(M\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left(\widetilde{Z}^D\right)^{\sigma-1} + M^I\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left(\widetilde{Z}^I\right)^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} \\
&= \left(M\left[\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\frac{1}{\widetilde{Z}^D}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}\right]^{\frac{\sigma-1}{\sigma}} + M^I\left[\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\frac{1}{\widetilde{Z}^I}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}\right]^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(M\left[y^D\left(\widetilde{Z}^D\right)\right]^\rho + M^I\left[y^I\left(\widetilde{Z}^I\right)\right]^\rho\right)^{\frac{1}{\rho}} \\
&= \left(M\left(\frac{\tau_L^D}{\tau_V^D}\frac{1}{\widetilde{Z}^D}\right)^{1-\sigma} + M^I\left(\frac{\tau_L^I}{\tau_V^I}\frac{1}{\widetilde{Z}^I}\right)^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{1}{\rho}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} \\
&= \left(M^{DI}\right)^{\frac{1}{\rho}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} = \left(M^{DI}\right)^{\frac{1}{\rho}-1}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{-1}\nu\theta\frac{PC}{W} \\
&= \underbrace{\left(M^{DI}\right)^{\frac{1}{\sigma-1}}}_{\text{Variety Effect}}\underbrace{\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{-1}}_{\text{Productivity Effect}}\underbrace{\nu\theta\left(L+\frac{T}{W}\right)}_{\text{Income Effect}}
\end{aligned}
$$

where variety effect increases the welfare due to the love of variety (extensive margin); productivity effect lowers the marginal cost and the overall price; inccome effect raises output per each variety due to the increase in total demand (intensive margin). Note that the price index for Home domestic composite consumption can be solved as:

$$\begin{aligned}
\left(\frac{P^H}{W}\right)^{1-\sigma} &= M\left(\frac{p^D(\widetilde{Z}^D)}{W}\right)^{1-\sigma} + M^I\left(\frac{p^I(\widetilde{Z}^I)}{W}\right)^{1-\sigma} \\
&= \left(M\left(\frac{\tau_L^D}{\tau_V^D}\frac{1}{\widetilde{Z}^D}\right)^{1-\sigma} + M^I\left(\frac{\tau_L^I}{\tau_V^I}\frac{1}{\widetilde{Z}^I}\right)^{1-\sigma}\right)\left(\frac{1}{\rho}\right)^{1-\sigma} \\
&= M^{DI}\left(\widetilde{Z}^{DI}\right)^{\sigma-1}\left(\frac{1}{\rho}\right)^{1-\sigma} \\
&= M^{DI}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{1-\sigma}
\end{aligned}$$

Likewise, we can decompose Home consumption index C^F on imported goods:

$$\begin{aligned}
C^F &= \left(\int_{\omega^*\in\Omega^*}\left[y^{D,X*}(\omega^*)\right]^{\rho} d\omega^*\right)^{\frac{1}{\rho}} \\
&= \left[M^*\int_{Z^{D*}}^{\infty}\left(\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{z}\right)^{-\sigma} Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right)^{\frac{\sigma-1}{\sigma}}\frac{dG^*(z)}{1-G^*(Z^{D*})}\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[M^*\int_{Z^{D*}}^{\infty} z^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^{D*})}\left(\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[M^*(\widetilde{Z}^{D*})^{\sigma-1}\left(\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[M^*\left(\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-\sigma} Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
&= (M^*)^{\frac{1}{\rho}}\, y^{D,X*}(\widetilde{Z}^{D*}) \\
&= (M^*)^{\frac{1}{\rho}}\left(Q_L\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W} \\
&= \underbrace{(M^*)^{\frac{1}{\sigma-1}}}_{\text{Variety Effect}}\underbrace{\left(Q_L\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-1}}_{\text{Productivity Effect}}\underbrace{(1-\nu)\theta\left(L+\frac{T}{W}\right)}_{\text{Income Effect}}
\end{aligned}$$

Its price index can be found to be:

$$\begin{aligned}
\left(\frac{P^F}{W}\right)^{1-\sigma} &= \int_{\omega^*\in\Omega^*}\left(Q_L\frac{p^{D*}(\omega^*)}{W^*}\right)^{1-\sigma}d\omega^* = M^*\left(Q_L\frac{p^{D*}(\widetilde{Z}^{D*})}{W^*}\right)^{1-\sigma} = M^*\left(Q_L\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{1-\sigma}\\
\frac{P^F}{W}C^F &= \int_{\omega^*\in\Omega^*}Q_L\frac{p^{D*}(\omega^*)}{W^*}y^{D,X*}(\omega^*)d\omega^* = M^*\left(Q_L\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})}\right)\\
&= (1-\nu)\theta\frac{PC}{W}
\end{aligned}$$

Therefore, the composite consumption of differentiated goods in Home is given by:

$$\begin{aligned}
C_1 &= \frac{\left(C^H\right)^{\nu}\left(C^F\right)^{1-\nu}}{(\nu)^{\nu}(1-\nu)^{1-\nu}}\\
&= \frac{\left(\left(M^{DI}\right)^{\frac{1}{\rho}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}\right)^{\nu}\left((M^*)^{\frac{1}{\rho}}\left(Q_L\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right)^{1-\nu}}{(\nu)^{\nu}(1-\nu)^{1-\nu}}\\
&= \left(\left(M^{DI}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\theta\frac{PC}{W}\right)^{\nu}\left((M^*)^{\frac{\sigma}{\sigma-1}}\left(Q_L\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}\theta\frac{PC}{W}\right)^{1-\nu}\\
&= \left(\left(M^{DI}\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{-1}\theta\frac{PC}{W}\right)^{\nu}\left((M^*)^{\frac{1}{\sigma-1}}\left(Q_L\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-1}\theta\frac{PC}{W}\right)^{1-\nu}\\
&= \left[\left(M^{DI}\right)^{\nu}(M^*)^{1-\nu}\right]^{\frac{1}{\sigma-1}}\left[\frac{1}{\rho}\left(\frac{1}{\widetilde{Z}^{DI}}\right)^{\nu}\left(Q_L\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{1-\nu}\right]^{-1}\theta\frac{PC}{W}\\
&= \underbrace{\left[M^{HF}\right]^{\frac{1}{\sigma-1}}}_{\text{Variety Effect: Extensive Margin}}\underbrace{\left[\frac{1}{\rho}\left(\frac{1}{\widetilde{Z}^{HF}}\right)\right]^{-1}}_{\text{Productivity Effect}}\underbrace{\theta\left(L+\frac{T}{W}\right)}_{\text{Income Effect: Intensive Margin}}
\end{aligned}$$

where variety effect increases the welfare since consumers have more varieties to consume and their utilities feature the love of variety (extensive margin); productivity effect reduces the marginal cost and the aggregate price decreases; income effect raises output per each variety due to the rise in total demand (intensive margin). The price index in Home is given by:

$$
\begin{aligned}
\frac{P_1}{W} &= \left(\frac{P^H}{W}\right)^{\nu}\left(\frac{P^F}{W}\right)^{1-\nu} \\
&= \left[\left(M^{DI}\right)^{\frac{1}{1-\sigma}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)\right]^{\nu}\left[\left(M^{*}\right)^{\frac{1}{1-\sigma}}\left(Q_L\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)\right]^{1-\nu} \\
&= \left[\left(M^{DI}\right)^{\nu}\left(M^{*}\right)^{1-\nu}\right]^{\frac{1}{1-\sigma}}\frac{1}{\rho}\left(\frac{1}{\widetilde{Z}^{DI}}\right)^{\nu}\left(Q_L\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{1-\nu} \\
&= \underbrace{\left[\frac{1}{M^{HF}}\right]^{\frac{1}{\sigma-1}}}_{\text{Competition Effect}}\underbrace{\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{HF}}\right)}_{\text{Productivity Effect}}
\end{aligned}
$$

where $V = C = \Phi\left(C_0\right)^{\theta_0}\left(C_1\right)^{\theta_1}$ and $P = \left(P_0\right)^{\theta_0}\left(P_1\right)^{\theta}$.

Welfare Decomposition for Foreign consumption:

Symmetrically, we can decompose Foreign consumption index C^{F*} of locally-produced goods by:

$$
\begin{aligned}
C^{F*} &= \left(\int_{\omega^*\in\Omega^*}\left[y^{D*}(\omega^*)\right]^{\rho}d\omega^*\right)^{\frac{1}{\rho}} \\
&= \left[M^*\int_{Z^{D*}}^{\infty}\left(\left(\frac{p^{D*}(z)}{W^*}\right)^{-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}\right)^{\frac{\sigma-1}{\sigma}}\frac{dG^*(z)}{1-G^*(Z^{D*})}\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[M^*\int_{Z^{D*}}^{\infty}\left(\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{z}\right)^{-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}\right)^{\frac{\sigma-1}{\sigma}}\frac{dG^*(z)}{1-G^*(Z^{D*})}\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[M^*\left(\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left(M^*\right)^{\frac{1}{\rho}}y^{D*}(\widetilde{Z}^{D*}) \\
&= \left(M^*\right)^{\frac{1}{\rho}}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*} \\
&= \left(M^*\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-1}\nu^*\theta\left(L^*+\frac{T^*}{W^*}\right)
\end{aligned}
$$

The price index for Foreign domestic composite consumption can be solved as:

$$\begin{aligned}\left(\frac{P^{F*}}{W^*}\right)^{1-\sigma} &= \int_{\omega^*\in\Omega^*}\left(\frac{p^{D*}(\omega^*)}{W^*}\right)^{1-\sigma} d\omega^* = M^*\left(\frac{p^{D*}(\widetilde{Z}^{D*})}{W^*}\right)^{1-\sigma} = M^*\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{1-\sigma} \\ \frac{P^{F*}}{W^*}C^{F*} &= \int_{\omega^*\in\Omega^*}\frac{p^{D*}(\omega^*)}{W^*}y^{D*}(\omega^*)d\omega^* = M^*\left(\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})}\right) = \nu^*\theta\frac{P^*C^*}{W^*}\end{aligned}$$

Likewise, we can decompose Foreign consumption index C^{H*} on imported goods:

$$\begin{aligned}C^{H*} &= \left(M\left[y^{D,X}(\widetilde{Z}^D)\right]^{\rho} + M^I\left[y^{I,X}(\widetilde{Z}^I)\right]^{\rho}\right)^{\frac{1}{\rho}} \\ &= \left(\int_{\omega\in\Omega}\left[y^{D,X}(\omega)\right]^{\rho}d\omega + \int_{\omega^*\in\Omega^I}\left[y^{I,X}(\omega^*)\right]^{\rho}d\omega^*\right)^{\frac{1}{\rho}} \\ &= \begin{pmatrix} \int_{Z^D}^{\infty}\left(\frac{p^D(z)}{W}\right)^{1-\sigma}\frac{MdG(z)}{(1-G(Z^D))} \\ +\quad \int_{Z^I}^{\infty}\left(\frac{p^I(\alpha z)}{W}\right)^{1-\sigma}\frac{M^I dG^*(z)}{(1-G^*(Z^I))}\end{pmatrix}^{\frac{\sigma}{\sigma-1}} {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*} \\ &= \left(M\left(\frac{p^D(\widetilde{Z}^D)}{W}\right)^{1-\sigma} + M^I\left(\frac{p^I(\widetilde{Z}^I)}{W}\right)^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}} {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*} \\ &= \left(M^{DI}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}} {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*} \\ &= \left(M^{DI}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\frac{1}{Q_L}\right)^{-\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*} \\ &= \left(M^{DI}\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\frac{1}{Q_L}\right)^{-1}(1-\nu^*)\theta\left(L^* + \frac{T^*}{W^*}\right)\end{aligned}$$

Its price index can be found to be:

$$\begin{aligned}\left(\frac{P^{H*}}{W^*}\right)^{1-\sigma} &= \int_{\omega\in\Omega}\left(\frac{1}{Q_L}\frac{p^D(\omega)}{W}\right)^{1-\sigma}d\omega + \int_{\omega^*\in\Omega^I}\left(\frac{1}{Q_L}\frac{p^I(\omega^*)}{W}\right)^{1-\sigma}d\omega^* \\ &= M\left(\frac{1}{Q_L}\frac{p^D(\widetilde{Z}^D)}{W}\right)^{1-\sigma} + M^I\left(\frac{1}{Q_L}\frac{p^I(\widetilde{Z}^I)}{W}\right)^{1-\sigma} \\ &= M^{DI}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\frac{1}{Q_L}\right)^{1-\sigma}\end{aligned}$$

Therefore, the composite consumption in Foreign is given by:

$$
\begin{aligned}
C_1^* &= \frac{\left(C^{F*}\right)^{\nu^*}\left(C^{H*}\right)^{1-\nu^*}}{\left(\nu^*\right)^{\nu^*}\left(1-\nu^*\right)^{1-\nu^*}} \\
&= \frac{\left(\left(M^*\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-1}\nu^*\theta\left(L^*+\frac{T^*}{W^*}\right)\right)^{\nu^*}\left(\left(M^{DI}\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\frac{1}{Q_L}\right)^{-1}(1-\nu^*)\theta\left(L^*+\frac{T^*}{W^*}\right)\right)^{1-\nu^*}}{\left(\nu^*\right)^{\nu^*}\left(1-\nu^*\right)^{1-\nu^*}} \\
&= \left(\left(M^*\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-1}\theta\left(L^*+\frac{T^*}{W^*}\right)\right)^{\nu^*}\left(\left(M^{DI}\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{Q_L}\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{-1}\theta\left(L^*+\frac{T^*}{W^*}\right)\right)^{1-\nu^*} \\
&= \left[\left(M^*\right)^{\nu^*}\left(M^{DI}\right)^{1-\nu^*}\right]^{\frac{1}{\sigma-1}}\left[\frac{1}{\rho}\left(\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{\nu^*}\left(\frac{1}{Q_L}\frac{1}{\widetilde{Z}^{DI}}\right)^{1-\nu^*}\right]^{-1}\theta\left(L^*+\frac{T^*}{W^*}\right) \\
&= \underbrace{\left[M^{HF*}\right]^{\frac{1}{\sigma-1}}}_{\text{Variety Effect: Extensive Margin}}\underbrace{\left[\frac{1}{\rho}\left(\frac{1}{\widetilde{Z}^{HF*}}\right)\right]^{-1}}_{\text{Productivity Effect}}\underbrace{\theta\left(L^*+\frac{T^*}{W^*}\right)}_{\text{Income Effect: Intensive Margin}}
\end{aligned}
$$

where variety effect increases the welfare since consumers have more varieties to consume and their utilities feature the love of variety (extensive margin); productivity effect reduces the marginal cost and the aggregate price decreases; income effect raises output per each variety due to the rise in total demand (intensive margin). The price index in Foreign is derived as:

$$
\begin{aligned}
\frac{P_1^*}{W^*} &= \left(\frac{P^{F*}}{W^*}\right)^{\nu^*}\left(\frac{P^{H*}}{W^*}\right)^{1-\nu^*} \\
&= \left[\left(M^*\right)^{\frac{1}{1-\sigma}}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)\right]^{\nu^*}\left[\left(M^{DI}\right)^{\frac{1}{1-\sigma}}\left(\frac{1}{Q_L}\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)\right]^{1-\nu^*} \\
&= \left[\left(M^*\right)^{\nu^*}\left(M^{DI}\right)^{1-\nu^*}\right]^{\frac{1}{1-\sigma}}\frac{1}{\rho}\left(\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{\nu^*}\left(\frac{1}{Q_L}\frac{1}{\widetilde{Z}^{DI}}\right)^{1-\nu^*} \\
&= \underbrace{\left[\frac{1}{M^{HF*}}\right]^{\frac{1}{\sigma-1}}}_{\text{Competition Effect}}\underbrace{\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{HF*}}\right)}_{\text{Productivity Effect}}
\end{aligned}
$$

where $V^* = C^* = \Phi\left(C_0^*\right)^{\theta_0}\left(C_1^*\right)^{\theta_1}$ and $P^* = \left(P_0^*\right)^{\theta_0}\left(P_1^*\right)^{\theta}$.

Aggregate Measures across Firms:

Aggregate labor demand, revenue, profits, and costs are given by

$$
\begin{aligned}
L^D &= \int_{Z^D}^{\infty} l^D(z) \frac{M dG(z)}{(1-G(Z^D))} = M \cdot l^D(\widetilde{Z}^D) = M \cdot \frac{1}{\tau_L^D} \frac{\sigma-1}{\sigma} \frac{r^D(\widetilde{Z}^D)}{W} \\
\frac{R^D}{W} &= \int_{Z^D}^{\infty} \tau_C^D \frac{r^D(z)}{W} \frac{M dG(z)}{(1-G(Z^D))} = M \cdot \tau_C^D \frac{r^D(\widetilde{Z}^D)}{W} \\
\frac{\Pi^D}{W} &= \int_{Z^D}^{\infty} \frac{\pi^D(z)}{W} \frac{M dG(z)}{(1-G(Z^D))} = M \cdot \frac{\pi^D(\widetilde{Z}^D)}{W} = M \cdot \tau_C^D \left[\frac{1}{\sigma} \frac{r^D(\widetilde{Z}^D)}{W} - f^D \right] \\
\frac{\Xi^D}{W} &= \int_{Z^D}^{\infty} \frac{\xi^D(z)}{W} \frac{M dG(z)}{(1-G(Z^D))} = M \cdot \frac{\xi^D(\widetilde{Z}^D)}{W} = M \cdot \tau_C^D \left[\frac{\sigma-1}{\sigma} \frac{r^D(\widetilde{Z}^D)}{W} + f^D \right]
\end{aligned}
$$

$$
\begin{aligned}
L^I &= \int_{Z^I}^{\infty} l^I(z) \frac{M^I dG^*(z)}{(1-G^*(Z^I))} = M^I \cdot l^I(\widetilde{Z}^I) = M^I \cdot \frac{1}{\tau_L^I} \frac{\sigma-1}{\sigma} \frac{r^I(\widetilde{Z}^I)}{W} \\
\frac{R^I}{W} &= \int_{Z^I}^{\infty} \tau_C^I \frac{r^I(z)}{W} \frac{M^I dG^*(z)}{(1-G^*(Z^I))} = M^I \cdot \tau_C^I \frac{r^I(\widetilde{Z}^I)}{W} \\
\frac{\Pi^I}{W} &= \int_{Z^I}^{\infty} \frac{\pi^I(z)}{W} \frac{M^I dG^*(z)}{(1-G^*(Z^I))} = M^I \cdot \frac{\pi^I(\widetilde{Z}^I)}{W} = M^I \cdot \tau_C^I \left[\frac{1}{\sigma} \frac{r^I(\widetilde{Z}^I)}{W} - f^I \right] \\
\frac{\Xi^I}{W} &= \int_{Z^I}^{\infty} \frac{\xi^I(z)}{W} \frac{M^I dG^*(z)}{(1-G^*(Z^I))} = M^I \cdot \frac{\xi^I(\widetilde{Z}^I)}{W} = M^I \cdot \tau_C^I \left[\frac{\sigma-1}{\sigma} \frac{r^I(\widetilde{Z}^I)}{W} + f^I \right]
\end{aligned}
$$

$$
\begin{aligned}
L^{D*} &= \int_{Z^{D*}}^{\infty} l^{D*}(z) \frac{M^* dG^*(z)}{(1-G^*(Z^{D*}))} = M^* \cdot l^{D*}(\widetilde{Z}^{D*}) = M^* \cdot \frac{1}{\tau_L^{D*}} \frac{\sigma-1}{\sigma} \frac{r^{D*}(\widetilde{Z}^{D*})}{W^*} \\
\frac{R^{D*}}{W^*} &= \int_{Z^{D*}}^{\infty} \tau_C^{D*} \frac{r^{D*}(z)}{W^*} \frac{M^* dG^*(z)}{(1-G^*(Z^{D*}))} = M^* \cdot \tau_C^{D*} \frac{r^{D*}(\widetilde{Z}^{D*})}{W^*} \\
\frac{\Pi^{D*}}{W^*} &= \int_{Z^{D*}}^{\infty} \frac{\pi^{D*}(z)}{W^*} \frac{M^* dG^*(z)}{(1-G^*(Z^{D*}))} = M^* \cdot \frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} = M^* \cdot \tau_C^{D*} \left[\frac{1}{\sigma} \frac{r^{D*}(\widetilde{Z}^{D*})}{W^*} - f^{D*} \right] \\
\frac{\Xi^{D*}}{W^*} &= \int_{Z^{D*}}^{\infty} \frac{\xi^{D*}(z)}{W^*} \frac{M^* dG^*(z)}{(1-G^*(Z^{D*}))} = M^* \cdot \frac{\xi^{D*}(\widetilde{Z}^{D*})}{W^*} = M^* \cdot \tau_C^{D*} \left[\frac{\sigma-1}{\sigma} \frac{r^{D*}(\widetilde{Z}^{D*})}{W^*} + f^{D*} \right]
\end{aligned}
$$

Market Share of Home local firms and Home FDI firms among Home firms, excluding Foreign local firms are defined as:

$$
MS^{H,D} = \frac{\frac{R^D}{W}}{\frac{R^D}{W} + \frac{R^I}{W}} \quad \text{and} \quad MS^{H,I} = \frac{\frac{R^I}{W}}{\frac{R^D}{W} + \frac{R^I}{W}}
$$

Market Share of Home local firms, Home FDI firms, and Foreign local firms in two countries are defined as:

$$
\begin{aligned}
MS^D &= \frac{\frac{R^D}{W}}{\frac{R^D}{W} + \frac{R^I}{W} + Q_L \frac{R^{D^*}}{W^*}} \\
MS^I &= \frac{\frac{R^I}{W}}{\frac{R^D}{W} + \frac{R^I}{W} + Q_L \frac{R^{D^*}}{W^*}} \\
MS^{D*} &= \frac{Q_L \frac{R^{D^*}}{W^*}}{\frac{R^D}{W} + \frac{R^I}{W} + Q_L \frac{R^{D^*}}{W^*}}
\end{aligned}
$$

A.2.2 Equilibrium Conditions and Proof of Propositions

A.2.2.1 Total Equilibrium Conditions

Recall we assume there is only one differentiated-good sector. Due to the presence of the homogeneous-good sector, we get $W = \epsilon W^* = P_0 = \epsilon P_0^*$ and $Q_L = \frac{\epsilon W^*}{W} = 1$. Since the labor real exchange rate Q_L is already pinned down, the resource constraint (A.2.25) needs not to be included in the equilibrium system. Instead of using labor market clearing conditions, we use budget constraints of households to find the equilibrium allocation: $\frac{PC}{W} = L + \frac{T}{W}$ and $\frac{P^*C^*}{W^*} = L^* + \frac{T^*}{W^*}$. We solve for eighteen endogenous variables: Z^D, Z^I, Z^{D*}, M, M^*, M^I, M^E, M^{E*}, $\frac{P^H}{W}$, $\frac{P^{H*}}{W^*}$, $\frac{P^{F*}}{W^*}$, $\frac{P^F}{W}$, $\frac{P}{W}$, $\frac{P^*}{W^*}$, C, C^*, $\frac{T}{W}$, $\frac{T^*}{W^*}$. When we take the lump-sum transfer to be chosen exogenously, then one of wedges among $\left[\tau_C^D, \tau_V^D, \tau_L^D, \tau_C^I, \tau_V^I, \tau_L^I\right]$ will be endogenously determined through Home government budget balance (A.2.26).

Definitions & Substitutes:

$$
\begin{aligned}
\sigma &= \frac{1}{1-\rho} \text{ and } \rho = \frac{\sigma-1}{\sigma} \\
A &\equiv \left[\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta\frac{PC}{W} + Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W} + Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
A^* &\equiv \left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1} \nu^*\theta\frac{P^*C^*}{W^*} + Q_L{}^{-\sigma}\left(\frac{P^{F}}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right] \\
&= \left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\left[\nu^*\theta\frac{P^*C^*}{W^*} + \frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right]
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{\eta} &\equiv \frac{\eta\,(z_{min})^{\eta}}{\eta-\sigma+1} \\
\widetilde{\eta}^* &\equiv \frac{\eta^*\,(z^*_{min})^{\eta^*}}{\eta^*-\sigma+1} \\
1-G(z) &= (z_{min})^{\eta} z^{-\eta} \\
1-G^*(z) &= (z^*_{min})^{\eta^*} z^{-\eta^*} \\
J(z) &= \widetilde{\eta}\,\{z\}^{-\eta+\sigma-1} \\
J^*(z) &= \widetilde{\eta}^*\,\{z\}^{-\eta^*+\sigma-1} \\
\frac{J(z)}{1-G(z)} &= \frac{\eta}{\eta-\sigma+1}\,(z)^{\sigma-1} \\
\frac{J^*(z)}{1-G^*(z)} &= \frac{\eta^*}{\eta^*-\sigma+1}\,(z)^{\sigma-1}
\end{aligned}
$$

$$
\begin{aligned}
M^{DI} &\equiv M + M^{I} \\
\widetilde{Z}^{DI} &\equiv \left[\frac{1}{M^{DI}} \left(M \left(\frac{\tau_L^D}{\tau_V^D} \frac{1}{\widetilde{Z}^D} \right)^{1-\sigma} + M^I \left(\frac{\tau_L^I}{\tau_V^I} \frac{1}{\widetilde{Z}^I} \right)^{1-\sigma} \right) \right]^{\frac{1}{\sigma-1}} \\
\log\left(M^{HF}\right) &\equiv \nu \log\left(M^{DI}\right) + (1-\nu)\log\left(M^*\right) \\
\log\left(\frac{1}{\widetilde{Z}^{HF}}\right) &\equiv \nu \log\left(\frac{1}{\widetilde{Z}^{DI}}\right) + (1-\nu)\log\left(Q_L \frac{\tau_L^{D*}}{\tau_V^{D*}} \frac{1}{\widetilde{Z}^{D*}}\right) \\
\log\left(M^{HF*}\right) &\equiv \nu^* \log\left(M^*\right) + (1-\nu^*)\log\left(M^{DI}\right) \\
\log\left(\frac{1}{\widetilde{Z}^{HF*}}\right) &\equiv \nu^* \log\left(\frac{\tau_L^{D*}}{\tau_V^{D*}} \frac{1}{\widetilde{Z}^{D*}}\right) + (1-\nu^*)\log\left(\frac{1}{Q_L} \frac{1}{\widetilde{Z}^{DI}}\right)
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{Z}^D &\equiv \left[\int_{Z^D}^{\infty} z^{\sigma-1} \frac{dG(z)}{1-G(Z^D)} \right]^{\frac{1}{\sigma-1}} = \left[\frac{J(Z^D)}{1-G(Z^D)} \right]^{\frac{1}{\sigma-1}} = \left[\frac{\eta}{\eta-\sigma+1} \right]^{\frac{1}{\sigma-1}} Z^D \\
\widetilde{Z}^I &\equiv \left[\int_{Z^I}^{\infty} (\alpha z)^{\sigma-1} \frac{dG^*(z)}{1-G^*(Z^I)} \right]^{\frac{1}{\sigma-1}} = \alpha \left[\frac{J^*(Z^I)}{1-G^*(Z^I)} \right]^{\frac{1}{\sigma-1}} = \alpha \left[\frac{\eta^*}{\eta^*-\sigma+1} \right]^{\frac{1}{\sigma-1}} Z^I \\
\widetilde{Z}^{D*} &\equiv \left[\int_{Z^{D*}}^{\infty} z^{\sigma-1} \frac{dG^*(z)}{1-G^*(Z^{D*})} \right]^{\frac{1}{\sigma-1}} = \left[\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})} \right]^{\frac{1}{\sigma-1}} = \left[\frac{\eta^*}{\eta^*-\sigma+1} \right]^{\frac{1}{\sigma-1}} Z^{D*} \\
\frac{J(Z^D)}{1-G(Z^D)} &= \left(\widetilde{Z}^D\right)^{\sigma-1} \text{ and } \left(\frac{\widetilde{Z}^D}{Z^D}\right)^{\sigma-1} = \int_{Z^D}^{\infty} \left(\frac{z}{Z^D}\right)^{\sigma-1} \frac{dG(z)}{1-G(Z^D)} = \frac{\eta}{\eta-\sigma+1} \\
\alpha^{\sigma-1} \frac{J^*(Z^I)}{1-G^*(Z^I)} &= \left(\widetilde{Z}^I\right)^{\sigma-1} \text{ and } \left(\frac{\widetilde{Z}^I}{\alpha Z^I}\right)^{\sigma-1} = \int_{Z^I}^{\infty} \left(\frac{z}{Z^I}\right)^{\sigma-1} \frac{dG^*(z)}{1-G^*(Z^I)} = \frac{\eta^*}{\eta^*-\sigma+1} \\
\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})} &= \left(\widetilde{Z}^{D*}\right)^{\sigma-1} \text{ and } \left(\frac{\widetilde{Z}^{D*}}{Z^{D*}}\right)^{\sigma-1} = \int_{Z^{D*}}^{\infty} \left(\frac{z}{Z^{D*}}\right)^{\sigma-1} \frac{dG^*(z)}{1-G^*(Z^{D*})} = \frac{\eta^*}{\eta^*-\sigma+1}
\end{aligned}
$$

Zero Profit Cutoff Productivity Conditions:

$$
\begin{aligned}
&(Z^D)^{\sigma-1} \tfrac{\tau_V^D}{\sigma} \left(\tfrac{1}{\rho} \tfrac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} \left[\left(\tfrac{P^H}{W}\right)^{\sigma-1} \nu\theta \tfrac{PC}{W} + Q_L{}^{\sigma} \left(\tfrac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \tfrac{P^*C^*}{W^*} \right] \\
= \;& \left[f^D + \left(\tfrac{1}{\lambda} - 1\right)\left(\zeta f^D - \chi F^D\right) \right]
\end{aligned}
\qquad \text{(A.2.9)}
$$

$$(\alpha Z^I)^{\sigma-1}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+{Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]=f^I \tag{A.2.10}$$

$$(Z^{D*})^{\sigma-1}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}+{Q_L}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right]=f^{D*} \tag{A.2.11}$$

Free Entry for Home firms:

$$F^D = \left(\tfrac{1-G(Z^D)}{\delta}\right)\tau_C^D\left\{\tfrac{J(Z^D)}{1-G(Z^D)}\tfrac{\tau_V^D}{\sigma}\left(\tfrac{1}{\rho}\tfrac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left[\begin{array}{c}\left(\tfrac{P^H}{W}\right)^{\sigma-1}\nu\theta\tfrac{PC}{W}\\ +\ \ {Q_L}^{\sigma}\left(\tfrac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\tfrac{P^*C^*}{W^*}\end{array}\right]-f^D\right\} \tag{A.2.12}$$

Free Entry for Foreign firms:

$$F^{D*} = \begin{array}{l}\left(\tfrac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}\left\{\tfrac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\tfrac{\tau_V^{D*}}{\sigma}\left(\tfrac{1}{\rho}\tfrac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left[\begin{array}{c}\left(\tfrac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\tfrac{P^*C^*}{W^*}\\ +\ \ {Q_L}^{-\sigma}\left(\tfrac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\tfrac{PC}{W}\end{array}\right]-f^{D*}\right\}\\ +\left(\tfrac{1-G^*(Z^I)}{Q_L\cdot\delta}\right)\tau_C^I\left\{\alpha^{\sigma-1}\tfrac{J^*(Z^I)}{1-G^*(Z^I)}\tfrac{\tau_V^I}{\sigma}\left(\tfrac{1}{\rho}\tfrac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left[\begin{array}{c}\left(\tfrac{P^H}{W}\right)^{\sigma-1}\nu\theta\tfrac{PC}{W}\\ +\ \ {Q_L}^{\sigma}\left(\tfrac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\tfrac{P^*C^*}{W^*}\end{array}\right]-f^I\right\}\end{array} \tag{A.2.13}$$

Price Index for Home:

$$\left(\frac{P^H}{W}\right)^{1-\sigma} = M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} + M^I\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} \quad \text{(A.2.14)}$$

$$\left(\frac{P^F}{W}\right) = Q_L\left(\frac{P^{F*}}{W^*}\right) \quad \text{(A.2.15)}$$

Price Index for Foreign:

$$\left(\frac{P^{F*}}{W^*}\right)^{1-\sigma} = M^*\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} \quad \text{(A.2.16)}$$

$$\left(\frac{P^{H*}}{W^*}\right) = \left(\frac{1}{Q_L}\right)\left(\frac{P^H}{W}\right) \quad \text{(A.2.17)}$$

The Evolution of the Mass of Firms:

$$M^E = \frac{\delta M}{(1-G(Z^D))} \quad \text{(A.2.18)}$$

$$M^{E*} = \frac{\delta M^*}{(1-G^*(Z^{D*}))} \quad \text{(A.2.19)}$$

$$M^I = \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)M^* = \left(\frac{Z^{D*}}{Z^I}\right)^{\eta^*}M^* \quad \text{(A.2.20)}$$

$$\frac{M^I}{1-G^*(Z^I)} = \frac{M^*}{1-G^*(Z^{D*})}$$

Aggregate Price Index:

$$\left(\frac{P}{W}\right)^{1-\sigma} = \left(\frac{P_0}{W}\right)^{\theta_0(1-\sigma)}\left(\frac{P^H}{W}\right)^{(1-\sigma)\nu\theta}\left(Q_L\frac{P^{F*}}{W^*}\right)^{(1-\sigma)(1-\nu)\theta} \quad \text{(A.2.21)}$$

$$\left(\frac{P^*}{W^*}\right)^{1-\sigma} = \left(\frac{P_0^*}{W^*}\right)^{\theta_0(1-\sigma)}\left(\frac{P^{F*}}{W^*}\right)^{(1-\sigma)\nu^*\theta}\left(\frac{1}{Q_L}\frac{P^H}{W}\right)^{(1-\sigma)(1-\nu^*)\theta} \quad \text{(A.2.22)}$$

where $W = P_0$, $W^* = P_0^*$, and $\theta_0 + \theta = 1$ hold.

Labor Market Clearing Condition in Home:

$$L - L_0 = \left(\begin{array}{ll} & M\left(\frac{\delta F^D}{1-G(Z^D)} + f^D\right) + M^I f^I \\ + & M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\ + & M^I\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \end{array} \right) \tag{A.2.23}$$

Labor Market Clearing Condition in Foreign:

$$L^* - L_0^* = \left(\begin{array}{ll} & M^*\left(\frac{\delta F^{D*}}{1-G^*(Z^{D*})} + f^{D*}\right) \\ + & M^*\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma}\left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*} + {Q_L}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right] \end{array} \right) \tag{A.2.24}$$

The Resource Constraint:

$$\begin{aligned} & \left(\frac{P_0}{W}C_0 - L_0\right) + M^* Q_L \frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\left[\frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right] \quad \text{(A.2.25)} \\ & + M^I \tau_C^I \left\{\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W} + Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] - f^I\right\} \\ = \; & M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\ & + M^I\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \end{aligned}$$

Home Government Budget Balance:

$$
\begin{aligned}
\frac{T}{W} &= M\left(\tau_L^D - 1\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W} + Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \qquad \text{(A.2.26)}\\
&+ M\left(1-\tau_V^D\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W} + Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]\\
&+ M\left(1-\tau_C^D\right)\left\{\frac{J(Z^D)}{1-G(Z^D)}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W} + Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] - f^D\right\}\\
&+ M^I\left(\tau_L^I - 1\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W} + Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]\\
&+ M^I\left(1-\tau_V^I\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W} + Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]\\
&+ M^I\left(1-\tau_C^I\right)\left\{\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W} + Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] - f^I\right\}
\end{aligned}
$$

Foreign Government Budget Balance:

$$
\begin{aligned}
\frac{T^*}{W^*} &= M^*\left(\tau_L^{D*} - 1\right)\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\left[\nu^*\theta\frac{P^*C^*}{W^*} + \frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right] \qquad \text{(A.2.27)}\\
&+ M^*\left(1-\tau_V^{D*}\right)\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\left[\nu^*\theta\frac{P^*C^*}{W^*} + \frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right]\\
&+ M^*\left(1-\tau_C^{D*}\right)\left\{\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\left[\nu^*\theta\frac{P^*C^*}{W^*} + \frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right] - f^{D*}\right\}
\end{aligned}
$$

A.2.2.2 Proof of Propositions

By combining equations for cutoff productivity (A.2.9) and (A.2.10) with equations for Home free entry condition (A.2.12), we can pin down Z^D and Z^I,

$$Z^D = z_{min} \left(\frac{\tau_C^D}{\delta}\right)^{\frac{1}{\eta}} \left(\frac{\eta}{F^D(\eta-\sigma+1)}\right)^{\frac{1}{\eta}} \left[\frac{f^D(\sigma-1)}{\eta} + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right]^{\frac{1}{\eta}} \quad \text{(A.2.28)}$$

$$\alpha Z^I = Z^D \left(\frac{f^I}{\left[f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right]}\right)^{\frac{1}{\sigma-1}} \left(\frac{\tau_V^D}{\tau_V^I}\right)^{\frac{\sigma}{\sigma-1}} \frac{\tau_L^I}{\tau_L^D} \quad \text{(A.2.29)}$$

$$= z_{min} \left(\frac{\tau_C^D}{\delta}\right)^{\frac{1}{\eta}} \left(\frac{\eta}{F^D(\eta-\sigma+1)}\right)^{\frac{1}{\eta}} \frac{\left[\frac{f^D(\sigma-1)}{\eta} + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right]^{\frac{1}{\eta}}}{\left[f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right]^{\frac{1}{\sigma-1}}} \left(f^I\right)^{\frac{1}{\sigma-1}} \left(\frac{\tau_V^D}{\tau_V^I}\right)^{\frac{\sigma}{\sigma-1}} \frac{\tau_L^I}{\tau_L^D}$$

Proposition 1. *If the term* $\left(\frac{f^I}{\left[f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right]}\right)^{\frac{1}{\sigma-1}} \left(\frac{\tau_V^D}{\tau_V^I}\right)^{\frac{\sigma}{\sigma-1}} \frac{\tau_L^I}{\tau_L^D}$ *is less than one, the cutoff productivity of FDI firms is lower than that of local firms:* $\alpha Z^I < Z^D$.

Proof. This proposition follows from the equation (A.2.29). □

Proposition 2. *Suppose there are no wedges from taxes:* $\tau_V^D = \tau_V^I = \tau_L^D = \tau_L^I = 1$ *in Home. In the case that there is no financial friction* ($\lambda = 1$)*, the cutoff productivity of FDI firms is not lower than that of domestic firms:* $\alpha Z^I \geq Z^D$.

Proof. This follows from the equation (A.2.29) with the assumption of fixed production costs: $f^I \geq f^D$. □

Proposition 3. *Suppose there are no wedges from taxes:* $\tau_V^D = \tau_V^I = \tau_L^D = \tau_L^I = 1$ *in Home. In the case that local firms and FDI firms have the same fixed production costs:* $f^I = f^D$*, the cutoff productivity of FDI firms is lower than that of domestic firms:* $\alpha Z^I < Z^D$ *under financial frictions,* $\lambda < 1$.

Proof. This follows from the equation (A.2.29) with the assumption of $\sigma > 1$, $k > 0$, $0 < \lambda < 1$, and $\zeta f^D - \chi F^D > 0$. □

A.2.2.3 Characterization of the equilibrium system

We restrict our model to have only one sector for the differentiated goods. The model is tractable enough to allow for the closed-form equilibrium allocation. Since there are tradable homogeneous goods which are produced in all coutries, we have $W = \epsilon W^* = P_0 = \epsilon P_0^*$ and $Q_L = \frac{\epsilon W^*}{W} = 1$. Firstly, by combining equations (A.2.9), (A.2.10), (A.2.11), (A.2.12), and (A.2.13), we can pin down Z^D, Z^I, and Z^{D*}:

$$
\begin{aligned}
Z^D &= \left\{ \left(\frac{\tau_C^D}{\delta}\right) \left(\frac{f^D}{F^D}\right) \left(\frac{(\sigma-1)\left(z_{min}\right)^{\eta}}{\eta-\sigma+1}\right) \left[1 + \frac{\eta}{\sigma-1}\left(\frac{1}{\lambda}-1\right)\left(\zeta - \chi\frac{F^D}{f^D}\right)\right]\right\}^{\frac{1}{\eta}} \\
\alpha Z^I &= Z^D \left(\frac{f^I}{\left[f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right]}\right)^{\frac{1}{\sigma-1}} \left(\frac{\tau_V^D}{\tau_V^I}\right)^{\frac{\sigma}{\sigma-1}} \frac{\tau_L^I}{\tau_L^D} \\
Z^{D*} &= \left\{ \frac{\tau_C^{D*} f^{D*} \frac{(\sigma-1)\left(z^*_{min}\right)^{\eta^*}}{\eta^*-\sigma+1}}{\delta F^{D*} - \left(\frac{1}{Q_L}\right)\tau_C^I f^I \frac{(\sigma-1)\left(z^*_{min}\right)^{\eta^*}}{\eta^*-\sigma+1}\left(Z^I\right)^{-\eta^*}} \right\}^{\frac{1}{\eta^*}}
\end{aligned}
$$

Therefore, we can find the market demand A and A^* from equations (A.2.9), (A.2.10), and (A.2.11):

$$
\begin{aligned}
A &= \left(\frac{P^H}{W}\right)^{\sigma-1} \left[\nu\theta\frac{PC}{W} + Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] = \frac{\left[f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right]}{\left(Z^D\right)^{\sigma-1}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}} \\
A^* &= \left(\frac{P^{F*}}{W^*}\right)^{\sigma-1} \left[\nu^*\theta\frac{P^*C^*}{W^*} + \frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right] = \frac{f^{D*}}{\left(Z^{D*}\right)^{\sigma-1}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}}
\end{aligned}
$$

where we have $\frac{\left[f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right]}{\left(Z^D\right)^{\sigma-1}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}} = \frac{f^I}{\left(\alpha Z^I\right)^{\sigma-1}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}}$.

Combining these with price indexes (A.2.14) and (A.2.16), we can obtain:

$$\begin{aligned}
\frac{1}{A^*}\left[\nu^*\theta\frac{P^*C^*}{W^*}+\frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right] &= \left(\frac{P^{F*}}{W^*}\right)^{1-\sigma} \\
&= M^*\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} \\
\frac{1}{A}\left[\nu\theta\frac{PC}{W}+Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] &= \left(\frac{P^H}{W}\right)^{1-\sigma} \\
&= \begin{array}{c} M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} \\ +\quad M^*\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\end{array}
\end{aligned}$$

That is,

$$\begin{aligned}
\left[\frac{(1-\nu)\theta}{Q_L}\frac{PC}{W}+\nu^*\theta\frac{P^*C^*}{W^*}\right] &= M^*\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^* \\
\left[\nu\theta\frac{PC}{W}+Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] &= \begin{array}{c} M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A \\ +\quad M^*\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A\end{array}
\end{aligned}$$

where we can solve for $\frac{PC}{W}$ and $\frac{P^*C^*}{W^*}$ by using:

$$\begin{pmatrix} \frac{(1-\nu)\theta}{Q_L} & \nu^*\theta \\ \nu\theta & Q_L(1-\nu^*)\theta \end{pmatrix}^{-1} = \frac{1}{\theta(\nu+\nu^*-1)}\begin{pmatrix} -Q_L(1-\nu^*) & \nu^* \\ \nu & -\frac{(1-\nu)}{Q_L} \end{pmatrix}$$

Therefore, we obtain:

$$\begin{aligned}
\frac{PC}{W} &= \frac{1}{\theta(\nu+\nu^*-1)}\begin{pmatrix} (M)(\nu^*)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A \\ +\quad (M^*)(-Q_L(1-\nu^*))\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^* \\ +\quad (M^*)(\nu^*)\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A \end{pmatrix} \\
\frac{P^*C^*}{W^*} &= \frac{1}{\theta(\nu+\nu^*-1)}\begin{pmatrix} (M)\left(\frac{-(1-\nu)}{Q_L}\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A \\ +\quad (M^*)(\nu)\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^* \\ +\quad (M^*)\left(\frac{-(1-\nu)}{Q_L}\right)\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A \end{pmatrix}
\end{aligned}$$

Note that Home and Foreign budget constraints are given by $\frac{PC}{W} = L + \frac{T}{W}$ and $\frac{P^*C^*}{W^*} = L^* + \frac{T^*}{W^*}$. By substituting out for $\frac{T}{W}$ and $\frac{T*}{W^*}$, we can rewrite government budget balance (A.2.26) and (A.2.27) as:

$$
\begin{aligned}
\frac{PC}{W} - L &= M\left(\tau_L^D - 1\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma} A \\
&+ M\left(1 - \tau_V^D\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} A \\
&+ M\left(1 - \tau_C^D\right)\left\{\frac{J(Z^D)}{1-G(Z^D)}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} A - f^D\right\} \\
&+ M^*\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(\tau_L^I - 1\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma} A \\
&+ M^*\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(1 - \tau_V^I\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} A \\
&+ M^*\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(1 - \tau_C^I\right)\left\{\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} A - f^I\right\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{P^*C^*}{W^*} - L^* &= M^*\left(\tau_L^{D*} - 1\right)\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} A^* \\
&+ M^*\left(1 - \tau_V^{D*}\right)\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} A^* \\
&+ M^*\left(1 - \tau_C^{D*}\right)\left\{\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} A^* - f^{D*}\right\}
\end{aligned}
$$

By feeding equations for $\frac{PC}{W}$ and $\frac{P^*C^*}{W^*}$ into government budget balance, we obtain two linear simultaneous equations for M and M^*:

$$\frac{1}{\theta(\nu+\nu^*-1)}\left(\begin{array}{cl} & (M)\,(\nu^*)\,\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A \\ + & (M^*)\,(-Q_L(1-\nu^*))\,\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^* \\ + & (M^*)\,(\nu^*)\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A \end{array}\right)-L$$

$$= M\left[\begin{array}{cl} & \left(\tau_L^D-1\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma}A \\ + & \left(1-\tau_V^D\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A \\ + & \left(1-\tau_C^D\right)\left\{\frac{J(Z^D)}{1-G(Z^D)}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A-f^D\right\} \end{array}\right]$$

$$+M^*\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left[\begin{array}{cl} & \left(\tau_L^I-1\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma}A \\ + & \left(1-\tau_V^I\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A \\ + & \left(1-\tau_C^I\right)\left\{\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A-f^I\right\} \end{array}\right]$$

and

$$\frac{1}{\theta(\nu+\nu^*-1)}\left(\begin{array}{cl} & (M)\left(\frac{-(1-\nu)}{Q_L}\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A \\ + & (M^*)\,(\nu)\,\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^* \\ + & (M^*)\left(\frac{-(1-\nu)}{Q_L}\right)\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A \end{array}\right)-L^*$$

$$= M^*\left[\begin{array}{cl} & \left(\tau_L^{D*}-1\right)\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma}A^* \\ + & \left(1-\tau_V^{D*}\right)\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^* \\ + & \left(1-\tau_C^{D*}\right)\left\{\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^*-f^{D*}\right\} \end{array}\right]$$

Define:

$$\Theta^D \equiv \frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A, \quad \Theta^I \equiv \frac{\alpha^{\sigma-1}J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A, \quad \Theta^{D*} \equiv \frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^*$$

Then we can find M and M^* by solving:

$$
L = M \left[\begin{array}{ll}
+ & \frac{(\nu^*)\Theta^D}{\theta(\nu+\nu^*-1)} \\
- & \left(\tau_L^D - 1\right)\Theta^D \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-1} \\
- & \left(1-\tau_V^D\right)\Theta^D \\
- & \left(1-\tau_C^D\right)\left\{\frac{\tau_V^D}{\sigma}\Theta^D - f^D\right\}
\end{array} \right] + M^* \left[\begin{array}{ll}
+ & \frac{(-Q_L(1-\nu^*))\Theta^{D*}}{\theta(\nu+\nu^*-1)} \\
+ & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{(\nu^*)\Theta^I}{\theta(\nu+\nu^*-1)} \\
- & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(\tau_L^I - 1\right)\Theta^I \left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-1} \\
- & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(1-\tau_V^I\right)\Theta^I \\
- & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(1-\tau_C^I\right)\left\{\frac{\tau_V^I}{\sigma}\Theta^I - f^I\right\}
\end{array} \right]
$$

$$
L^* = M \left[\begin{array}{ll}
+ & \frac{\left(\frac{-(1-\nu)}{Q_L}\right)\Theta^D}{\theta(\nu+\nu^*-1)}
\end{array} \right] + M^* \left[\begin{array}{ll}
+ & \frac{(\nu)\Theta^{D*}}{\theta(\nu+\nu^*-1)} \\
+ & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{\left(\frac{-(1-\nu)}{Q_L}\right)\Theta^I}{\theta(\nu+\nu^*-1)} \\
- & \left(\tau_L^{D*} - 1\right)\Theta^{D*} \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-1} \\
- & \left(1-\tau_V^{D*}\right)\Theta^{D*} \\
- & \left(1-\tau_C^{D*}\right)\left\{\frac{\tau_V^{D*}}{\sigma}\Theta^{D*} - f^{D*}\right\}
\end{array} \right]
$$

which corresponds to:

$$
\left[\begin{array}{cc}
\left(\begin{array}{ll}
+ & \frac{(\nu^*)\Theta^D}{\theta(\nu+\nu^*-1)} \\
- & \left(\tau_L^D - 1\right)\Theta^D \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-1} \\
- & \left(1-\tau_V^D\right)\Theta^D \\
- & \left(1-\tau_C^D\right)\left\{\frac{\tau_V^D}{\sigma}\Theta^D - f^D\right\}
\end{array} \right) &
\left(\begin{array}{ll}
+ & \frac{(-Q_L(1-\nu^*))\Theta^{D*}}{\theta(\nu+\nu^*-1)} \\
+ & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{(\nu^*)\Theta^I}{\theta(\nu+\nu^*-1)} \\
- & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(\tau_L^I - 1\right)\Theta^I \left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-1} \\
- & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(1-\tau_V^I\right)\Theta^I \\
- & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(1-\tau_C^I\right)\left\{\frac{\tau_V^I}{\sigma}\Theta^I - f^I\right\}
\end{array} \right) \\
\left(\begin{array}{ll}
+ & \frac{\left(\frac{-(1-\nu)}{Q_L}\right)\Theta^D}{\theta(\nu+\nu^*-1)}
\end{array} \right) &
\left(\begin{array}{ll}
+ & \frac{(\nu)\Theta^{D*}}{\theta(\nu+\nu^*-1)} \\
+ & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{\left(\frac{-(1-\nu)}{Q_L}\right)\Theta^I}{\theta(\nu+\nu^*-1)} \\
- & \left(\tau_L^{D*} - 1\right)\Theta^{D*} \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-1} \\
- & \left(1-\tau_V^{D*}\right)\Theta^{D*} \\
- & \left(1-\tau_C^{D*}\right)\left\{\frac{\tau_V^{D*}}{\sigma}\Theta^{D*} - f^{D*}\right\}
\end{array} \right)
\end{array} \right]
\left(\begin{array}{c} M \\ M^* \end{array} \right) = \left(\begin{array}{c} L \\ L^* \end{array} \right)
$$

where $M^I = \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) M^* = \left(\frac{Z^{D*}}{Z^I}\right)^{\eta^*} M^*$, $M^E = \frac{\delta M}{(1-G(Z^D))}$, and $M^{E*} = \frac{\delta M^*}{(1-G^*(Z^{D*}))}$.

Therefore, we have found Z^D, Z^I, Z^{D*}, A, A^*, M, M^*, M^I, M^E, and M^{E*}. We can derive the rest of endogenous variables as follows.

$$\left(\frac{P^H}{W}\right) = \left\{ M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} + M^I\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\right\}^{\frac{1}{1-\sigma}},$$

$$\left(\frac{P^{F*}}{W^*}\right) = (M^*)^{\frac{1}{1-\sigma}}\left(\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\right)^{\frac{1}{1-\sigma}}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right),$$

where $\left(\frac{P^F}{W}\right) = Q_L\left(\frac{P^{F*}}{W^*}\right)$ and $\left(\frac{P^{H*}}{W^*}\right) = \left(\frac{1}{Q_L}\right)\left(\frac{P^H}{W}\right)$.

$$\left(\frac{P}{W}\right) = \left(\frac{P_0}{W}\right)^{\theta_0}\left(\frac{P^H}{W}\right)^{\nu\theta}\left(Q_L\frac{P^{F*}}{W^*}\right)^{(1-\nu)\theta}$$

$$\left(\frac{P^*}{W^*}\right) = \left(\frac{P_0^*}{W^*}\right)^{\theta_0}\left(\frac{P^{F*}}{W^*}\right)^{\nu^*\theta}\left(\frac{1}{Q_L}\frac{P^H}{W}\right)^{(1-\nu^*)\theta}$$

where $W = P_0$, $W^* = P_0^*$, and $\theta_0 + \theta = 1$ hold. Then we have:

$$\frac{PC}{W} = \left(\begin{array}{cl} & (M)\,\frac{(\nu^*)\Theta^D}{\theta(\nu+\nu^*-1)} \\ + & (M^*)\,\frac{(-Q_L(1-\nu^*))\Theta^{D*}}{\theta(\nu+\nu^*-1)} \\ + & (M^I)\,\frac{(\nu^*)\Theta^I}{\theta(\nu+\nu^*-1)} \end{array}\right), \quad \text{and} \quad \frac{P^*C^*}{W^*} = \left(\begin{array}{cl} & (M)\,\frac{\left(\frac{-(1-\nu)}{Q_L}\right)\Theta^D}{\theta(\nu+\nu^*-1)} \\ + & (M^*)\,\frac{(\nu)\Theta^{D*}}{\theta(\nu+\nu^*-1)} \\ + & (M^I)\,\frac{\left(\frac{-(1-\nu)}{Q_L}\right)\Theta^I}{\theta(\nu+\nu^*-1)} \end{array}\right),$$

$\frac{T}{W} = \frac{PC}{W} - L$, $\frac{T^*}{W^*} = \frac{P^*C^*}{W^*} - L^*$, $C = \frac{\left(\frac{PC}{W}\right)}{\left(\frac{P}{W}\right)}$, $C^* = \frac{\left(\frac{P^*C^*}{W^*}\right)}{\left(\frac{P^*}{W^*}\right)}$, $C_0 = \left(\frac{P_0}{W}\right)^{-1}\theta_0\left(\frac{PC}{W}\right)$, $C_0^* = \left(\frac{P_0^*}{W^*}\right)^{-1}\theta_0\left(\frac{P^*C^*}{W^*}\right)$,

$$L_0 = L - \left(\begin{array}{cl} & M\left(\frac{\delta F^D}{1-G(Z^D)} + f^D\right) + M^I f^I \\ + & M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma} A \\ + & M^I\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma} A \end{array}\right)$$

$$L_0^* = L^* - \left(\begin{array}{cl} & M^*\left(\frac{\delta F^{D*}}{1-G^*(Z^{D*})} + f^{D*}\right) \\ + & M^*\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} A^* \end{array}\right)$$

where $W = P_0$ and $W^* = P_0^*$.

A.2.3 Average Measures under Pareto distribution

If we assume that productivity of firms follows the Pareto distribution, then it makes average measures constant. Note that we have derived the market demand as:

$$
\begin{aligned}
A &\equiv \left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]=\left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W}+Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \frac{\left[f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right]}{(Z^D)^{\sigma-1}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}}=\frac{f^I}{(\alpha Z^I)^{\sigma-1}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}} \\
A^* &\equiv \left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}+Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right]=\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\left[\nu^*\theta\frac{P^*C^*}{W^*}+\frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right] \\
&= \frac{f^{D*}}{(Z^{D*})^{\sigma-1}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}}
\end{aligned}
$$

Also, average productivity is linear in cutoff productivity:

$$
\begin{aligned}
\widetilde{Z}^D &= \left[\int_{Z^D}^{\infty} z^{\sigma-1}\frac{dG(z)}{1-G(Z^D)}\right]^{\frac{1}{\sigma-1}}=\left[\frac{\eta}{\eta-\sigma+1}\right]^{\frac{1}{\sigma-1}}Z^D \\
\widetilde{Z}^I &= \left[\int_{Z^I}^{\infty}(\alpha z)^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^I)}\right]^{\frac{1}{\sigma-1}}=\alpha\left[\frac{\eta^*}{\eta^*-\sigma+1}\right]^{\frac{1}{\sigma-1}}Z^I \\
\widetilde{Z}^{D*} &= \left[\int_{Z^{D*}}^{\infty} z^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^{D*})}\right]^{\frac{1}{\sigma-1}}=\left[\frac{\eta^*}{\eta^*-\sigma+1}\right]^{\frac{1}{\sigma-1}}Z^{D*}
\end{aligned}
$$

Then we can characterize average labor as follows:

$$
\begin{aligned}
l^D(\widetilde{Z}^D) &= \left(\widetilde{Z}^D\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma} A = \left(\frac{\eta}{\eta-\sigma+1}\right)\left(Z^D\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma}\frac{\left(f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right)}{\left(Z^D\right)^{\sigma-1}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}} \\
&= \left(f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right)\left(\frac{\sigma-1}{\tau_L^D}\right)\left(\frac{\eta}{\eta-\sigma+1}\right) \\
l^I\left(\widetilde{Z}^I\right) &= \left(\widetilde{Z}^I\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma} A = \alpha^{\sigma-1}\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)\left(Z^I\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma}\left(\frac{f^I}{\left(\alpha Z^I\right)^{\sigma-1}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}}\right) \\
&= f^I\left(\frac{\sigma-1}{\tau_L^I}\right)\left(\frac{\eta^*}{\eta^*-\sigma+1}\right) \\
l^{D*}\left(\widetilde{Z}^{D*}\right) &= \left(\widetilde{Z}^{D*}\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} A^* = \left(\frac{\eta^*}{\eta^*-\sigma+1}\right)\left(Z^{D*}\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma}\left(\frac{f^{D*}}{\left(Z^{D*}\right)^{\sigma-1}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}}\right) \\
&= f^{D*}\left(\frac{\sigma-1}{\tau_L^{D*}}\right)\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)
\end{aligned}
$$

And average revenues are given by:

$$
\begin{aligned}
\tau_C^D\frac{r^D(\widetilde{Z}^D)}{W} &= \tau_C^D\left(\frac{\tau_L^D}{\rho}\right)l^D(\widetilde{Z}^D) = \tau_C^D\sigma\left(f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right)\left(\frac{\eta}{\eta-\sigma+1}\right) \\
\tau_C^I\frac{r^I(\widetilde{Z}^I)}{W} &= \tau_C^I\left(\frac{\tau_L^I}{\rho}\right)l^I(\widetilde{Z}^I) = \tau_C^I\sigma f^I\left(\frac{\eta^*}{\eta^*-\sigma+1}\right) \\
\tau_C^{D*}\frac{r^{D*}(\widetilde{Z}^{D*})}{W^*} &= \tau_C^{D*}\left(\frac{\tau_L^{D*}}{\rho}\right)l^{D*}(\widetilde{Z}^{D*}) = \tau_C^{D*}\sigma f^{D*}\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)
\end{aligned}
$$

And average profits are given by:

$$
\begin{aligned}
\frac{\pi^D(\widetilde{Z}^D)}{W} &= \tau_C^D\left[\frac{\tau_L^D}{\sigma-1}l^D(\widetilde{Z}^D)-f^D\right] = \tau_C^D\left[\left(f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right)\left(\frac{\eta}{\eta-\sigma+1}\right)-f^D\right] \\
\frac{\pi^I(\widetilde{Z}^I)}{W} &= \tau_C^I\left[\frac{\tau_L^I}{\sigma-1}l^I(\widetilde{Z}^I)-f^I\right] = \tau_C^I\left[f^I\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)-f^I\right] \\
\frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} &= \tau_C^{D*}\left[\frac{\tau_L^{D*}}{\sigma-1}l^{D*}(\widetilde{Z}^{D*})-f^{D*}\right] = \tau_C^{D*}\left[f^{D*}\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)-f^{D*}\right]
\end{aligned}
$$

And average costs are given by:

$$\frac{\xi^D(\widetilde{Z}^D)}{W} = \tau_C^D\left[\frac{\sigma-1}{\sigma}\frac{r^D(\widetilde{Z}^D)}{W}+f^D\right] = \tau_C^D\left[(\sigma-1)\left(f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)\right)\left(\frac{\eta}{\eta-\sigma+1}\right)+f^D\right]$$

$$\frac{\xi^I(\widetilde{Z}^I)}{W} = \tau_C^I\left[\frac{\sigma-1}{\sigma}\frac{r^I(\widetilde{Z}^I)}{W}+f^I\right] = \tau_C^I\left[(\sigma-1)f^I\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)+f^I\right]$$

$$\frac{\xi^{D*}(\widetilde{Z}^{D*})}{W^*} = \tau_C^{D*}\left[\frac{\sigma-1}{\sigma}\frac{r^{D*}(\widetilde{Z}^{D*})}{W^*}+f^{D*}\right] = \tau_C^{D*}\left[(\sigma-1)f^{D*}\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)+f^{D*}\right]$$

A.3 Empirical Target Moments in Calibration

This section presents the details of empirical target moments used in our model calibration. We take our model to the Chinese data in 2000. We define a FDI firm as a firm of which capital is occupied by foreigners by more than 10%. In calibrating the ratio of fixed production costs between FDI and local firms, we take the empirical tangible asset ratio between these firms averaged over eight years, 1.115. We assume the asset depreciation is the same across all firms.

Table A.3: Empirical Tangible Asset Ratio for Calibration

	Foreign Capital ≥ 10%	Foreign Capital ≥ 25%
Year	$\frac{\text{FDI fixed assets per FDI firm}}{\text{Local fixed assets per local firm}}$	$\frac{\text{FDI fixed assets per FDI firm}}{\text{Local fixed assets per local firm}}$
2000	1.288	1.217
2001	1.185	1.110
2002	1.074	1.010
2003	1.020	0.975
2004	1.072	1.024
2005	1.065	0.994
2006	1.111	1.057
2007	1.103	1.059
Average	1.115	1.056

All data are from manufacturing sectors and from Chinese firm data.

Model Ratios in Aggregate: For calibration, the model moments are defined as follows:

(1) Ratio of the value of exports to the value of imports in aggregate

$$\frac{\left[\int_{\omega\in\Omega}\frac{P^D(\omega)}{W}y^{D,X}(\omega)d\omega+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I,X}(\omega^*)d\omega^*\right]}{\left[\int_{\omega^*\in\Omega^*}Q_L\frac{P^{D*}(\omega^*)}{W^*}y^{D,X*}(\omega^*)d\omega^*\right]}$$

$$=\quad\frac{M\int_{Z^D}^{\infty}\frac{p^D(z)}{W}\left(y^{D,X}(z)\right)\frac{dG(z)}{1-G(Z^D)}+M^I\int_{Z^I}^{\infty}\frac{p^I(\alpha z)}{W}\left(y^{I,X}(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)}}{Q_LM^*\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})}}$$

(2) Ratio of the value of exports to the value of domestic sales in aggregate

$$\frac{\left[\int_{\omega\in\Omega}\frac{P^D(\omega)}{W}y^{D,X}(\omega)d\omega+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I,X}(\omega^*)d\omega^*\right]}{\left[\int_{\omega\in\Omega}\frac{P^D(\omega)}{W}y^{D}(\omega)d\omega+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I}(\omega^*)d\omega^*\right]}$$

(3) Ratio of the value of imports to the value of total products in aggregate

$$\frac{\left[\int_{\omega^*\in\Omega^*}Q_L\frac{P^{D*}(\omega^*)}{W^*}y^{D,X*}(\omega^*)d\omega^*\right]}{\left[\int_{\omega\in\Omega}\frac{P^D(\omega)}{W}y^{D}(\omega)d\omega+\int_{\omega\in\Omega}\frac{P^D(\omega)}{W}y^{D,X}(\omega)d\omega+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I}(\omega^*)d\omega^*+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I,X}(\omega^*)d\omega^*\right]}$$

(4) Ratio of the value of FDI-firm products to the value of total products in aggregate

$$\frac{\left[\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I}(\omega^*)d\omega^*+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I,X}(\omega^*)d\omega^*\right]}{\left[\int_{\omega\in\Omega}\frac{P^D(\omega)}{W}y^{D}(\omega)d\omega+\int_{\omega\in\Omega}\frac{P^D(\omega)}{W}y^{D,X}(\omega)d\omega+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I}(\omega^*)d\omega^*+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I,X}(\omega^*)d\omega^*\right]}$$

A.4 More Details on Counterfactuals

A.4.1 Removing tax benefits of FDI firms

Welfare gains from the tax reform exhibit a humped shape. In the second row of Figure A.1, we plotted Home consumption C on the left[3] and its percent changes relative to the benchmark level under $\tau_V^I = 0.85$ on the right. To see why welfare gains are not monotone, observe the third row of Figure A.1. The effect from the varieties raises consumption for all tax rates in the experiment, however, the effect from aggregate productivity exhibits a humped shape: it increases up to 31% of tax on FDI and then decreases. Adverse effect from decreasing productivity becomes more dominant as the tax on FDI rises further beyond 31% and eventually consumption declines when the tax rate exceeds 33%.

The gains from product varieties are positive under the tax reform. Due to the tax-cut on local firms, more and more Home domestic incumbents operate their businesses while more and more FDI firms exit. Their net effect is the gain in total varieties. However, aggregate productivity also shows a humped-shape pattern like consumption. In the net effect, the welfare initially increases until it reaches the maximum at around 33% of tax on revenues of FDI firms, and then it monotonically decreases.

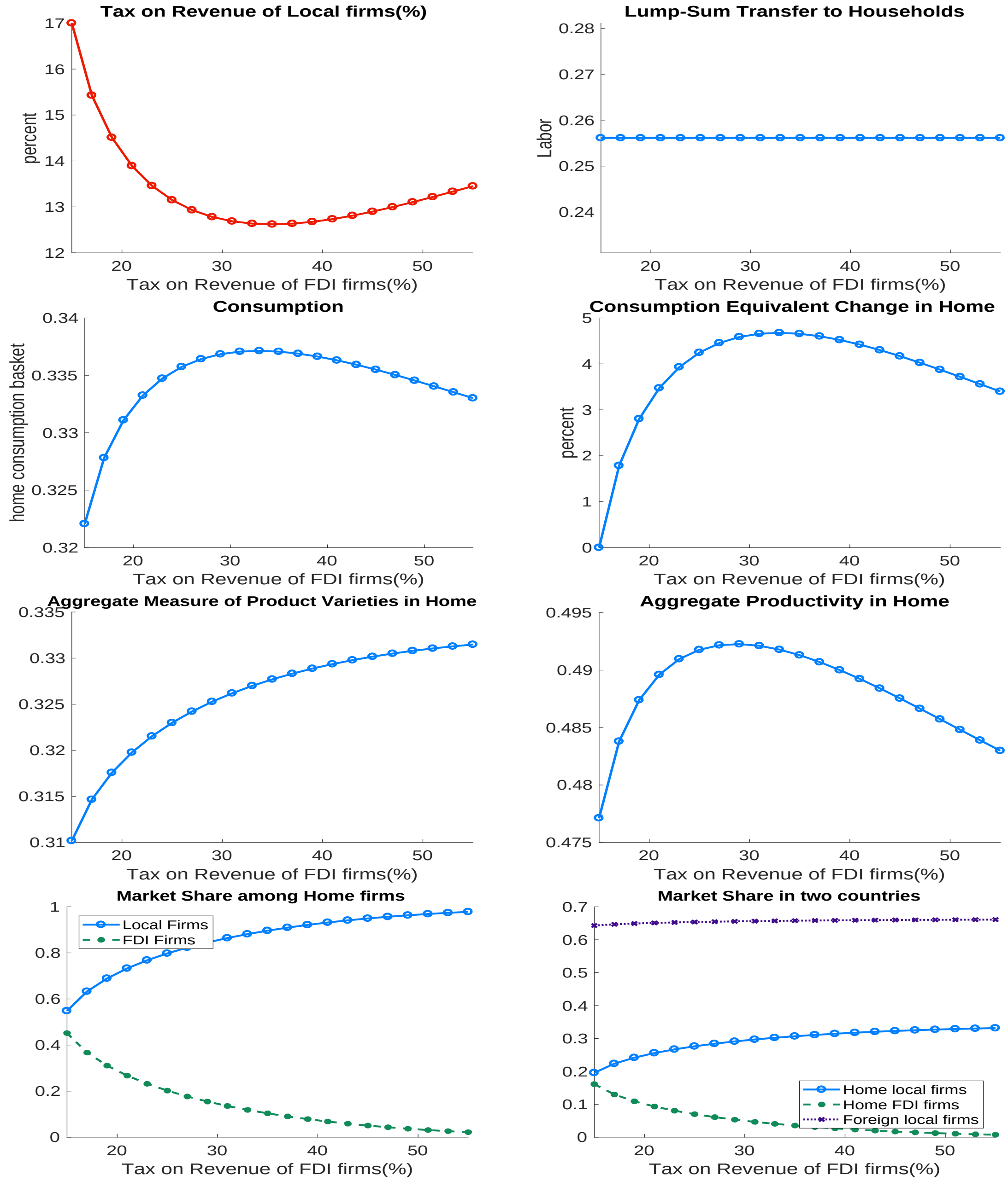
The market share of FDI firms is replaced with that of local firms and the market share of Foreign local firms increases. The tax reform drives out low-productivity FDI firms by raising its cutoff productivity, but the revenue tax changes do not affect the cutoff productivity of Home local firms and it attracts more low-productivity local firms in Foreign.

A humped shape in aggregate productivity: The appendix A.2.1 shows that the aggregate productivity in Home, $\widetilde{Z}^{HF}$, is defined as

$$\log\left(\widetilde{Z}^{HF}\right) \equiv \nu \log\left(\widetilde{Z}^{DI}\right) + (1-\nu)\log\left(\widetilde{Z}^{D*}\right),$$

[3] Note that aggregate consumption, C, is periodic utility itself since our framework is static: $C = V = \Phi {C_0}^{\theta_0} {C_1}^{\theta} = \Phi\left(C_0\right)^{\theta_0}\left(\left(M^{HF}\right)^{\frac{1}{\sigma-1}}\left(\rho\widetilde{Z}^{HF}\right)\theta\left(L+\frac{T}{W}\right)\right)^{\theta}$.

Figure A.1: Value-Added Tax Reform with varying τ_V^D under $\lambda = 0.70$, $\tau_C^D = 0.67$, and $\tau_C^I = 0.85$



The Home aggregate productivity, $\widetilde{Z}^{HF}$, puts a larger weight on average productivity measures of Home local and FDI firms relative to that of Foreign local firms due to the presence of home bias, $\nu = 0.83$. Among Home local and FDI firms, the productivity aggregator, $\widetilde{Z}^{DI}$, gives more importance on those firms who have larger mass.

Figure A.2 shows the effect of the tax reform on productivity. The top two subfigures plot the Home aggregate productivity, $\widetilde{Z}^{HF}$, on the left and productivity aggregator over local and FDI firms, $\widetilde{Z}^{DI}$ on the right. Subfigures in the second row present effective productivity measures which are adjusted by after-tax wedges: $\tau_V^D \widetilde{Z}^D$ and $\tau_V^I \widetilde{Z}^I$. We plot such revenue wedges on local and FDI firms in the third row: τ_V^D on the left and τ_V^I on the right. The bottom two subfigures plot cutoff productivity levels of Home local and FDI firms, and Foreign local firms.

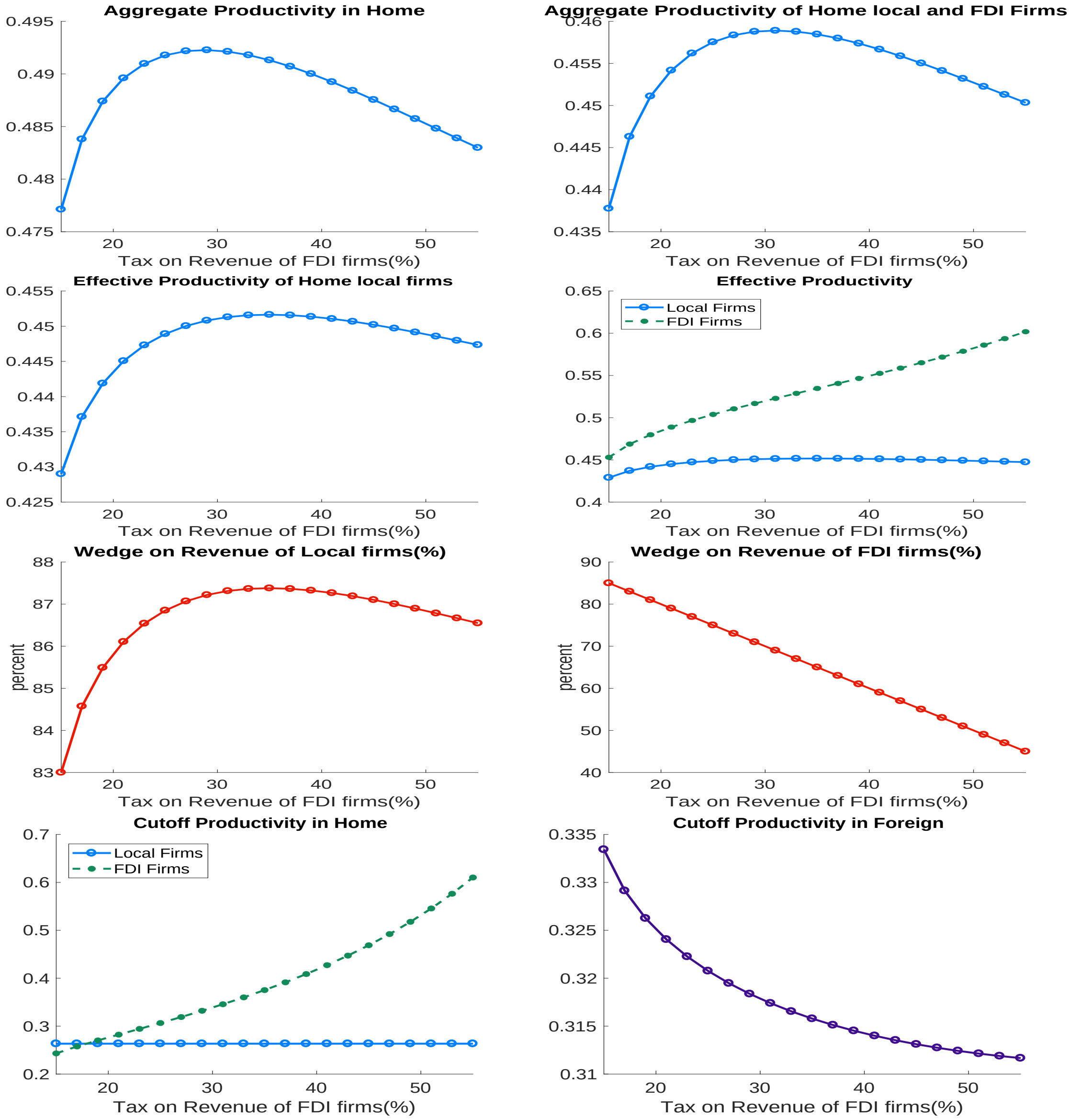
Be reminded that unlike the standard trade literature, we assume exporting is costless and our model abstracts from trade costs. We adopt this approach since our focus is to evaluate the effect of government policies on reallocations between local and FDI firms under financial market imperfection and tax distortions in the FDI host country. Tax on revenue, $1 - \tau_V$, reduces marginal revenue of firms, and hence it aggravates effective productivity by requiring more labor in producing one unit of product. Higher revenue tax, or lower wedge on revenue ($\tau_V \Downarrow$), negatively affects aggregate productivity $\widetilde{Z}^{HF}$.

Under the tax reform, effective productivity of Home local firms dominates those of FDI and Foreign local firms due to their larger mass and home bias. The variation in taxes on Home local and FDI firms does not change local firms' cutoff productivity. However, the change in the wedge on local firms' revenue mainly drives the change in Home effective productivity. Hence, the non-monotone feature of aggregate productivity is mainly driven by the wedge on revenue of Home local firms.

The appendix A.2 derives cutoff productivity levels of all firms. For Home firms, they are given by

$$
\begin{aligned}
Z^D &= \left\{ \left(\frac{\tau_C^D}{\delta}\right) \left(\frac{f^D}{F^D}\right) \left(\frac{(\sigma-1)(z_{min})^\eta}{\eta-\sigma+1}\right) \left[1 + \frac{\eta}{\sigma-1}\left(\frac{1}{\lambda}-1\right)\left(\zeta - \chi\frac{F^D}{f^D}\right)\right]\right\}^{\frac{1}{\eta}}, \\
\alpha Z^I &= Z^D \left(\frac{f^I}{\left[f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right]}\right)^{\frac{1}{\sigma-1}} \left(\frac{\tau_V^D}{\tau_V^I}\right)^{\frac{\sigma}{\sigma-1}}.
\end{aligned}
$$

Figure A.2: Value-Added Tax Reform with varying τ_V^D under $\lambda = 0.70$, $\tau_C^D = 0.67$, and $\tau_C^I = 0.85$



It shows that the increase in revenue tax on FDI relative to that on local firms, $\frac{\tau_V^I}{\tau_V^D} \Downarrow$, raises FDI cutoff productivity, $Z^I \Uparrow$. Higher cutoff for FDI firms, $Z^I \Uparrow$, leads to lower cutoff for Foreign firms, $Z^{D*} \Downarrow$, as shown by the free entry condition in Foreign, given by:

$$F^{D*} = \left(\frac{1 - G^*(Z^{D*})}{\delta}\right) \frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} + \left(\frac{1 - G^*(Z^I)}{\delta}\right) \frac{\pi^I(\widetilde{Z}^I)}{W}.$$

This condition implies that if the market environment requires FDI firms to be highly productive for their survival ($Z^I \Uparrow$), then more low-productivity local firms in Foreign can enter ($Z^{D*} \Downarrow$).[4] Therefore, the rise of revenue tax on FDI firms has negative spillover effect on cutoff productivity of Foreign firms.

Decrease in composite varieties: Now we move on to the competition effect. We define the aggregate mass of Home firms as

$$\log\left(M^{HF}\right) \equiv \nu \log\left(M + M^I\right) + (1-\nu)\log\left(M^*\right).$$

The top-left subfigure in Figure A.3 shows the aggregate measure for Home variety, M^{HF}, strictly increases. This is due to the dominant effect of the increase in the mass of Home local firms, M. The combination of labor market clearing conditions and free entry conditions leads to the following equilibrium conditions as shown in the appendix A.2:

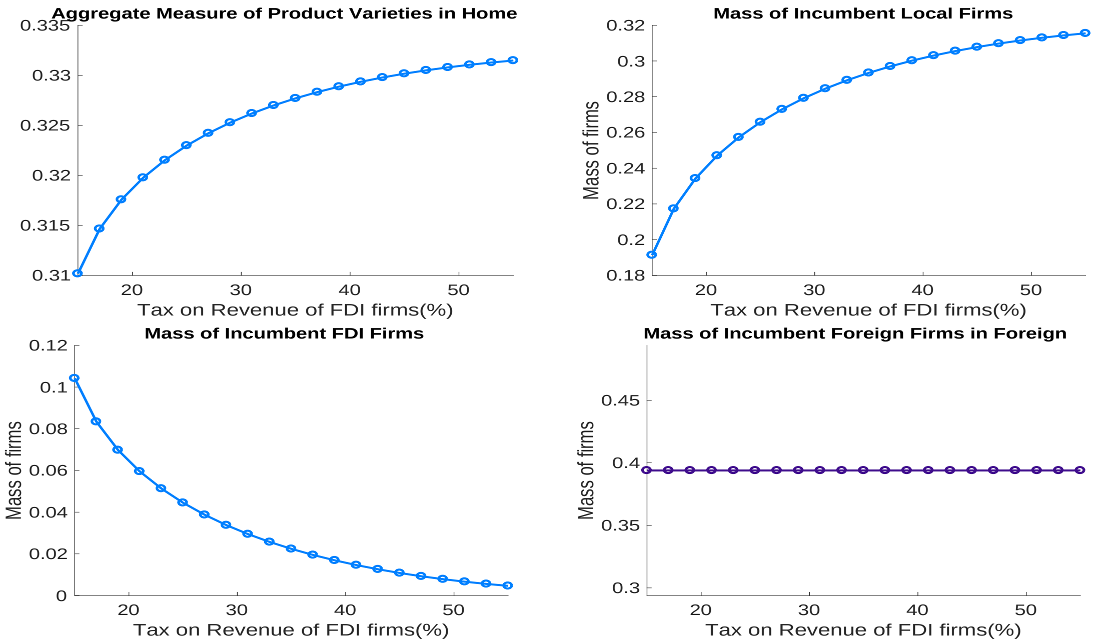
$$\begin{aligned} M &= \frac{(L-L_0) - \boldsymbol{M^I}\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]}{\left(\frac{\tau_C^D}{\sigma-1}+1\right)\boldsymbol{l^D}\left(\boldsymbol{\widetilde{Z}^D}\right) + (1-\tau_C^D)\,f^D}, \\ M^* &= \frac{L^* - \boldsymbol{L_0^*}}{\left\{\left(\frac{\tau_C^{D*}}{\sigma-1}+1\right) l^{D*}(\widetilde{Z}^{D*}) + (1-\tau_C^{D*})\,f^{D*} + \left(\frac{\boldsymbol{1-G^*(Z^I)}}{\boldsymbol{1-G^*(Z^{D*})}}\right)\frac{\tau_C^I}{Q_L}\left[\frac{1}{\sigma-1} l^I(\widetilde{Z}^I) - f^I\right]\right\}}, \end{aligned}$$

where $\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) = \left(\frac{Z^{D*}}{Z^I}\right)^{\eta^*}$ holds. The average labor demand can be derived as $l^D(\widetilde{Z}^D) = \left(f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right)(\sigma-1)\left(\frac{\eta}{\eta-\sigma+1}\right)$ for Home local firms, $l^I\left(\widetilde{Z}^I\right) = f^I(\sigma-1)\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)$ for Home FDI firms, and $l^{D*}\left(\widetilde{Z}^{D*}\right) = f^{D*}(\sigma-1)\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)$ for Foreign domestic firms (see the appendix A.2.3).

[4] In the appendix A.2.3, we show average profits can be derived as $\frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} = \tau_C^{D*} f^{D*}\left(\frac{\sigma-1}{\eta^*-\sigma+1}\right)$ and $\frac{\pi^I(\widetilde{Z}^I)}{W} = \tau_C^I f^I\left(\frac{\sigma-1}{\eta^*-\sigma+1}\right)$. That is, average profits do not depend on cutoff productivity. To be concrete, the cutoff productivity for Foreign firms is determined by $Z^{D*} = \left\{\frac{\tau_C^{D*} f^{D*}\frac{(\sigma-1)(z^*_{min})^{\eta^*}}{\eta^*-\sigma+1}}{\delta F^{D*} - \tau_C^I f^I \frac{(\sigma-1)(z^*_{min})^{\eta^*}}{\eta^*-\sigma+1}\boldsymbol{\left(\frac{1}{Z^I}\right)^{\eta^*}}}\right\}^{\frac{1}{\eta^*}}$, which shows its negative association with Z^I clearly.

There are two determinants for the mass of Home local firms, M. Firstly, when average labor demand among Home local firms, $l^D(\widetilde{Z}^D)$, is higher, it makes competition among local firms in hiring labor harder, and thus the mass of firms decreases. Second, if there are more FDI firms in operation, $M^I \Uparrow$, then this leads to stronger competition for hiring Home labor and so the mass of Home local firms gets smaller. In the counterfactual experiment, the mass of FDI firms decreases due to the rise in its cutoff productivity[5] and average labor demand stays constant. Therefore, the mass of Home local firms, M, increases under the tax reform. The mass of Foreign firms, M^*, stays constant since the effect from the increase in the labor demand in the homogeneous sector, $L_0^* \Uparrow$, and the effect from the decrease in $\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})} \Downarrow$ are cancelled out.

Figure A.3: Value-Added Tax Reform with varying τ_V^D under $\lambda = 0.70$, $\tau_C^D = 0.67$, and $\tau_C^I = 0.85$



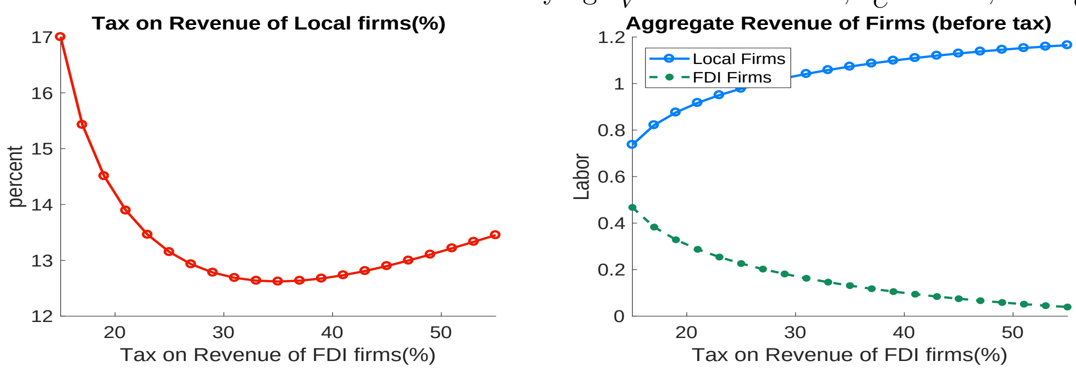
[5] The mass of FDI firms is defined as $M^I \equiv M^*\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)$. The negative relation of the mass M^I to its cutoff Z^I can be shown as $\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) = \frac{(z^*_{min})^{\eta^*}\frac{\pi^{D*}(\tilde{Z}^{D*})}{W^*}}{(Z^I)^{\eta^*}\delta F^{D*}-\left(\frac{1}{Q_L}\right)(z^*_{min})^{\eta^*}\frac{\pi^I(\tilde{Z}^I)}{W}} = \frac{\tau_C^{D*} f^{D*}\frac{(\sigma-1)(z^*_{min})^{\eta^*}}{\eta^*-\sigma+1}}{(Z^I)^{\eta^*}\delta F^{D*}-\left(\frac{1}{Q_L}\right)\tau_C^I f^I\frac{(\sigma-1)(z^*_{min})^{\eta^*}}{\eta^*-\sigma+1}}$.

A humped shape in wedge on revenue of Home local firms, τ_V^D: We find that revenue tax on Home local firms, $1-\tau_V^D$, exhibits a humped shape as reproduced in Figure A.4. To see why this occurs, we also plot before-tax aggregate revenues of local and FDI firms. The appendix A.2.3 shows before-tax revenues can be written out as:

$$\begin{aligned}\left(\frac{R^D}{W}\right)^{B.T} &= \frac{M}{\tau_V^D}\sigma\left(f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)\right)\left(\frac{\eta}{\eta-\sigma+1}\right),\\ \left(\frac{R^I}{W}\right)^{B.T} &= \frac{M^I}{\tau_V^I}\sigma f^I\left(\frac{\eta^*}{\eta^*-\sigma+1}\right).\end{aligned}$$

The mechanism is clear. Before-tax aggregate revenue of Home local firms increases, but that

Figure A.4: Value-Added Tax Reform with varying τ_V^D under $\lambda = 0.70$, $\tau_C^D = 0.67$, and $\tau_C^I = 0.85$



of FDI firms declines. Keeping the same amount of government revenues, the government may reduce the tax on local firms initially as it imposes larger tax on FDI firms. However, as more and more FDI firms exit and hence tax revenues from FDI firms declines, the government eventually needs to finance revenues by increasing tax on local firms after some threshold at around 33% tax on FDI firms.

A.4.2 Financial Market Reform under Tax Distortions

Increase in composite varieties: We define the composite mass of Home firms as $\log\left(M^{HF}\right) \equiv \nu \log\left(M + M^{I}\right) + (1-\nu)\log\left(M^{*}\right)$. The left subfigure in the middle of Figure A.5 shows M^{HF} increases. This is due to the dominant effect of the increase in M. The combination of labor market clearing conditions and free entry conditions leads to the following equilibrium conditions:

$$\begin{aligned} M &= \frac{(L-L_0) - \boldsymbol{M^I}\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]}{\left(\frac{\tau_C^D}{\sigma-1}+1\right)\boldsymbol{l^D}\left(\boldsymbol{\widetilde{Z}^D}\right) + \left(1-\tau_C^D\right)f^D}, \\ M^* &= \frac{L^* - \boldsymbol{L_0^*}}{\left\{\left(\frac{\tau_C^{D*}}{\sigma-1}+1\right)l^{D*}(\widetilde{Z}^{D*}) + \left(1-\tau_C^{D*}\right)f^{D*} + \left(\frac{\boldsymbol{1-G^*(Z^I)}}{\boldsymbol{1-G^*(Z^{D*})}}\right)\frac{\tau_C^I}{Q_L}\left[\frac{1}{\sigma-1}l^I(\widetilde{Z}^I) - f^I\right]\right\}}, \end{aligned}$$

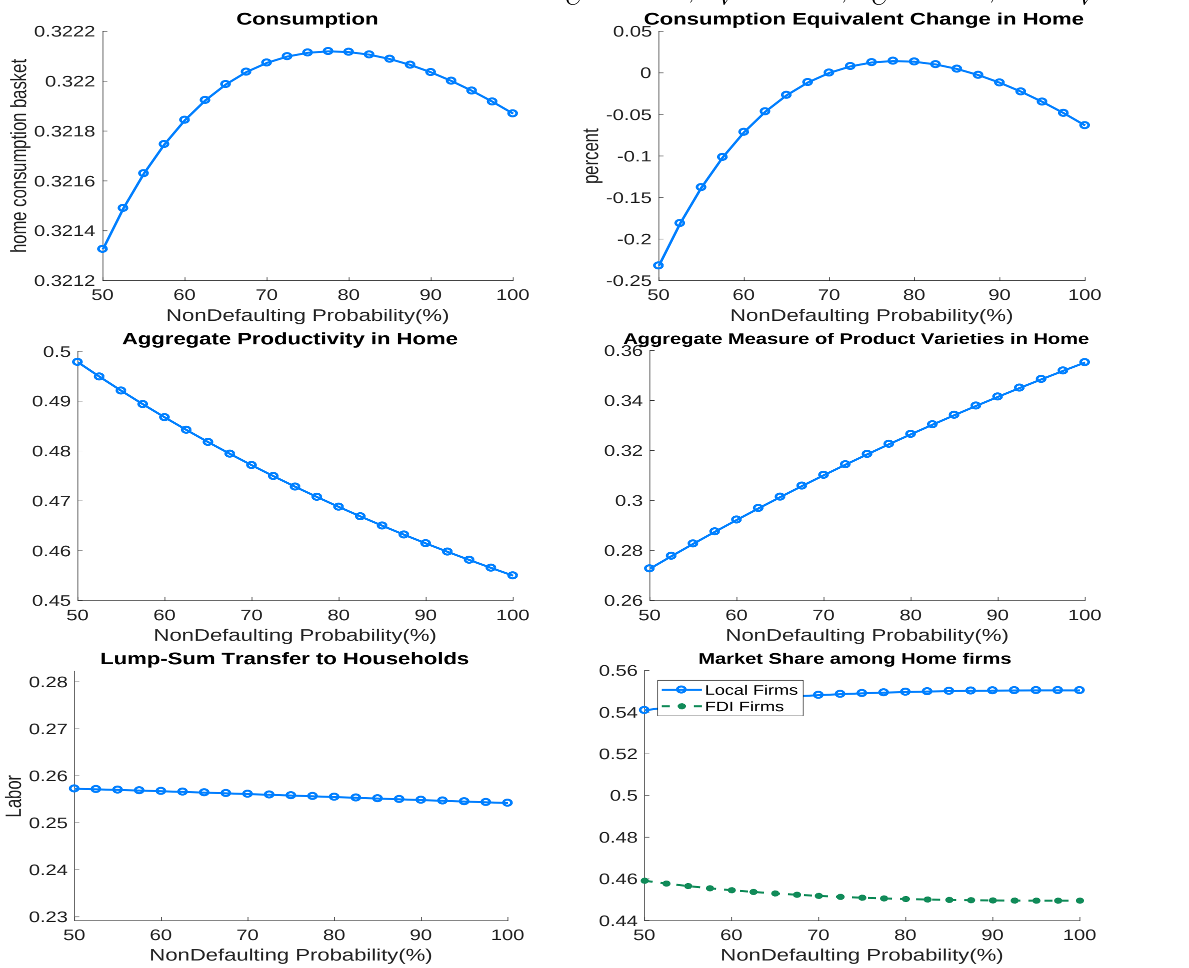
where $\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) = \left(\frac{Z^{D*}}{Z^I}\right)^{\eta^*}$ holds. The average labor demand can be derived as $l^D(\widetilde{Z}^D) = \left(f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right)(\sigma-1)\left(\frac{\eta}{\eta-\sigma+1}\right)$ for Home local firms, $l^I\left(\widetilde{Z}^I\right) = f^I(\sigma-1)\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)$ for Home FDI firms, and $l^{D*}\left(\widetilde{Z}^{D*}\right) = f^{D*}(\sigma-1)\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)$ for Foreign domestic firms (see the appendix A.2.3).

There are two determinants for the mass of Home local firms, M_k. Firstly, when average labor demand among Home local firms, $l^D(\widetilde{Z}^D)$, is higher, it makes competition among local firms in hiring labor harder, and thus the mass of firms decreases. Second, if there are more FDI firms in operation, $M^I \Uparrow$, then this leads to harder competition for hiring Home labor and the mass of Home local firms gets smaller. In the counterfactual experiment, both competition effects get smaller due to the decrease in average labor demand, $l^D(\widetilde{Z}^D)$, and the decrease in the mass of FDI firms. Therefore, M increases when λ rises as shown in Figure A.6.

The mass of Foreign firms, M^*, gets smaller mainly due to the increase in labor demand in the homogeneous sector, $L_0^* \Uparrow$, but its movement is negligible. Since the FDI cutoff Z^I

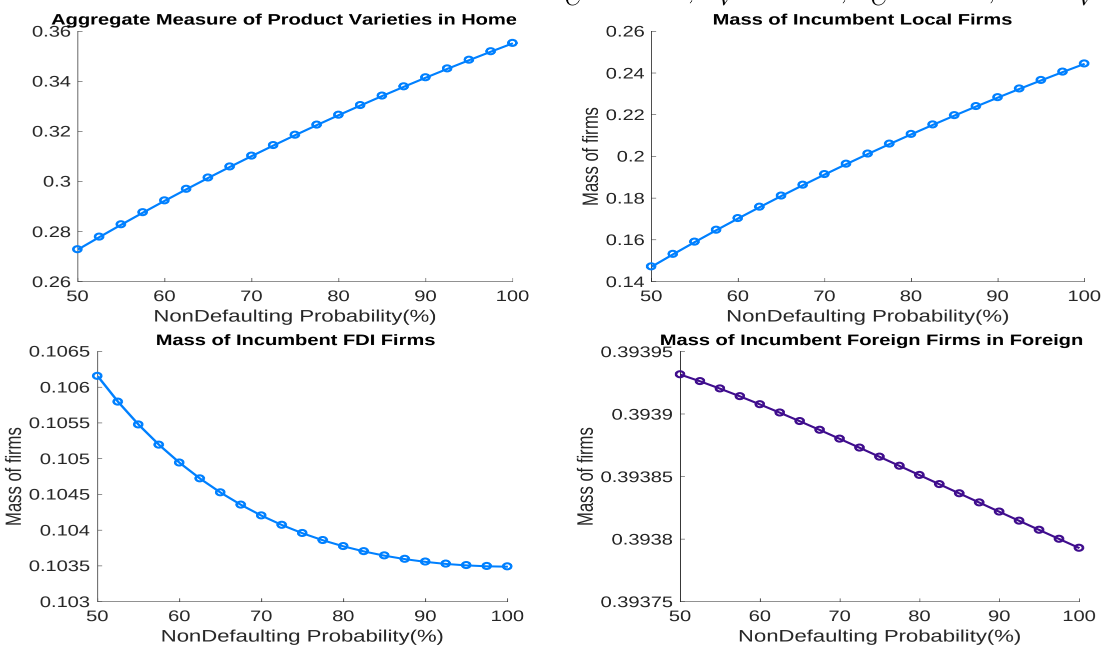
slightly increases, the mass of FDI firms decreases, $M^I \Downarrow$.[6]

Figure A.5: Financial Market Reform under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$



[6] The mass of FDI firms is defined as $M^I \equiv M^*\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)$. The negative relation of the mass M^I to its cutoff Z^I can be shown as $\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) = \frac{(z^*_{min})^{\eta^*}\frac{\pi^{D*}(\tilde{Z}^{D*})}{W^*}}{(Z^I)^{\eta^*}\delta F^{D*}-\left(\frac{1}{Q_L}\right)(z^*_{min})^{\eta^*}\frac{\pi^I(\tilde{Z}^I)}{W}} = \frac{\tau_C^{D*} f^{D*}\frac{(\sigma-1)(z^*_{min})^{\eta^*}}{\eta^*-\sigma+1}}{(Z^I)^{\eta^*}\delta F^{D*}-\left(\frac{1}{Q_L}\right)\tau_C^I f^I\frac{(\sigma-1)(z^*_{min})^{\eta^*}}{\eta^*-\sigma+1}}$.

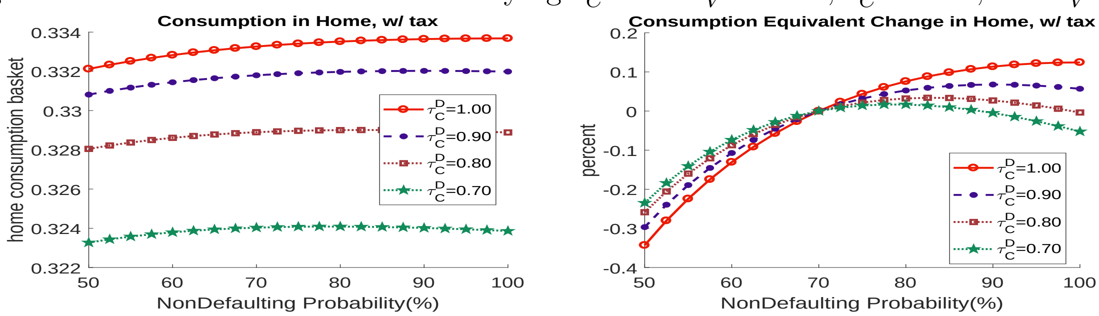
Figure A.6: Financial Market Reform under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$



A.4.3 Financial Reform with varying taxes on local firms' profits

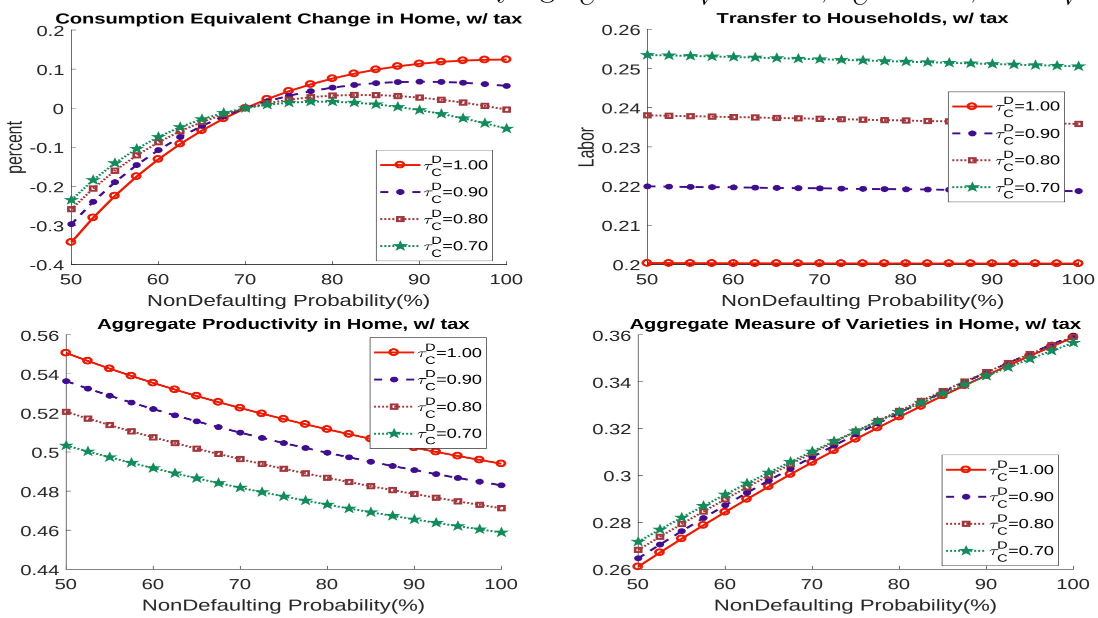
Figure A.7 shows changes in consumption according to financial reform with respect to different levels of a profit tax on Home local firms. When we normalize each line by dividing it by the value on $\lambda = 70\%$, then the consumption equivalent change clearly exhibits the pattern which becomes "humped" less and less as τ_C^D increases (decreasing profit tax on local firms).

Figure A.7: Financial Reform under varying τ_C^D with $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$



Observation of Figure A.8 leads us to the main culprit for this pattern: variety effect. Variety effect increases with a steeper slope as τ_C^D increases(decreasing profit tax). Also, lump-sum transfer and aggregate productivity are all less than one, and it dampens the increase in the variety effects and consumption plot becomes smoother than the variety effect plot: $C_1 = \left[M^{HF}\right]^{\frac{1}{\sigma-1}} \rho \widetilde{Z}^{HF} \theta \left(L + \frac{T}{W}\right)$.

Figure A.8: Financial Reform under varying τ_C^D with $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$



So why does the variety effect increase with a steeper slope as τ_C^D increases(decreasing profit tax)? This is mainly because the mass of Home local firms increases with a steeper slope as τ_C^D increases.

$$
\begin{aligned}
M &= \frac{(L-L_0) - \boldsymbol{M^I}\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]}{\left(\frac{\tau_C^D}{\sigma-1}+1\right)\boldsymbol{l^D}\left(\widetilde{\boldsymbol{Z}}^{\boldsymbol{D}}\right) + (1-\tau_C^D)\, f^D}, \\
&= \underbrace{\left\{(L-L_0) - \boldsymbol{M^I}\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]\right\}}_{M_l}\underbrace{\left\{\left(\frac{\tau_C^D}{\sigma-1}+1\right)\boldsymbol{l^D}\left(\widetilde{\boldsymbol{Z}}^{\boldsymbol{D}}\right) + \left(1-\tau_C^D\right) f^D\right\}^{-1}}_{M_r}
\end{aligned}
$$

where $l^D(\widetilde{Z}^D) = \left(f^D + \left(\frac{1}{\boldsymbol{\lambda}} - 1\right)\left(\zeta f^D - \chi F^D\right)\right)(\sigma-1)\left(\frac{\eta}{\eta-\sigma+1}\right)$. The left-bottom chart in

Figure A.9: Financial Reform under varying τ_C^D with $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$

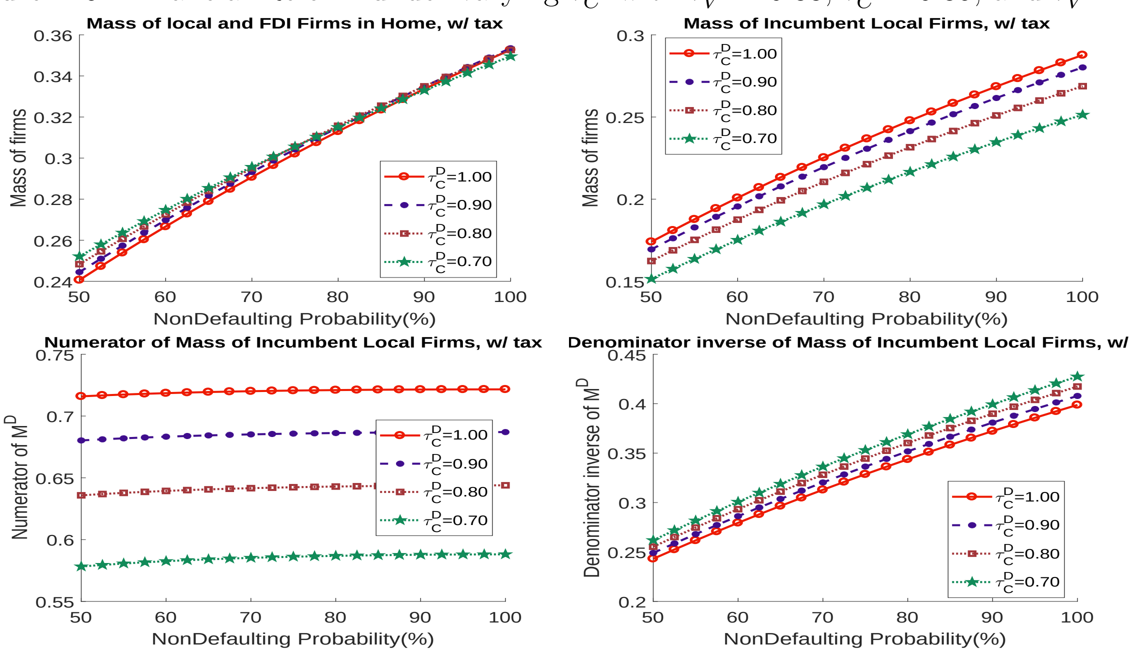


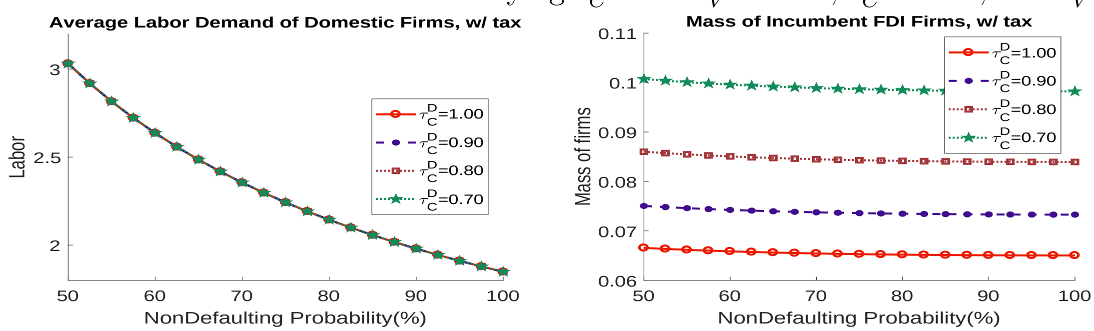
Fig A.9 plots $M_l = \left\{(L-L_0) - \boldsymbol{M^I}\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]\right\}$ and the right-bottom chart plots $M_r = \left\{\left(\frac{\tau_C^D}{\sigma-1}+1\right)\boldsymbol{l^D}\left(\widetilde{\boldsymbol{Z}}^{\boldsymbol{D}}\right) + \left(1-\tau_C^D\right) f^D\right\}^{-1}$. As λ increases, $l^D(\widetilde{Z}^D)$ decreases and thus M_r increases. It barely shifts according to the change in τ_C^D.

As τ_C^D increases (decreasing profit tax on local firms), M^I decreases due to the exit of

FDI firms, and M_l shifts up. Therefore, as τ_C^D increases, M_l acts as a multiplier which makes the slope of M_r steeper.

All in all, the main reason for the steeper slope of the variety effect is the decrease of FDI firm mass. According to the financial reform, λ increases and the average labor requirement $l^D(\widetilde{Z}^D)$ decreases since local firms pay less and less financial costs in labor term. The increase in M_r is multiplied by the upward shift in M_l as τ_C^D rises and this is mainly due to the exit of FDI firms: $M^I \downarrow$.

Figure A.10: Financial Reform under varying τ_C^D with $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$



A.4.4 Financial Reform with varying taxes on FDI firms' profits

Figure A.11 shows changes in consumption according to financial reform with respect to different levels of a profit tax on FDI firms. The Home consumption gets larger in level when profit taxes on FDI firms increases (τ_C^I decreases). This is mainly due to the income effect: as the Home government gathers large taxes from FDI firms, Home households can earn more lump-sum transfers.

Figure A.11: Financial Reform under varying τ_C^I with $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_V^I = 0.85$

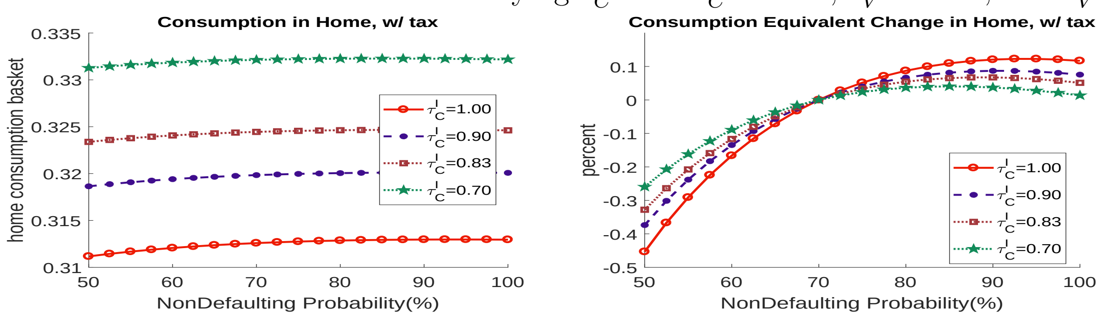


Figure A.12: Financial Reform under varying τ_C^I with $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_V^I = 0.85$

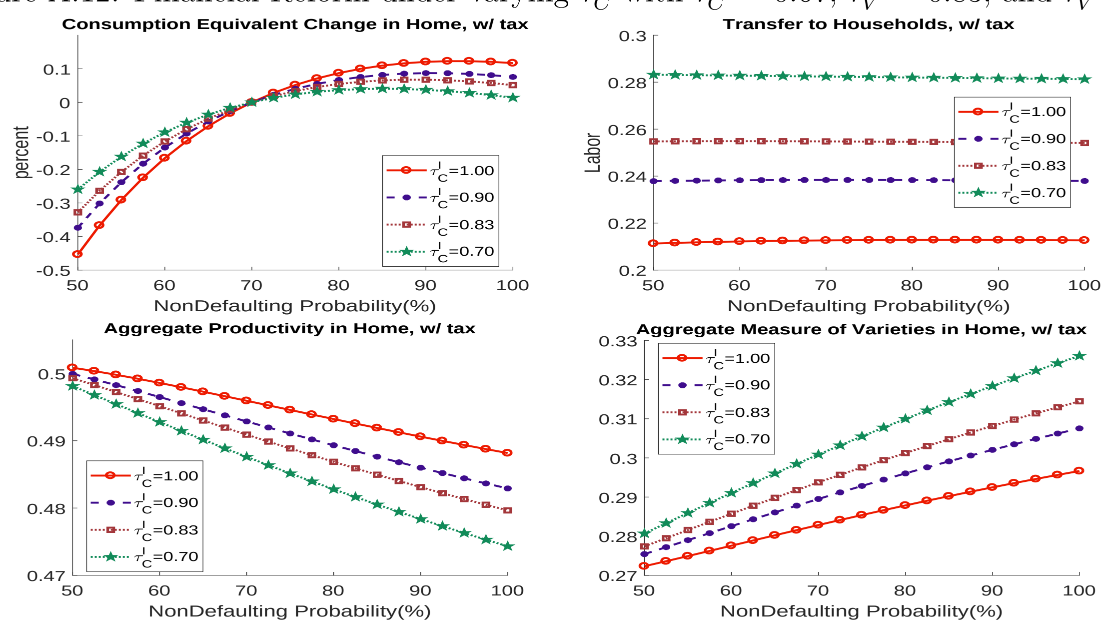


Figure A.13: Financial Reform under varying τ_C^I with $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_V^I = 0.85$

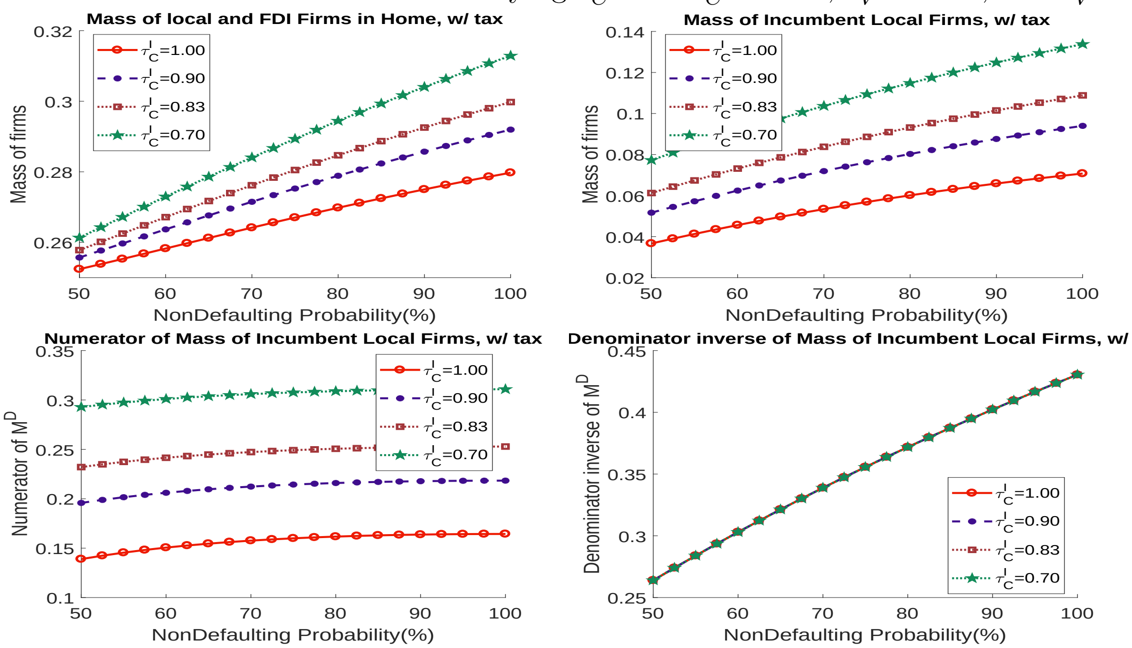
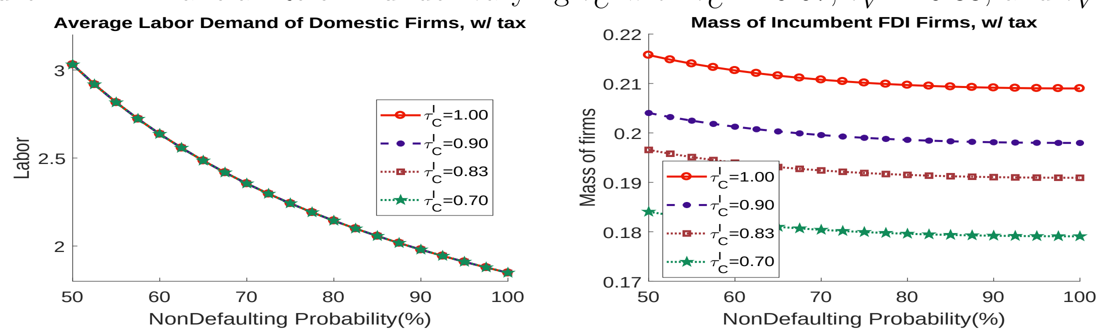


Figure A.14: Financial Reform under varying τ_C^I with $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_V^I = 0.85$



E Figure with $1 - \tau_V^I = 15\%$

Figure 15: Financial Market Reform under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$

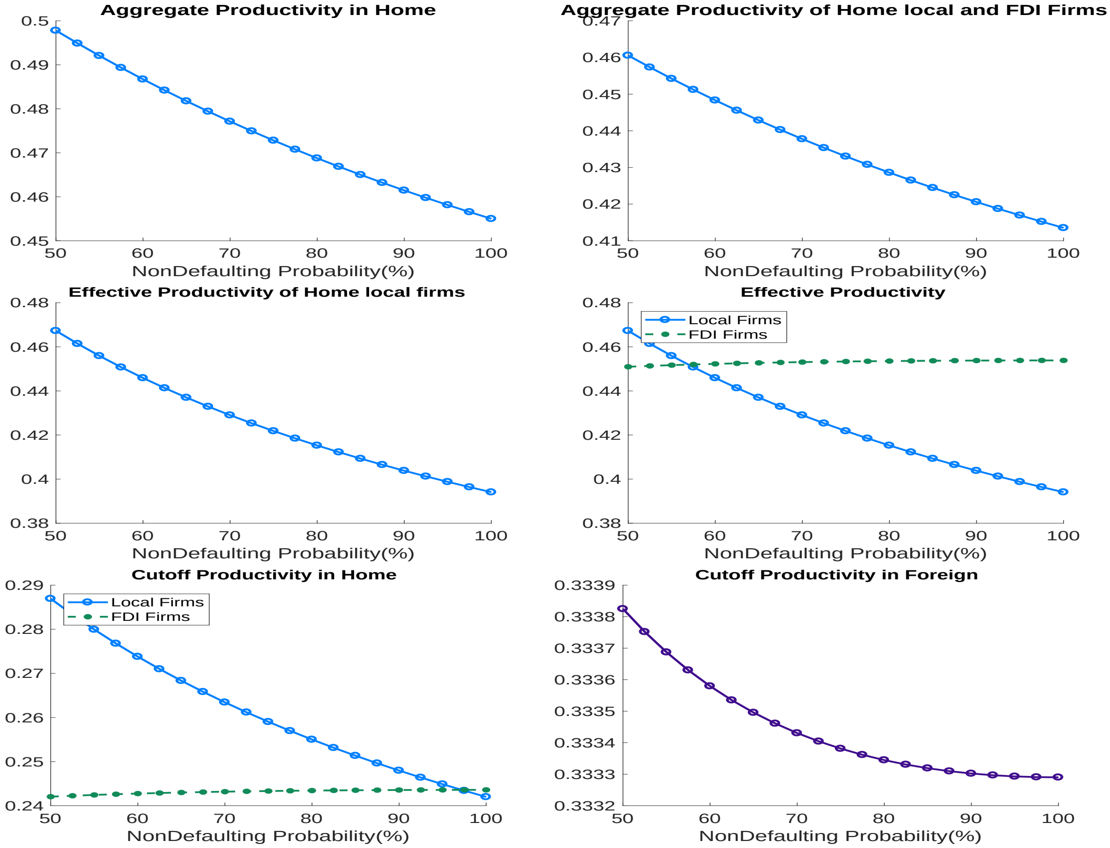


Figure 16: Financial Market Reform under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$

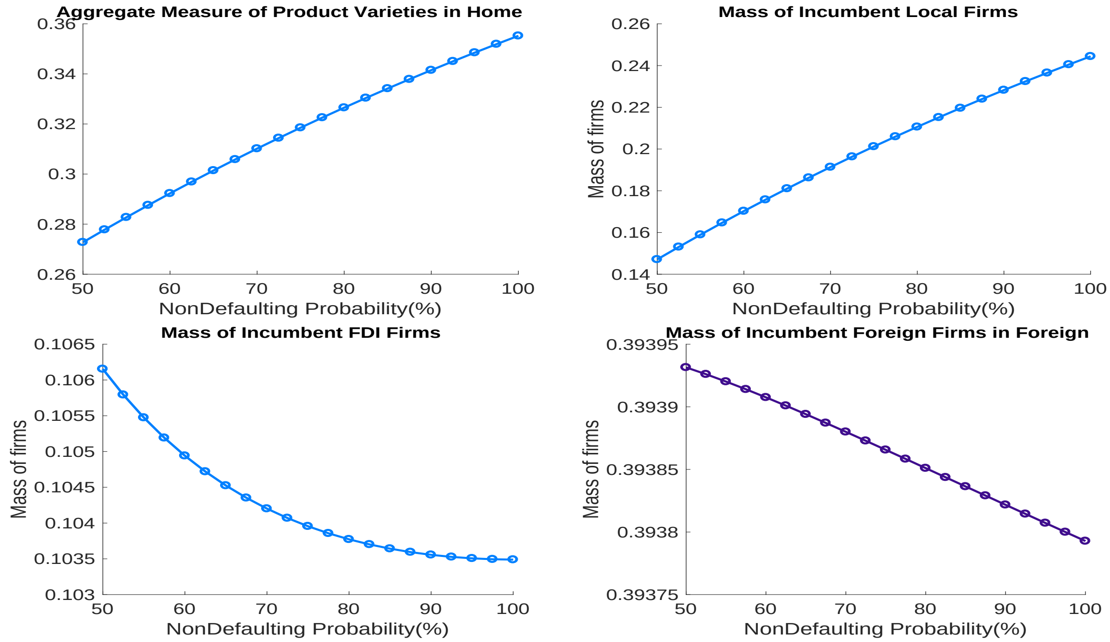
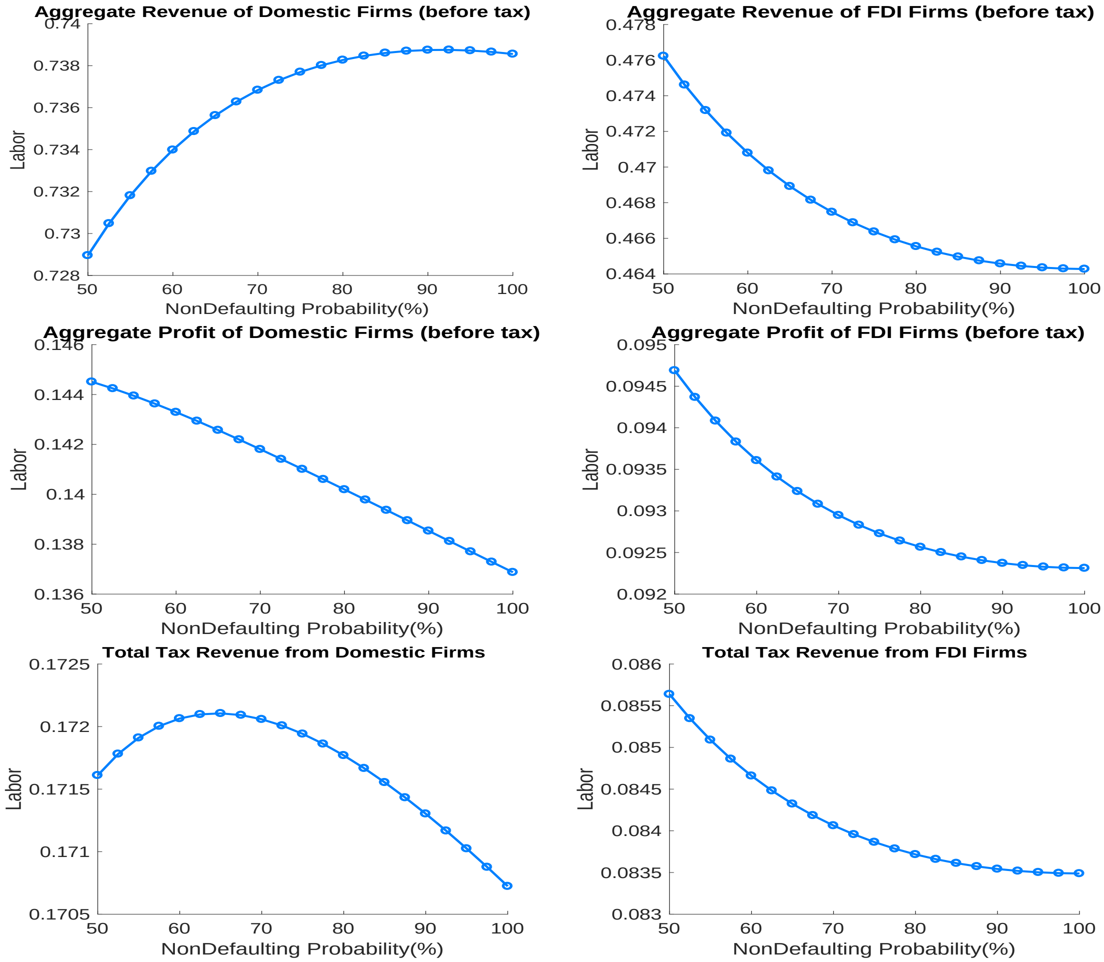


Figure 17: Financial Market Reform under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$



F Allocations with and without distortions

Figure 18: Value-Added Tax Reform under distortions and under no distortions.

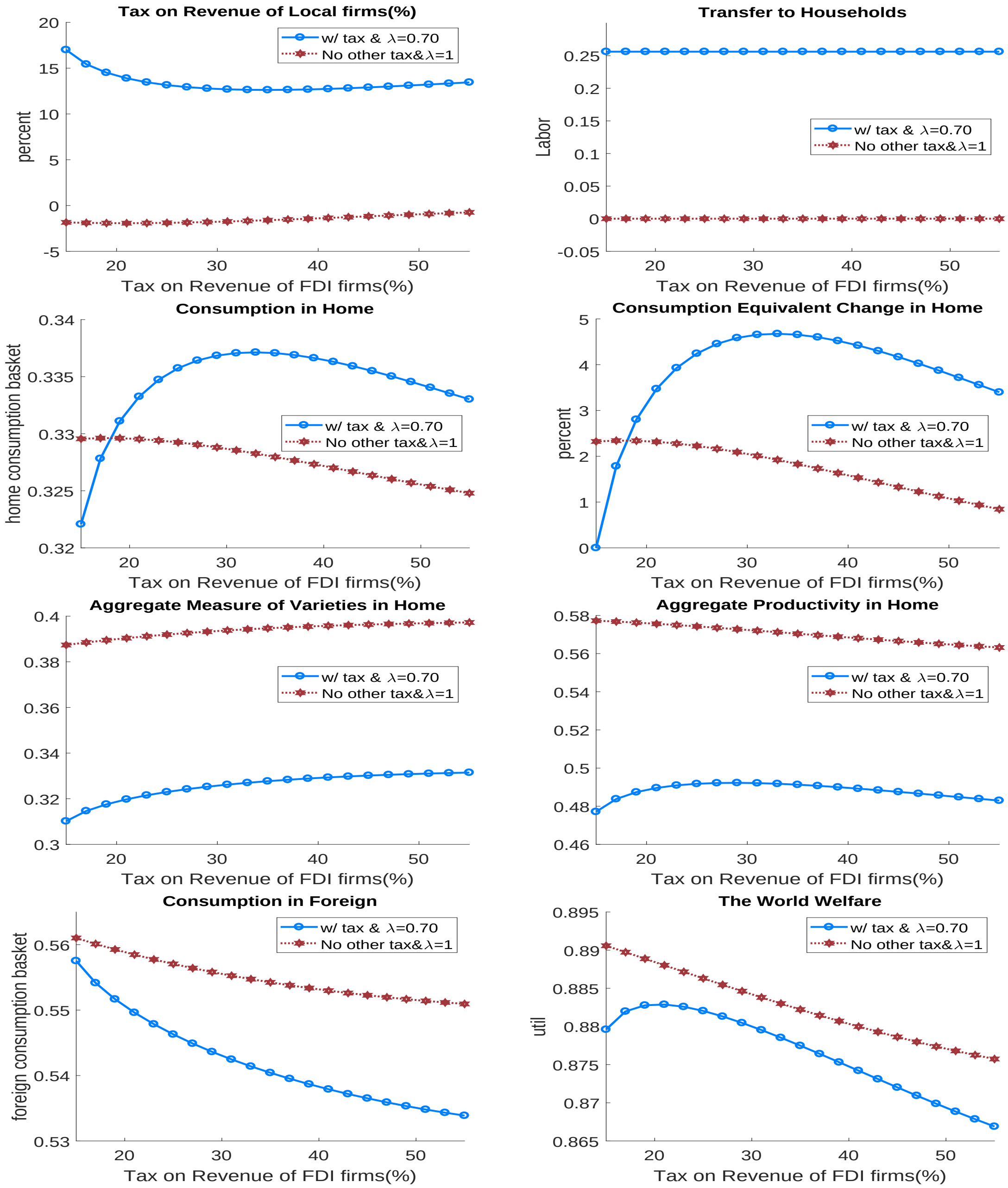


Figure 19: Value-Added Tax Reform under distortions and under no distortions.

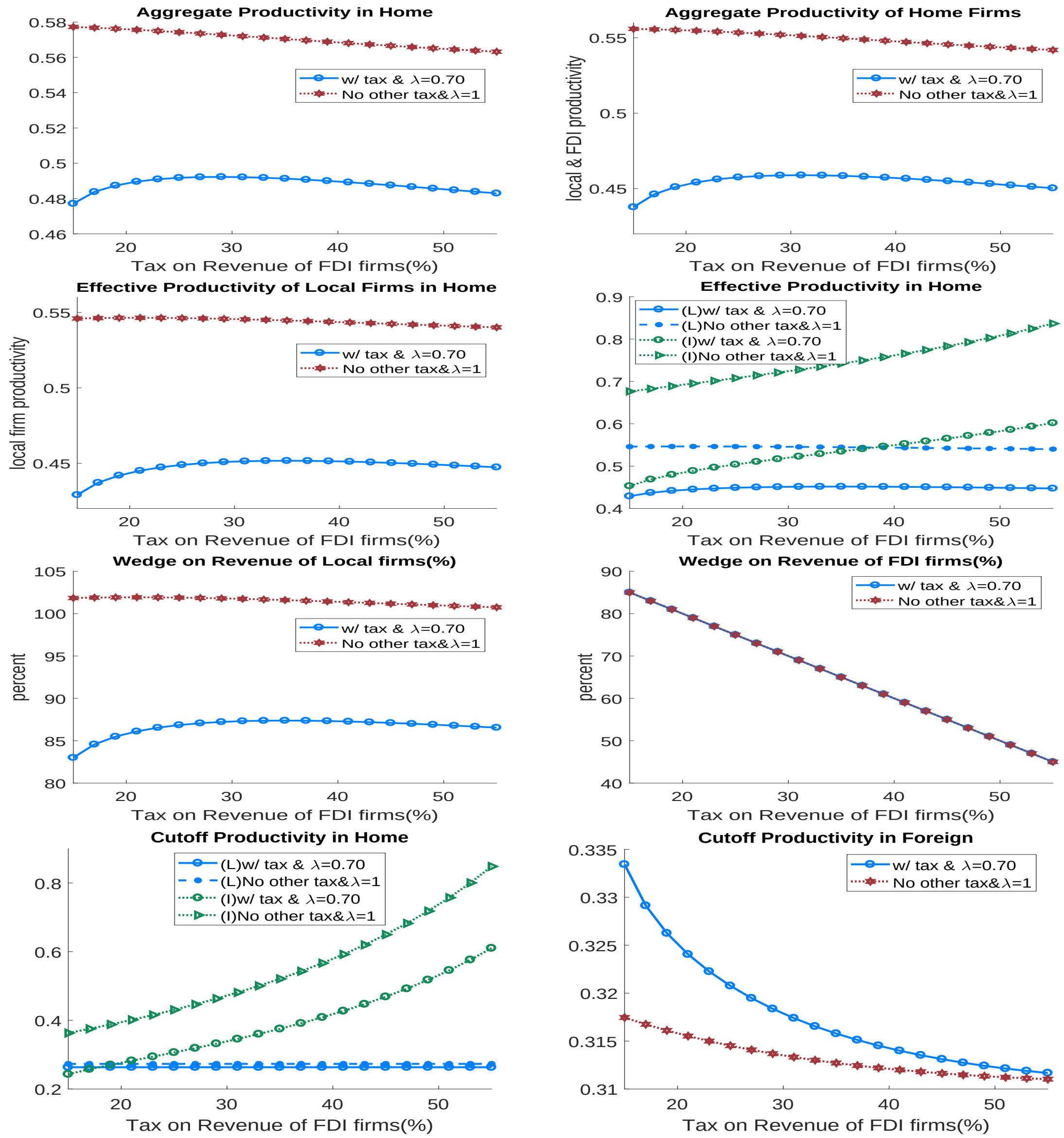


Figure 20: Value-Added Tax Reform under distortions and under no distortions.

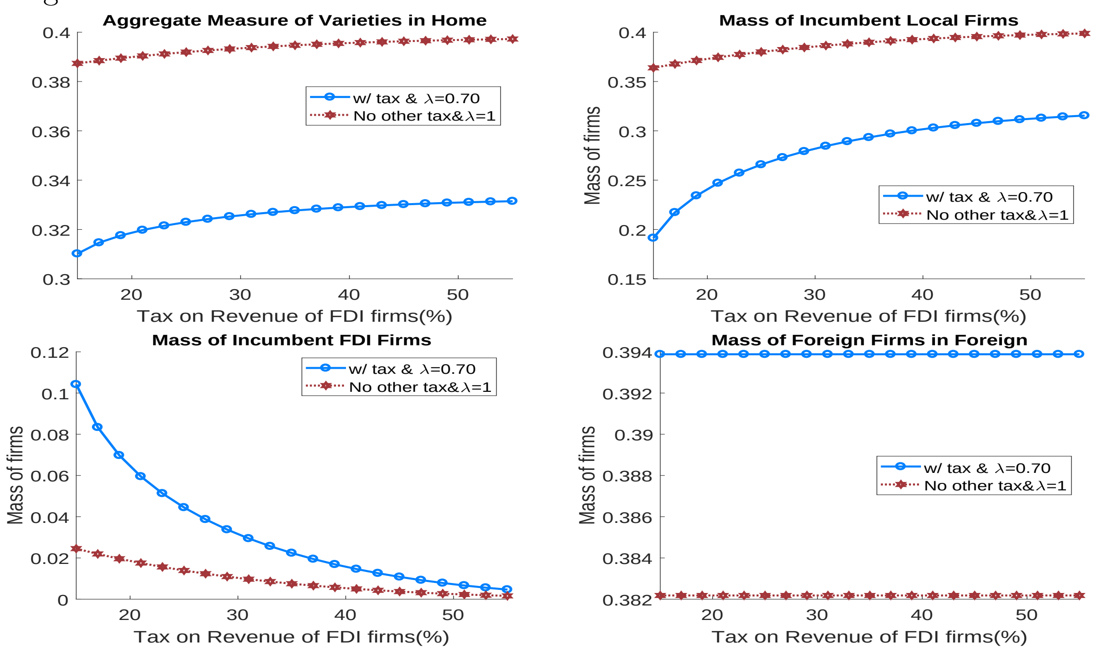


Figure 21: Financial Market Reform under distortions and under no distortions.

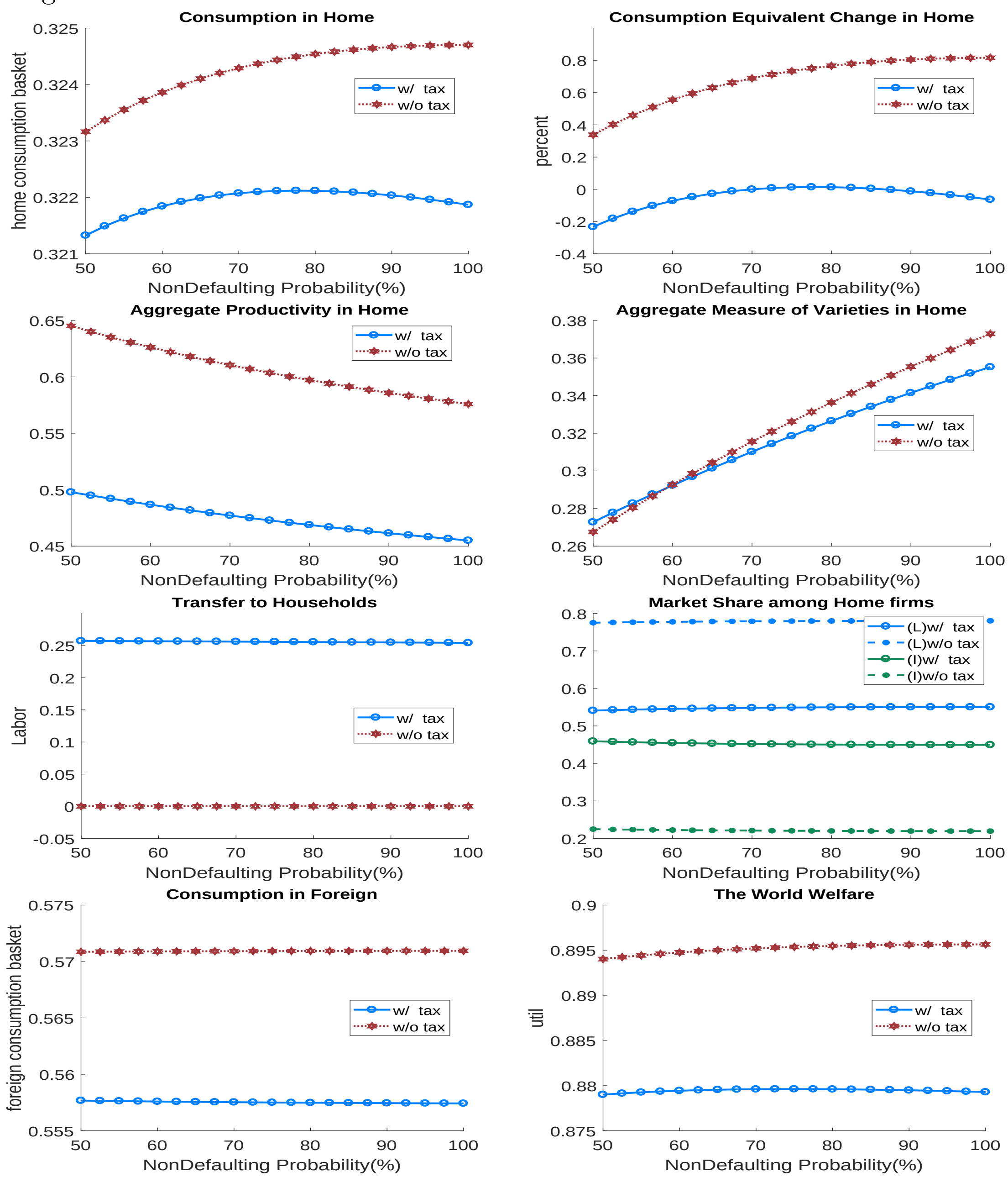


Figure 22: Financial Market Reform under distortions and under no distortions.

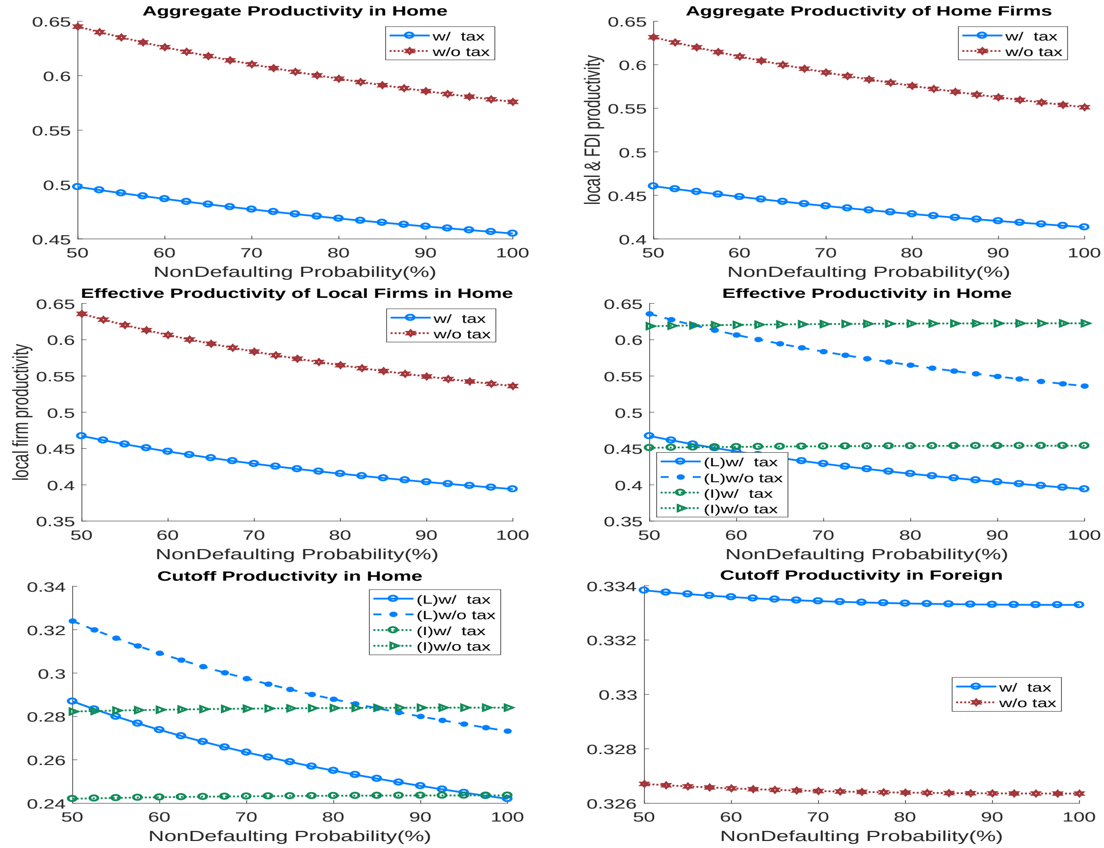


Figure 23: Financial Market Reform under distortions and under no distortions.

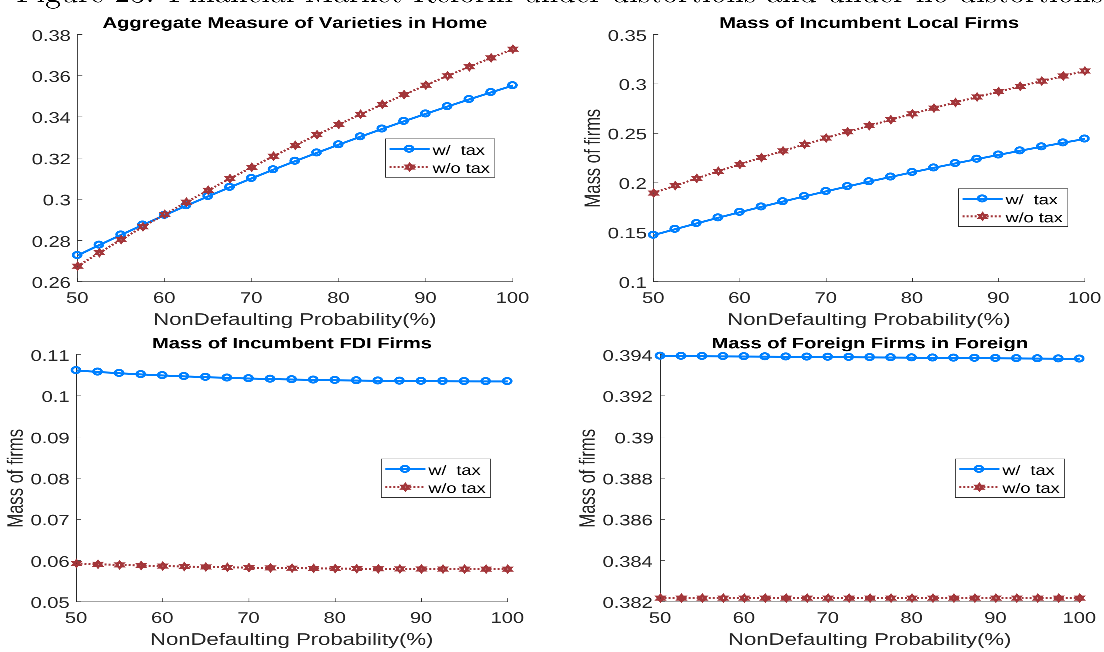
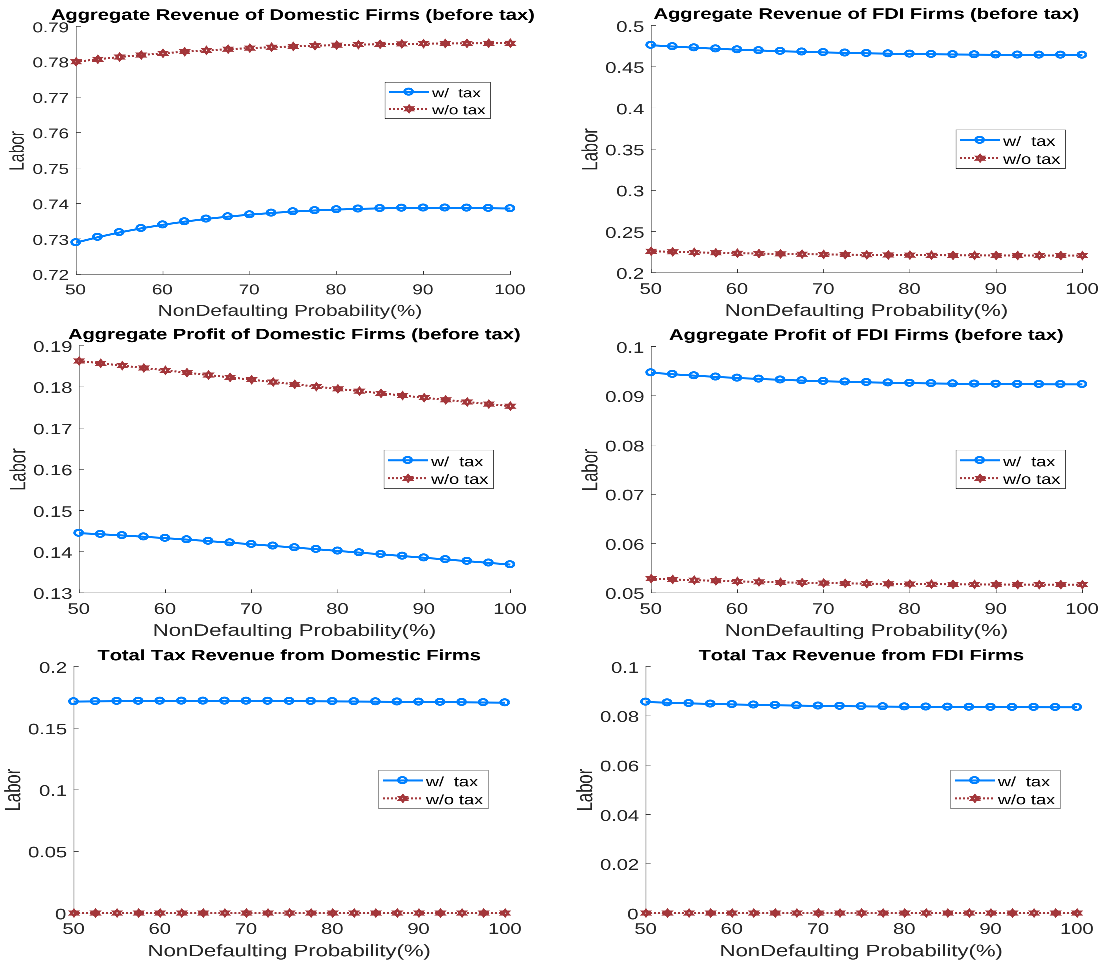


Figure 24: Financial Market Reform under distortions and under no distortions.



G Figure under varying degrees of frictions

Figure 25: Value-Added Tax Reform with varying λ and τ_V^D under $\tau_C^D = 0.67$, and $\tau_C^I = 0.85$.

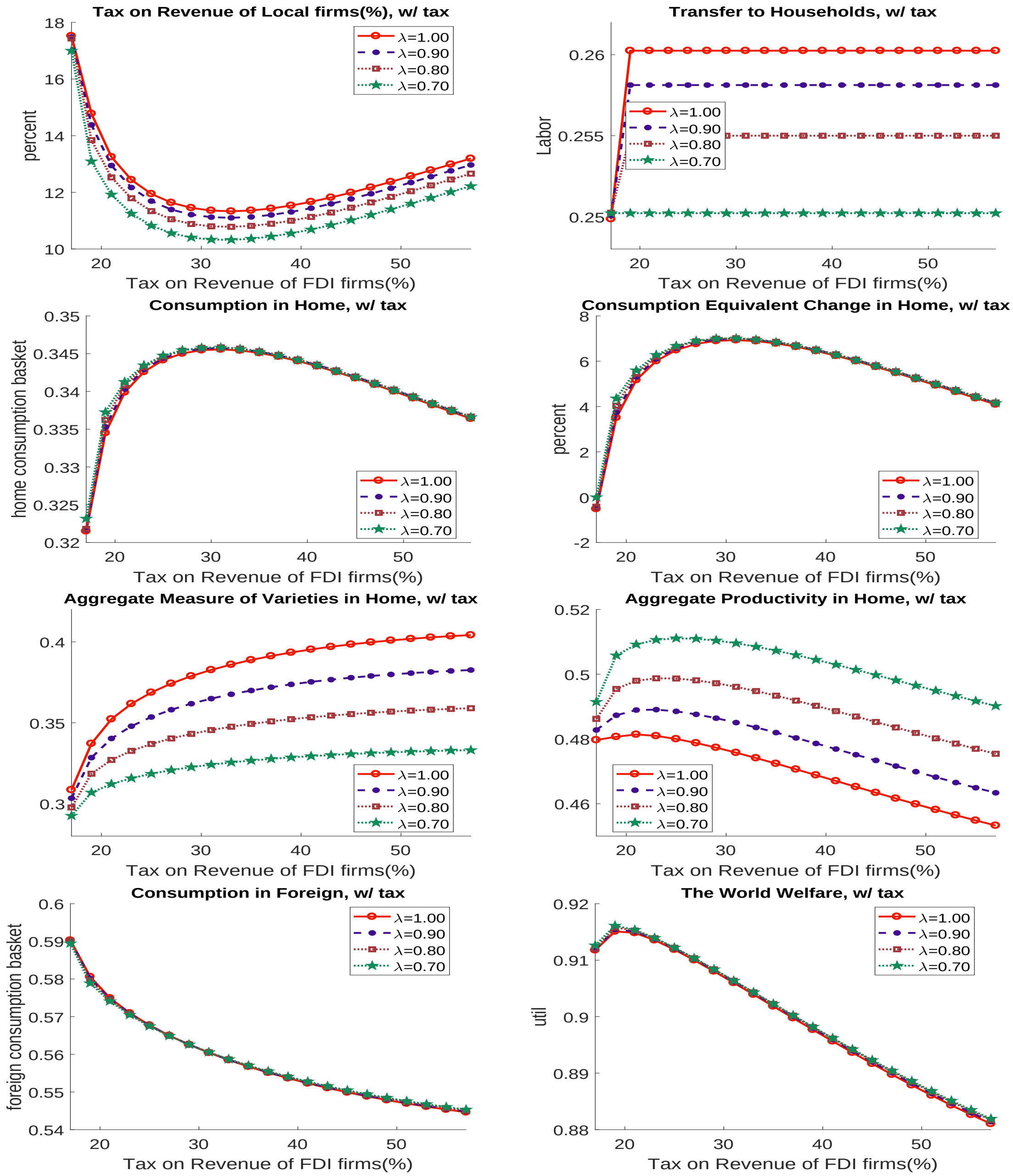


Figure 26: Value-Added Tax Reform with varying λ and τ_V^D under $\tau_C^D = 0.67$, and $\tau_C^I = 0.85$.

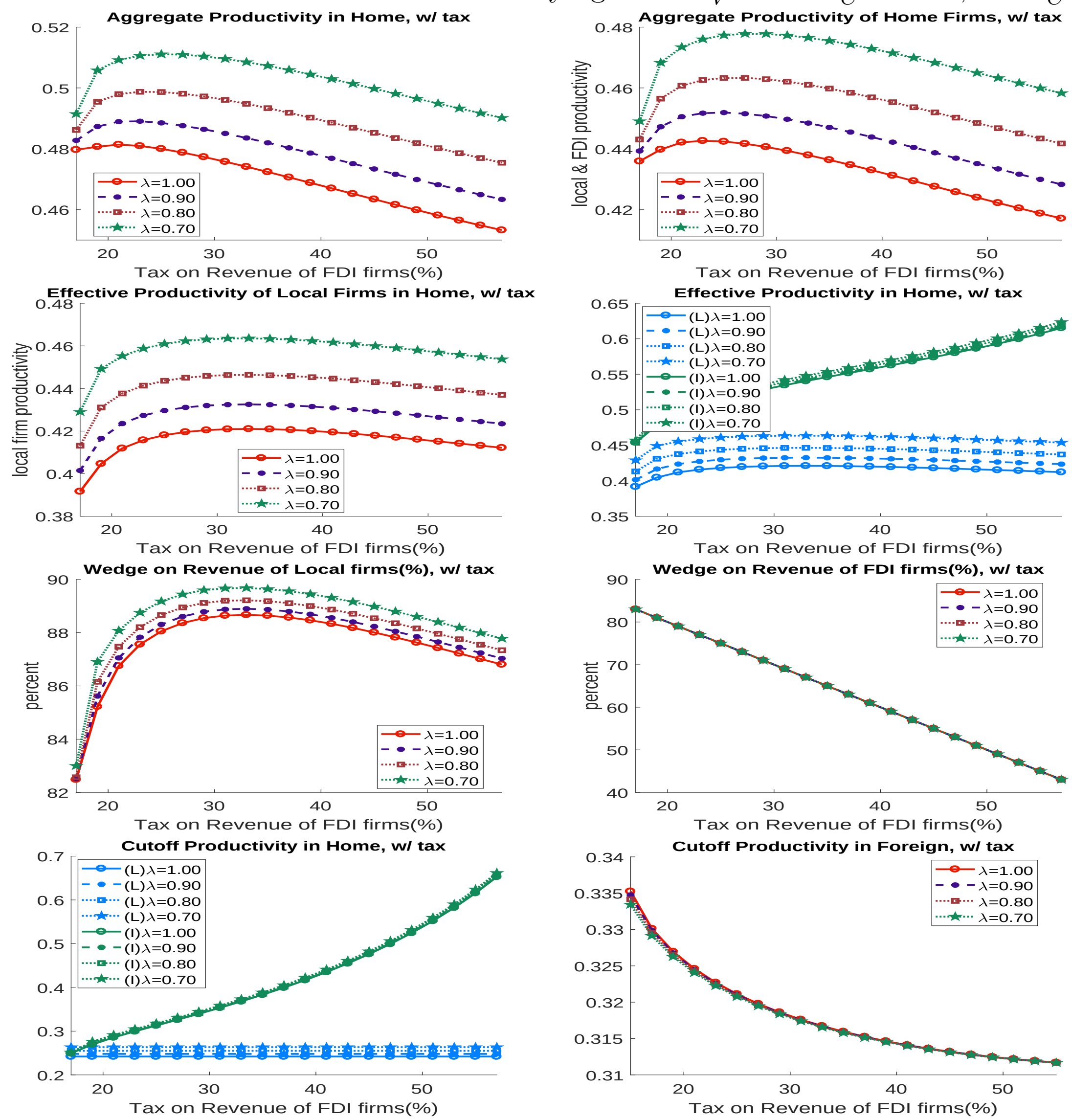


Figure 27: Value-Added Tax Reform with varying λ and τ_V^D under $\tau_C^D = 0.67$, and $\tau_C^I = 0.85$.

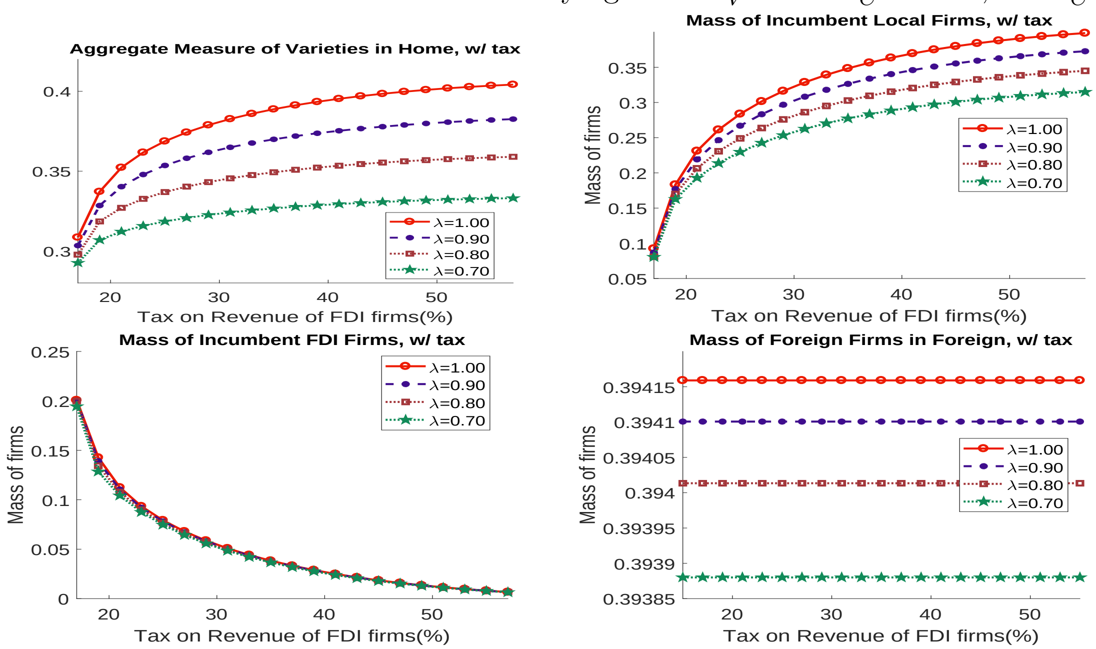


Figure 28: Financial Market Reform with varying τ_V^I under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_C^I = 0.85$.

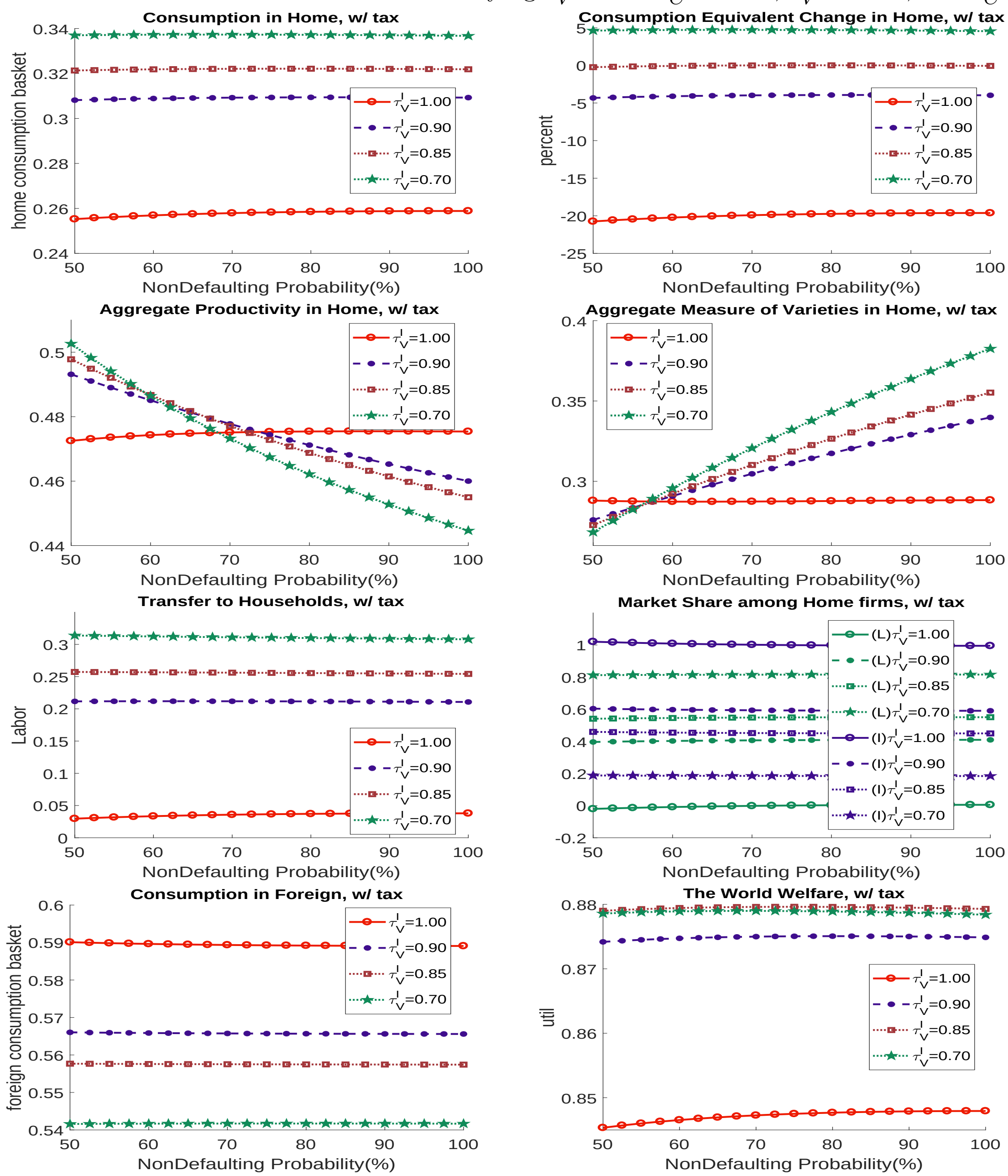


Figure 29: Financial Market Reform with varying τ_V^I under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_C^I = 0.85$.

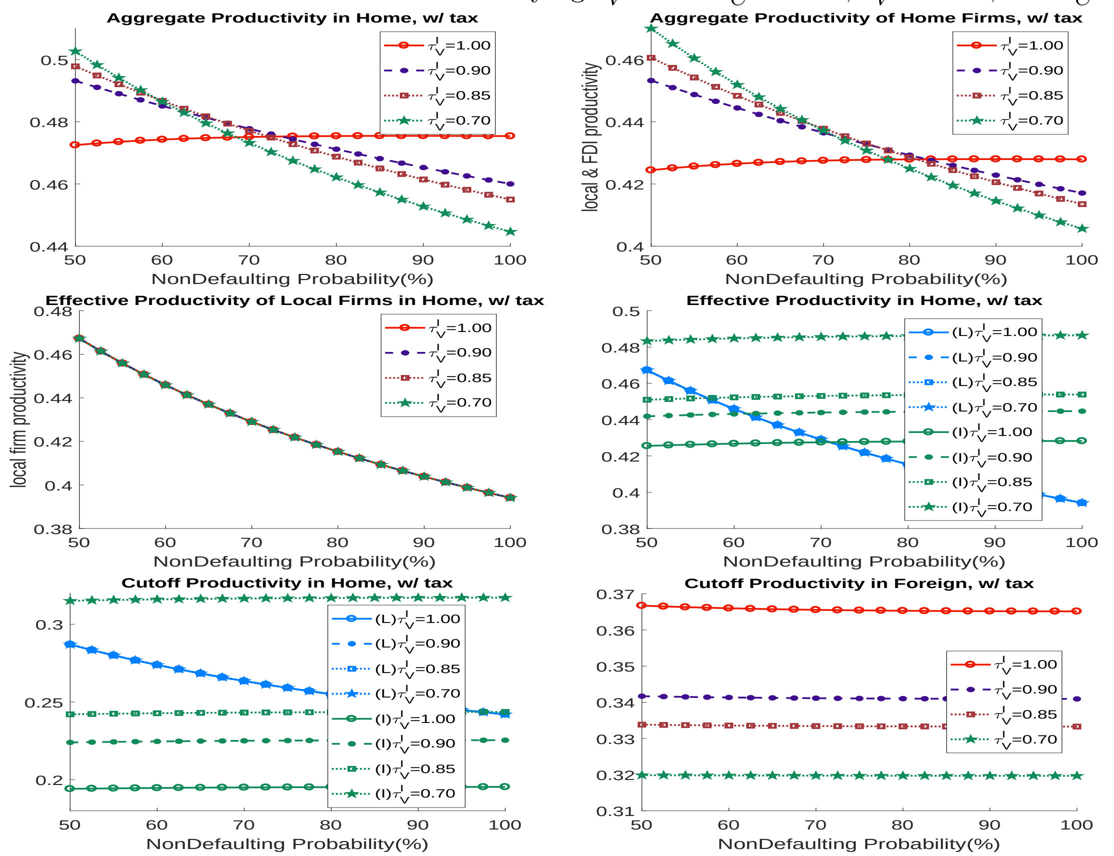


Figure 30: Financial Market Reform with varying τ_V^I under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_C^I = 0.85$.

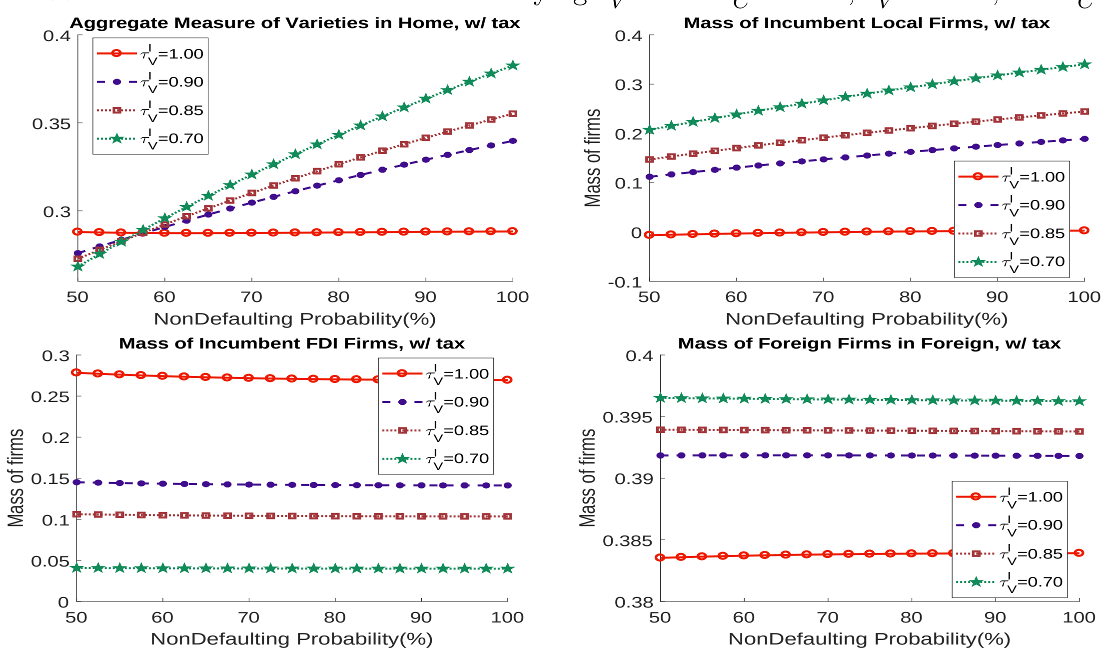
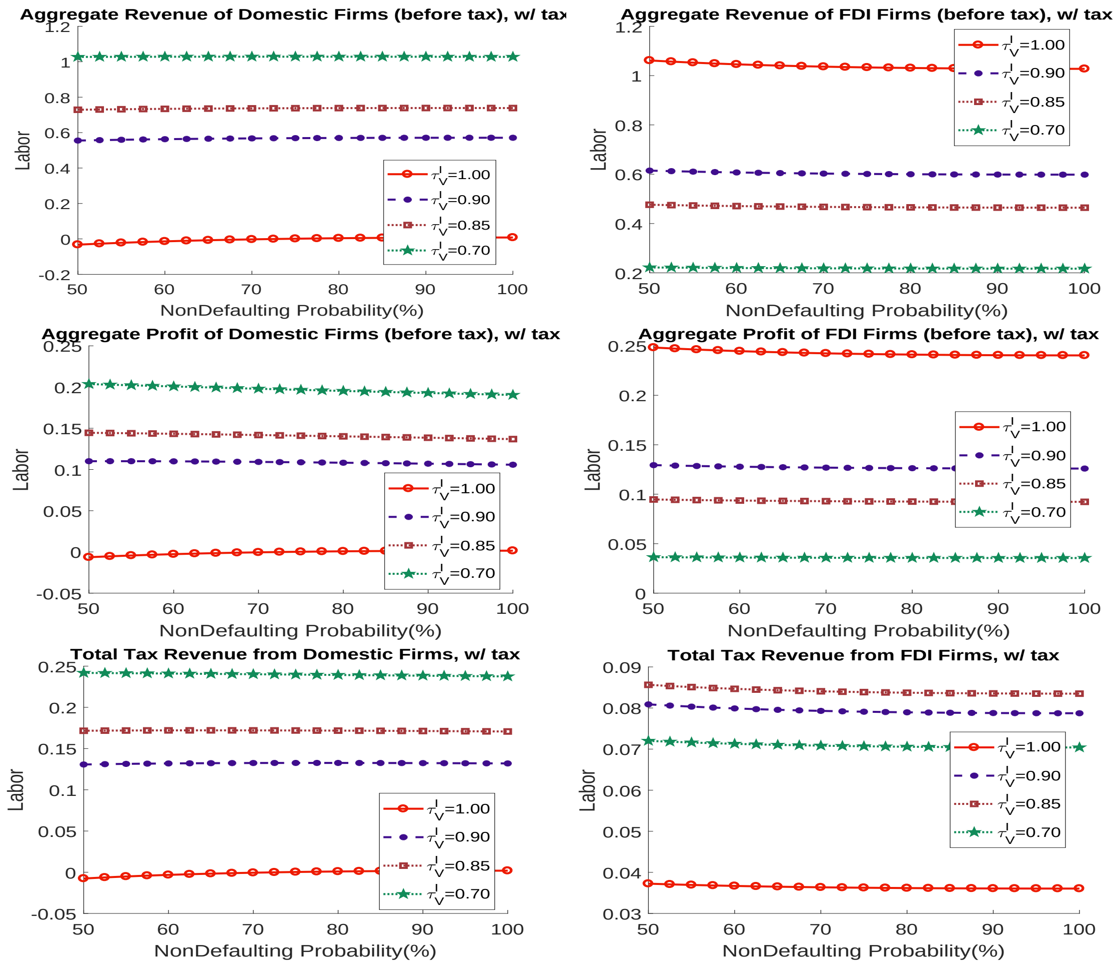


Figure 31: Financial Market Reform with varying τ_V^I under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_C^I = 0.85$.



H Figure under varying degrees of frictions with no other taxes

Figure 32: Value-Added Tax Reform with varying λ and τ_V^D under $\tau_C^D = 1.00$, and $\tau_C^I = 1.00$.

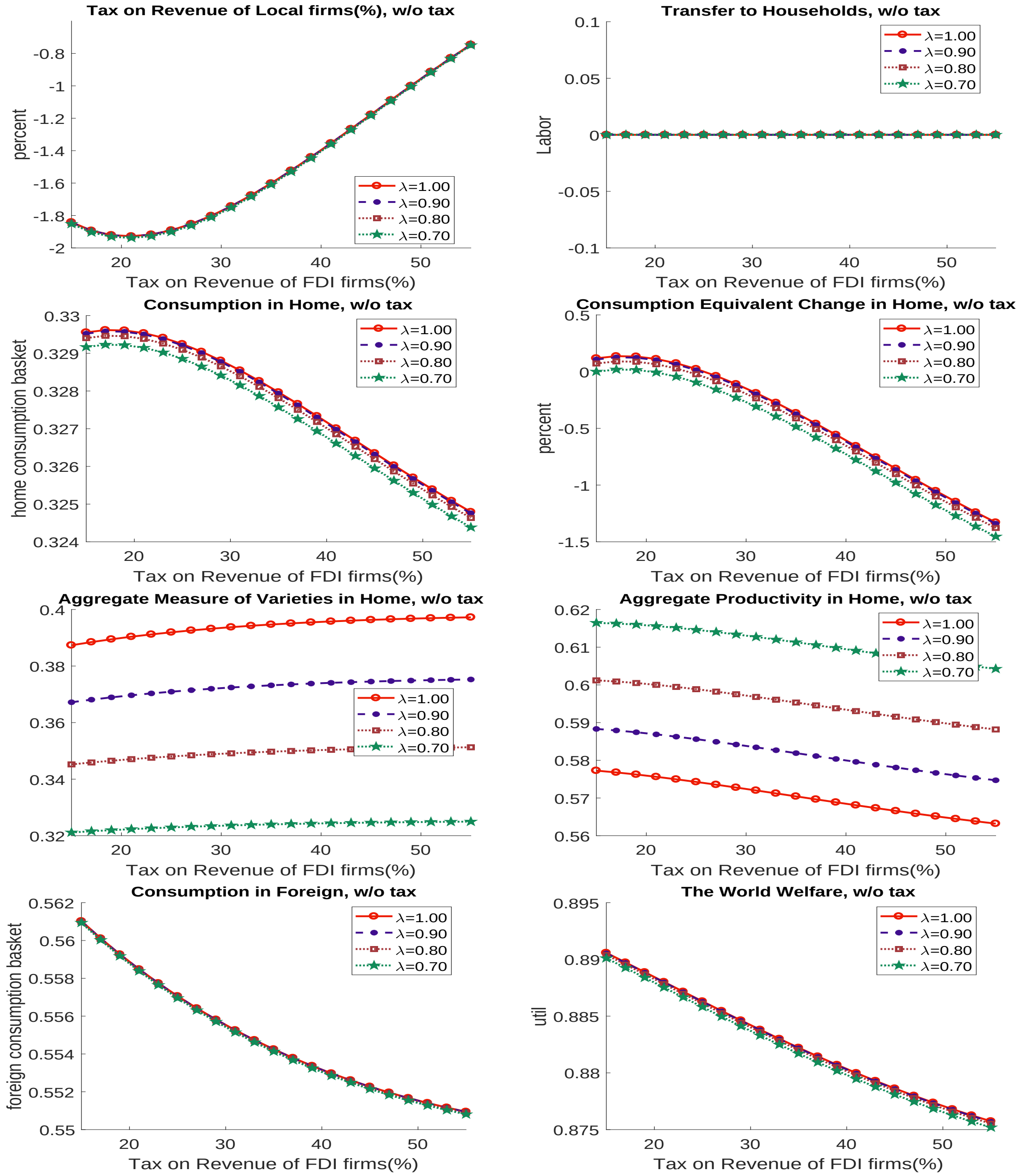


Figure 33: Value-Added Tax Reform with varying λ and τ_V^D under $\tau_C^D = 1.00$, and $\tau_C^I = 1.00$.

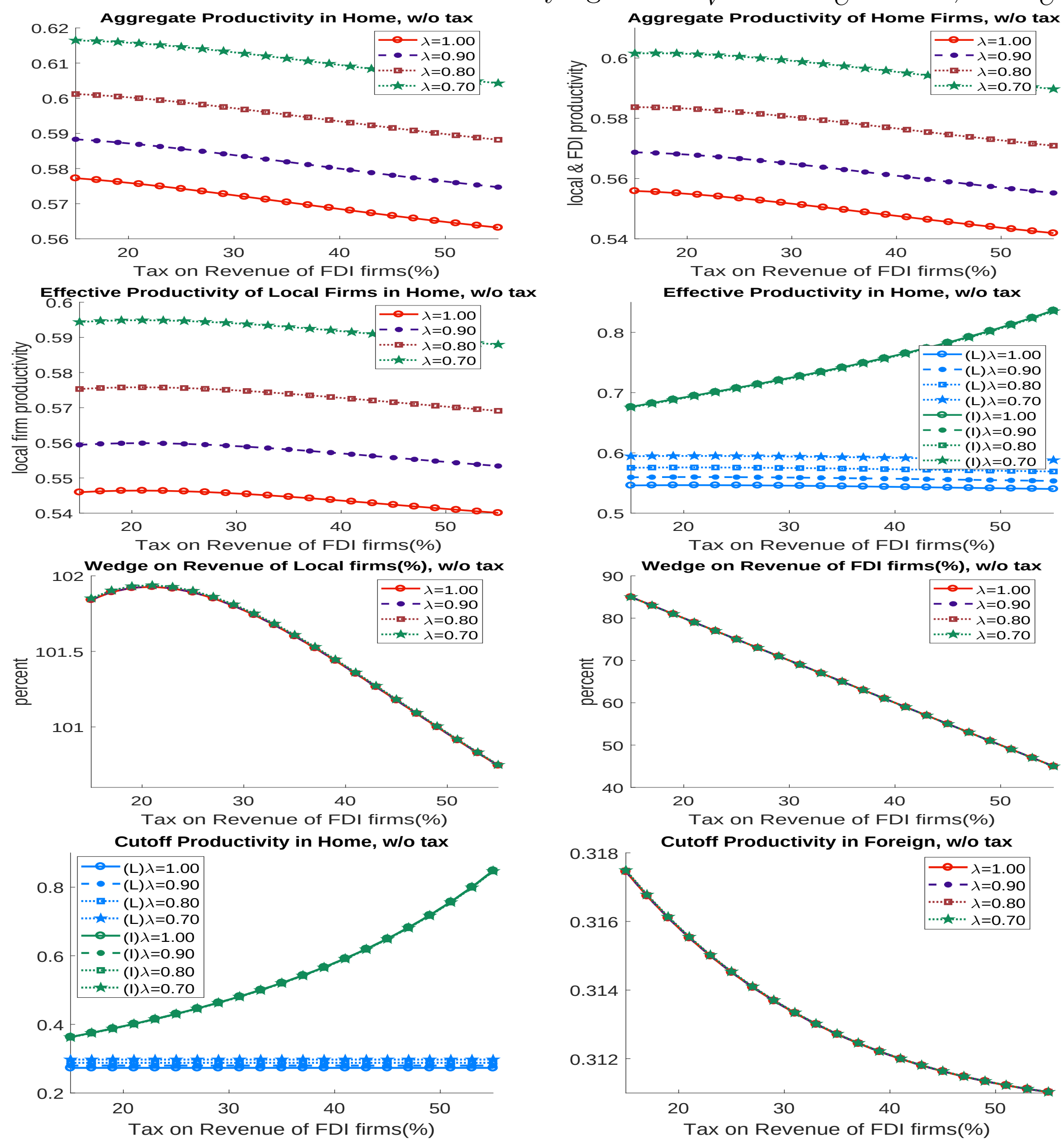


Figure 34: Value-Added Tax Reform with varying λ and τ_V^D under $\tau_C^D = 1.00$, and $\tau_C^I = 1.00$.

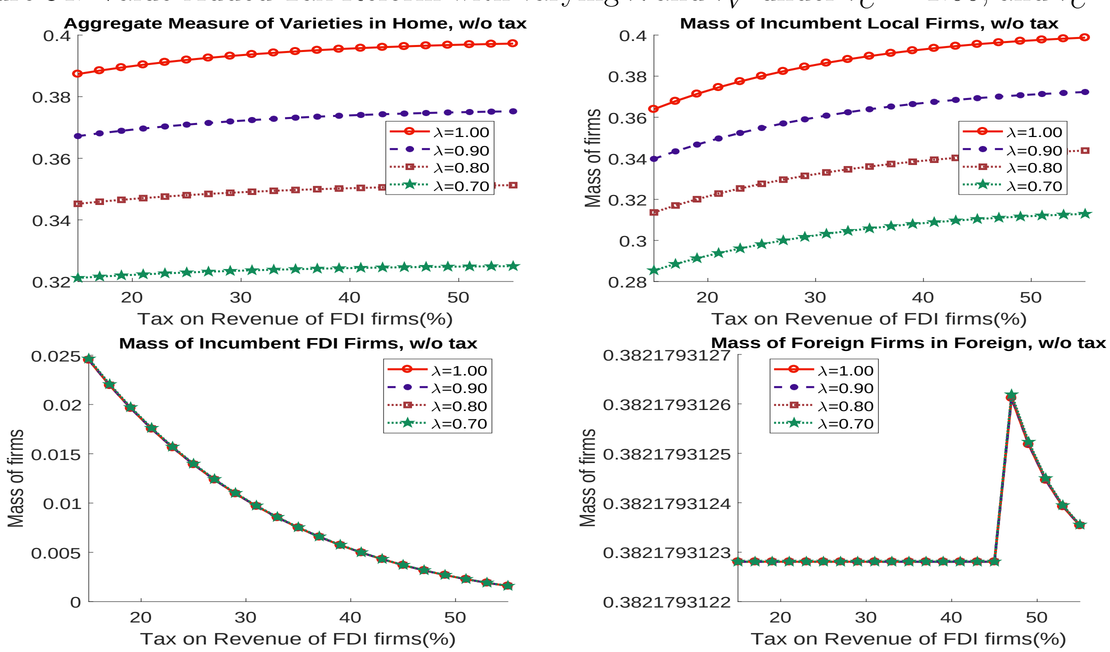


Figure 35: Financial Market Reform with varying τ_V^I under $\tau_C^D = 1.00$, $\tau_V^D = 1.00$, and $\tau_C^I = 1.00$.

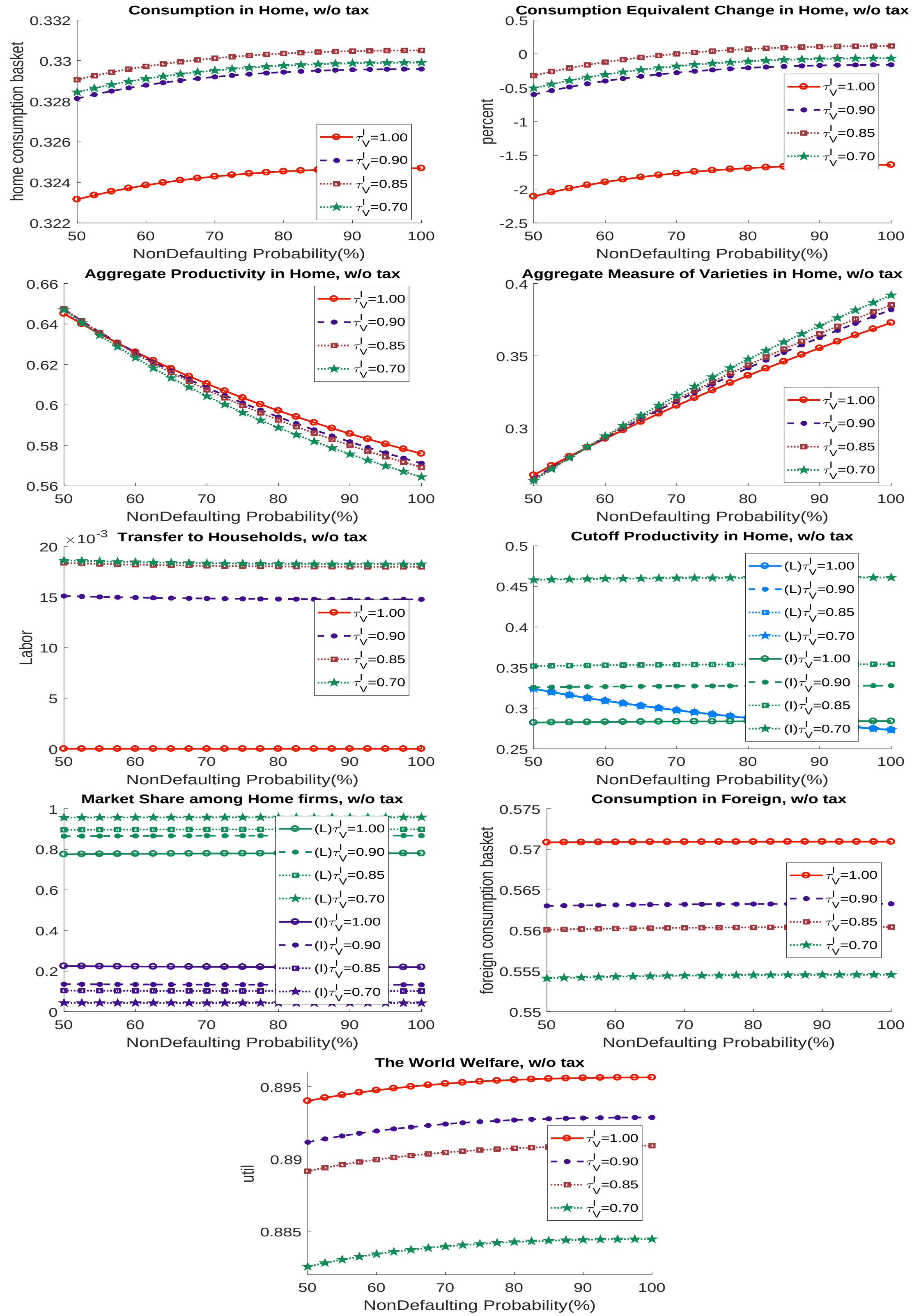


Figure 36: Financial Market Reform with varying τ_V^I under $\tau_C^D = 1.00$, $\tau_V^D = 1.00$, and $\tau_C^I = 1.00$.

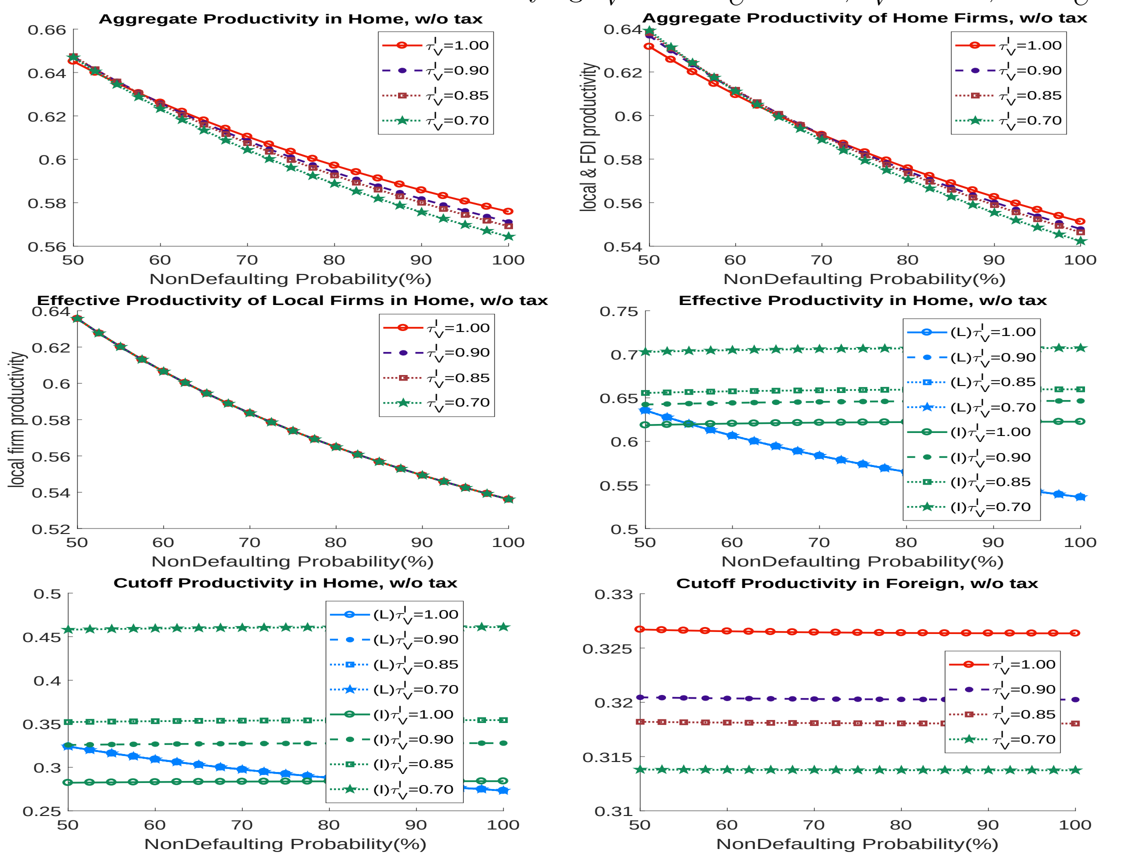


Figure 37: Financial Market Reform with varying τ_V^I under $\tau_C^D = 1.00$, $\tau_V^D = 1.00$, and $\tau_C^I = 1.00$.

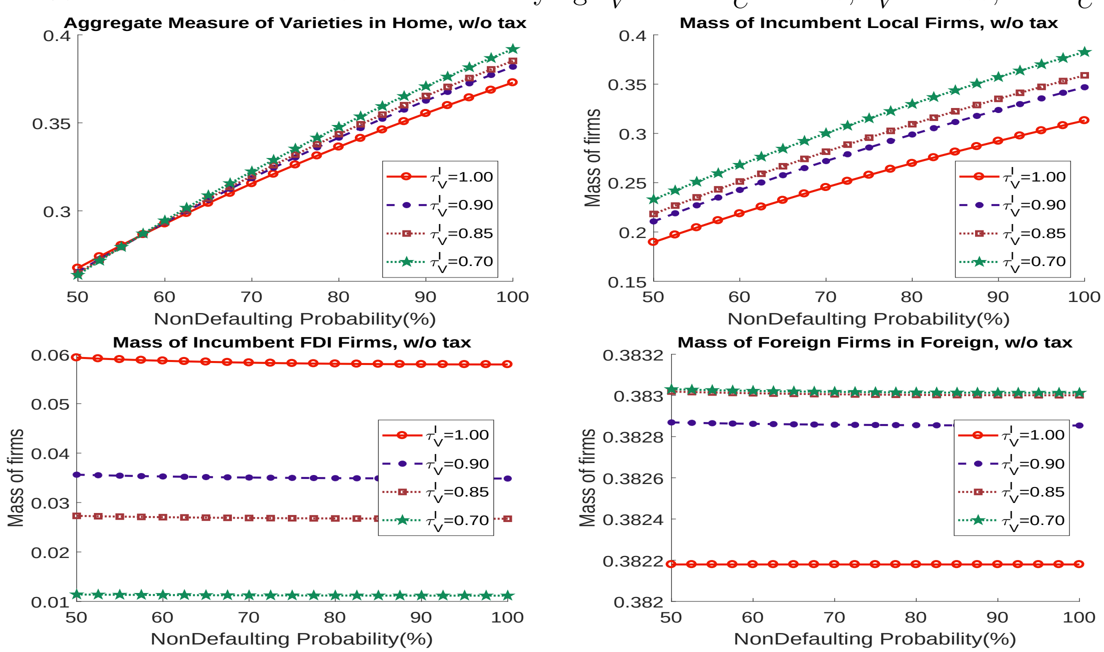


Figure 38: Financial Market Reform with varying τ_V^I under $\tau_C^D = 1.00$, $\tau_V^D = 1.00$, and $\tau_C^I = 1.00$.