Circuit Breakers and Contagion*

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ABSTRACT

Circuit breakers based on indices are commonly imposed in financial markets to reduce market crashes and market volatility in bad times. We develop a dynamic equilibrium model with multiple stocks to study how circuit breakers affect joint stock price dynamics. We show that in bad times, circuit breakers can cause crash contagion, volatility contagion, elevated market volatility, and higher correlations among otherwise independent stocks. Our analysis suggests that circuit breakers rules might have exacerbated the market plunges and the extreme volatilities at the beginning of the COVID-19 pandemic.

JEL classification: C02, G11

Keywords: Circuit breaker, crash contagion, volatility contagion, return correlation, market crash

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ABSTRACT

Circuit breakers based on indices are commonly imposed in financial markets to reduce market crashes and market volatility in bad times. We develop a dynamic equilibrium model with multiple stocks to study how circuit breakers affect joint stock price dynamics. We show that in bad times, circuit breakers can cause crash contagion, volatility contagion, elevated market volatility, and higher correlations among otherwise independent stocks. Our analysis suggests that circuit breakers rules might have exacerbated the market plunges and the extreme volatility at the beginning of the COVID-19 pandemic.

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1. Introduction

Circuit breakers in financial markets based on indices are widely implemented in many countries (e.g., the United States, France, Canada, and China) as one of the measures aimed at stabilizing market prices in bad times. In most of these markets, when the percentage decline in a market index reaches a regulatory threshold, the circuit breaker is triggered and trading is halted for a period of time for the entire market. Recent COVID-19 fears triggered circuit breakers multiple times across many countries including the United States, Japan, and South Korea. For example, circuit breakers on the S&P 500 were triggered twice during the week of March 9, 2020 and plunged almost 10% on March 12, 2020. In a dramatic move, Chinese regulators removed a four-day-old circuit breakers rule after it was triggered twice in the week of January 7, 2016. The existing literature on circuit breakers (e.g., Chen, Petukhov, and Wang (2017), Greenwald and Stein (1991), Subrahmanyam (1994)) has examined the impact of circuit breakers on the return and volatility of a stock index as a whole. One open question is whether circuit breakers can adversely affect stock return contagion and volatility contagion and thus increase the systemic risk in bad times. In this paper, we develop a continuous-time equilibrium model to shed some light on this important issue.

Contrary to the regulatory goals, we show that in bad times, circuit breakers can cause crash contagion, volatility contagion, and can increase cross-stock return correlations and market volatility. Our model suggests that market-wide circuit breakers may be a source of financial contagion and a channel through which idiosyncratic risks become systemic risks.[1] In particular, the circuit breakers might have significantly exacerbated the multiple market plunges and the extreme volatilities triggered by the COVID-19 pandemic. Our findings can also help explain the concurrence of the implementation of the circuit breakers rule and the significant market tumble in the week of January 7, 2016 in Chinese stock markets. We propose an alternative circuit breaker approach based on individual stocks (rather than an index) that does not cause either correlation or any contagion.[2]

In our model, investors can invest in one risk-free asset and two (groups of) risky assets ("stocks") with independent dividend processes to maximize expected utility from

[1]In a previous version, using a simple 3 period model we show that all the main results hold. To save space, we do not include it in this version, but available from the authors.

[2]Needless to say, circuit breakers may play a positive role in stabilizing markets. For example, they may reduce the effect of overreaction, panic, and herding on stock prices. Our model does not consider some potential benefits of market closure that can potentially justify the imposition of circuit breakers and is designed to shed light on some potential costs of imposing circuit breakers.

their final wealth at time T. Investors have stochastically heterogeneous beliefs about the expected growth rates of the dividends. To cleanly identify the role of circuit breakers in causing contagion and correlations, we assume that the investors have exponential preferences: As a result, in the absence of circuit breakers, the equilibrium stock returns would be independent. The stock market is subject to a market-wide circuit breaker rule which stipulates that if the sum of the two stock prices (the index) reaches a threshold, the entire stock market is closed until T.

The intuitions for our main results that circuit breakers can cause crash contagion and volatility contagion, and can increase return correlations and market volatility are as follows. After the circuit breaker is triggered, the market is closed, and thus risk sharing is reduced, which in turn causes stock prices to be likely lower than those without market closure. Before the circuit breaker is triggered, when an idiosyncratic negative shock to the price of one stock occurs, the sum of stock prices (in general, the index of the market) gets smaller, the probability of reaching the circuit breaker threshold increases, and thus the price of the other stock may also decrease in anticipation of the more likely market closure. This link through the circuit breaker induces the positive return correlation, even though stocks would be independent in the absence of the circuit breaker. When the idiosyncratic shock is large, and thus the index gets close to the circuit breaker, this increase in the correlation is even greater because the likelihood of a market closure is much higher. In the extreme case where one stock crashes and the circuit breaker is triggered, the price of the other stock with an otherwise continuous price process must jump down to the after-market-closure level. This results in crash contagion. After some stocks fall in prices, the index gets closer to the circuit breaker threshold, other stock prices also fall due to the fear of the more likely market closure, which in turn drives the index even closer to the threshold, and so on. It is this vicious cycle that may increase market volatility. In addition, as one stock becomes more volatile (e.g., due to an increase in the volatility of its dividend), the likelihood of triggering the circuit breaker becomes greater, and thus the prices of other stocks also become more volatile. This explains why a crash of one stock may cause another stock to crash and volatility can transmit across stocks even though stocks would be independent in the absence of circuit breakers. These contagion effects may transform idiosyncratic risks into systemic risks.

Our results suggest that to reduce the contagion effects and the systemic risks, it is better to impose circuit breakers on individual stocks. In this alternative approach, the threshold is based on individual stock returns: when a stock's circuit breaker is triggered, only trading in this single stock is halted. This alternative approach does not increase correlations or cause any form of contagion. We show that with this alternative approach, stock prices are generally higher, a market-wide large decline is less likely, and systemic risk is lower, compared to those with circuit breakers imposed on an index.

In the model, we assume there are only two stocks in the index on which the circuit breakers are based. One possible concern is that in practice indices typically consist of hundreds of stocks (if not more) and therefore it is unlikely that one stock's fall would trigger the fall of many other stocks. On the other hand, in bad times, markets typically focus on a small number of key factors such as Federal Reserve decisions and major economic news. Each of the two stocks in our model represent a large group of stocks that are significantly exposed to a common risk factor in bad times. When there is a bad shock in the risk factor, the prices of the large group of stocks go down, which can drag down another large group of stocks through the circuit breakers connection even though the latter group of stocks is not exposed to the risk factor.

Our paper is motivated by the seminal paper Chen, Petukhov, and Wang (2017). Using a dynamic asset pricing model with a single stock (index), Chen, Petukhov, and Wang (2017) are the first to show in a dynamic equilibrium setting that, contrary to some of the main goals of regulators, a downside circuit breaker may lower stock price, increase market volatility, and accelerate market decline (which they call the "magnet effect"). Different from their focus and findings, this paper focuses on the cross-stock contagion effect of circuit breaker rules using a dynamic equilibrium model with *multiple* stocks and possibly discontinuous stock prices. We show that, in the presence of jump risk, a crash in one stock can cause a crash in an otherwise independent stock. In addition, in Chen, Petukhov, and Wang (2017) the main mechanism through which circuit breakers affect price dynamics is the difference in leverage before and after market closure. Before market closure, investors face no constraints on leverage, but after market closure they cannot lever at all. As a result, investors need to completely unlever when the circuit breaker is triggered, which magnifies the effect of the market closure. In this paper, there is no constraint on leverage either before or after market closure. In contrast, the main economic mechanism that drives our results is the circuit breakers' contagion effect instead of the leverage constraint effect. Our results suggest that, even in the absence of leverage constraints, circuit breakers can still have a large impact on price dynamics.

Among other theoretical work related to circuit breakers, Greenwald and Stein (1991) show that in a market with limited participation, circuit breakers can help coordinate trading for market participants. Subrahmanyam (1994) demonstrates that circuit breakers can increase price volatility because investors may shift their trades to earlier periods with a lower liquidity supply if there is information asymmetry. Hong and Wang (2000) examine the impact of periodic exogenous market closure on asset prices and show that their model produces rich patterns of trading and returns consistent with empirical findings.

Many empirical studies find evidence against advocates of circuit breakers (including market-wide circuit breakers, price limits, and trading pauses). For example, exploiting Nasdaq order book data, Hautsch and Horvath (2019) show that trading pauses cause extra volatility and reduce price stability and liquidity after the pause, but enhance price discovery during the break. Kim and Rhee (1997) find evidence from Tokyo Stock Exchange data suggesting that the price limit system may be ineffective in the sense that price limits may cause higher volatility levels, prevent prices from efficiently reaching their equilibrium level, and interfere with trading. Lauterbach and Ben-Zion (1993) examine the behavior of the Israeli stock market to study the performance of circuit breakers during the October 1987 crash. They find that circuit breakers reduced the next-day opening order imbalance and the initial price loss; however, they had no effect on the long-run response. Lee, Ready, and Seguin (1994) examine the effect of firm-specific New York Stock Exchange (NYSE) trading halts and find that trading halts do not reduce either volume or price volatility during the post-halt period. Goldstein and Kavajecz (2004) focus on the NYSE during the October 1997 market break and demonstrate the magnet effect, that is, an acceleration of activity approaching the market-wide circuit breaker.[3]

Unlike the existing literature, this paper studies impacts of market-wide circuit breakers on the dynamic interactions among multiple stocks. Even though circuit breakers are designed almost exclusively to stabilize markets in bad states, we find that market-wide circuit breakers can have significant crash and volatility contagion effects, *especially in bad states*. To the best of our knowledge, this prediction is new to both the theoretical and the empirical literature on circuit breakers.

[3]A few other studies on market halts focus on other related issues. For example, Ackert, Church, and Jayaraman (2001) conduct an experimental study to analyze the effects of mandated market closures and temporary halts on market behavior. Corwin and Lipson (2000) study order submission strategies of traders around market halts, providing a detailed description of the mechanics of trading halts and identifying traders who provide liquidity. Christie, Corwin, and Harris (2002) study the impact on post-halt market prices of Nasdaq's alternative halt and reopening procedures. Their results are consistent with the hypothesis that increased information transmission during the halt reduces post-halt uncertainty.

2. The Model

We consider a continuous-time exchange economy over a finite time interval $[0,T]$. Investors can trade two risky assets, Stock 1 and Stock 2, and one risk-free asset. Each of the two stocks in our model represents a group of stocks that share the same significant risk exposure in bad times. The risk-free asset has a net supply of zero and the interest rate can be normalized to zero because there is no intertemporal consumption in our model. The total supply of each stock is one share and every stock pays only a terminal dividend at time T. The dividend processes are exogenous and publicly observed. Uncertainty about dividends is represented by a standard Brownian motion Z_t and an independent standard Poisson process N_t with jump intensity κ and jump size μ_J defined on a complete probability space $(\Omega, \mathcal{F}, \mathbf{P})$. An augmented filtration $\{\mathcal{F}_t\}_{t\geq 0}$ is generated by Z_t and N_t.

There is a continuum of investors of Types A and B in the economy, with a mass of 1 for each type. For $i = A, B$ and $j = 1, 2$, Type i investors are initially endowed with θ^i_{j0} shares of Stock j but no risk-free asset, with $0 \leq \theta^i_{j0} \leq 1$ and $\theta^A_{j0} + \theta^B_{j0} = 1$. The probability measure Type A investors use is $\mathbf{P}^A$, which is the same as the true probability measure $\mathbf{P}$. Under Type A's probability measure, Stock 1's dividend process evolves as:

$$dD_{1,t} = \mu_1 dt + \sigma dZ_t, \tag{1}$$

and Stock 2's dividend process follows a jump process with drift:

$$dD_{2,t} = \mu_2 dt + \mu_J(dN_t - \kappa^A dt), \tag{2}$$

where Stock 1's expected dividend growth rate μ_1, Stock 1's dividend volatility σ, Stock 2's expected dividend growth rate μ_2 and jump intensity κ^A are all constants, the compensated Poisson process $dN_t - \kappa^A dt$ is a martingale under $\mathbf{P}$, and $D_{j,0} = 1$ for $j = 1, 2$.

Relative to Type A investors, Type B investors have different beliefs about the dividend process $D_{1,t}$ and employ a different probability measure $\mathbf{P}^B$, under which the dividend process $D_{1,t}$ evolves as

$$dD_{1,t} = \mu^B_{1,t} dt + \sigma dZ^B_t, \tag{3}$$

where Z^B_t is a Brownian motion under measure $\mathbf{P}^B$, and $\mu^B_{1,t} \equiv \mu_1 + \delta_{1,t}$ for a stochastic process $\delta_{1,t}$ (specified below) that measures the disagreement between Type A and Type B investors about the growth rate of the dividend process $D_{1,t}$. Under $\mathbf{P}^B$, the dividend process $D_{2,t}$ evolves as

$$dD_{2,t} = \mu_{2,t}^B dt + \mu_J(dN_t^B - \kappa^B dt), \tag{4}$$

where under measure $\mathbf{P}^B$, N_t^B is a non-homogeneous Poisson process with jump intensity κ_t^B and jump size μ_J, and $\mu_{2,t}^B$ is Stock 2's expected dividend growth rate. For expositional simplicity, we assume $\kappa_t^B = \kappa^A \delta_{2,t}$, where $\delta_{2,t}$ (specified below) measures the disagreement between Type A and Type B investors about the jump intensity of process $D_{2,t}$. Similar to $D_{1,t}$, we assume that Type A and Type B investors also disagree on the expected growth rate of $D_{2,t}$. For simplicity, we assume that this disagreement only stems from the disagreement on the jump intensity. In other words, conditional on no jumps, Type A and B investors agree on Stock 2's expected growth rate, i.e., $\mu_{2,t}^B - \mu_J \kappa_{2,t}^B = \mu_2 - \mu_J \kappa^A$.

The Radon-Nikodym derivative between the two probability measures can therefore be written as.

$$\eta_T = \frac{d\mathbf{P}^B}{d\mathbf{P}^A}|_{\mathcal{F}_T} = e^{\int_0^T \frac{\delta_{1,t}}{\sigma} dZ_t - \int_0^T \frac{\delta_{1,t}^2}{2\sigma^2} dt} \cdot e^{\kappa^A \int_0^T (1-\delta_{2,t}) dt} \prod_{i=1}^{N_T} \delta_{2,t_i}, \tag{5}$$

where $t_i, i = 1, 2, ...$ are jump times before T. We define

$$\eta_{1,T} = e^{\int_0^T \frac{\delta_{1,t}}{\sigma} dZ_t - \int_0^T \frac{\delta_{1,t}^2}{2\sigma^2} dt}, \quad \eta_{2,T} = e^{\kappa^A \int_0^T (1-\delta_{2,t}) dt)} \prod_{i=1}^{N_T} \delta_{2,t_i}.$$

For the disagreement process $\delta_{1,t}$, we assume that under the probability measure $\mathbf{P}$:

$$d\delta_{1,t} = -k_1(\delta_{1,t} - \bar{\delta_1})dt + \nu_1 dZ_t, \tag{6}$$

where $\bar{\delta}_1$ is the constant long-time average of the disagreement (which could be zero), $k_1 > 0$ measures the speed of mean reversion in the disagreement, and ν_1 is the volatility of the disagreement.[4]

For the disagreement process $\delta_{2,t}$, we assume that under the probability measure $\mathbf{P}$:

$$d\delta_{2,t} = -k_2(\delta_{2,t} - \bar{\delta_2})dt + \nu_2 dN_t, \tag{7}$$

where $\bar{\delta}_2$ is the constant long-time average, $k_2 > 0$ is the speed of mean reversion, and ν_2

[4]In the Appendix, we show that this $\delta_{1,t}$ process is consistent with Kalman filtering when Type B investors do not know the expected growth rate of Stock 1's dividend.

is a constant jump size of the disagreement process.[5]

In this paper we focus on the market closure effect of circuit breakers, i.e., investors cannot trade for a period of time after circuit breakers are triggered. As we show later, stochastic disagreement is necessary for the presence of the market closure effect, because in the absence of stochastic disagreement, investors would not trade after time zero even when the market is always open and thus market closure would not have any impact on asset prices. To capture the market crash risk, the fundamentals of a group of stocks must jump down with a positive probability.[6] Therefore, we assume these two features in our model.

Hereafter, we use the notation $\mathbb{E}^i[\cdot]$ to denote the expectation under the probability measure $\mathbf{P}^i$ for $i \in \{A, B\}$.

To isolate the impact of circuit breakers on stock return correlations, we assume that, for $i \in \{A, B\}$, Type i investors have constant absolute risk averse (CARA) preferences over the terminal wealth W_T^i at time T:

$$u(W_T^i) = -\exp(-\gamma W_T^i),$$

where $\gamma > 0$ is the absolute risk aversion coefficient. With CARA preferences, there is no wealth effect and therefore in the absence of circuit breakers, it can be shown that returns of the two stocks would be independent.

Trading in the stocks is subject to a market-wide circuit breaker rule as explained next. Let $S_{j,t}$ denote the price of Stock $j = 1, 2$ at time $t \leq T$ and the index $S_t = S_{1,t} + S_{2,t}$ denote the sum of the two prices (equivalent to an equally weighted index).[7] Define the circuit breaker trigger time

$$\tau = \inf\{t : S_t \leq h, t \in [0, T)\},$$

where h is the circuit breaker threshold (hurdle). At the circuit breaker trigger time τ,

[5]With the specialized dynamics of κ_t^B, N_t^B is a Hawkes process in general. See Aït-Sahalia, Cacho-Diaz, and Laeven (2015) and Aït-Sahalia and Hurd (2016) for applications of Hawkes processes in finance.

[6]In the previous version, we show that when dividend $D_{2,t}$ is not a jump but a diffusion process with continuous paths, similar to $D_{1,t}$, our main results still hold such as increased correlations, volatility contagion and magnet effects in the presence of circuit breakers. The only exception is crash contagion, since no crash (a discrete change in a short time period) occurs in continuous changes of $D_{1,t}$ or $D_{2,t}$. To save space, we do not include these results in this version, but they are available from the authors.

[7]Using a different form of the combination of the stock prices as the index would not change our main results, as long as the index is increasing in both stock prices.

the market is closed until T,[8] which results in the market closure effect. In practice, the circuit breaker threshold h is typically equal to a percentage of the previous day's closing level. In this paper, we set $h = (1-\alpha)S_0$ for a constant α (e.g., $\alpha = 0.07$ for Level 2 market closure in the Chinese stock markets and for Level 1 market closure in the U.S. market).

3. Equilibrium without Circuit Breakers

As a benchmark case, we first solve for the equilibrium stock prices when there is no circuit breaker in place in the market. Because the market is complete in this case, it is convenient to solve the planner's problem:

$$\max_{W_T^A, W_T^B} \mathbb{E}_0^A[u(W_T^A) + \xi\eta_T u(W_T^B)], \tag{8}$$

subject to the budget constraint $W_T^A + W_T^B = D_{1,T} + D_{2,T}$, where ξ is a constant depending on the initial wealth weights of the two types of investors.

From the first order conditions, we obtain:

$$W_T^A = \frac{1}{2\gamma}\log(\frac{1}{\xi\eta_T}) + \frac{1}{2}(D_{1,T} + D_{2,T}), \tag{9}$$

$$W_T^B = -\frac{1}{2\gamma}\log(\frac{1}{\xi\eta_T}) + \frac{1}{2}(D_{1,T} + D_{2,T}). \tag{10}$$

Given the utility function $u(x) = -e^{-\gamma x}$, the state price density under Type A investors' beliefs is

$$\pi_t^A = \mathbb{E}_t^A[\zeta u'(W_T^A)] = \mathbb{E}_t^A[\gamma\zeta e^{-\gamma W_T^A}] = \gamma\zeta\xi^{\frac{1}{2}}\mathbb{E}_t^A[\eta_T^{\frac{1}{2}} \cdot e^{-\frac{\gamma}{2}(D_{1,T}+D_{2,T})}], \tag{11}$$

for some constant ζ. Therefore, the stock price in equilibrium is given by

$$\hat{S}_{j,t} = \frac{\mathbb{E}_t^A\left[\pi_T^A D_{j,T}\right]}{\mathbb{E}_t^A[\pi_T^A]} = D_{j,t} + \frac{\mathbb{E}_t^A\left[\pi_T^A(D_{j,T} - D_{j,t})\right]}{\mathbb{E}_t^A[\pi_T^A]}, \quad j = 1, 2. \tag{12}$$

[8] Assuming that markets can reopen after being halted for a period of time would not change the qualitative results on contagion. Quantitatively, the results are close in very bad times, because the fear of market closure is similar whether the closure is long or relatively short in very bad times.

Since the two dividend processes are independent, Equation (12) can be simplified into

$$\hat{S}_{1,t} = \frac{\mathbb{E}_t^A[\pi_{1,T}^A D_{1,T}]}{\mathbb{E}_t^A[\pi_{1,T}^A]}, \qquad \hat{S}_{2,t} = \frac{\mathbb{E}_t^A[\pi_{2,T}^A D_{2,T}]}{\mathbb{E}_t^A[\pi_{2,T}^A]}, \tag{13}$$

where $\pi_{1,t}^A = \mathbb{E}_t^A[\eta_{1,T}^{1/2} \cdot e^{-\frac{\gamma}{2}D_{1,T}}]$, $\pi_{2,t}^A = \mathbb{E}_t^A[\eta_{2,T}^{1/2} e^{-\frac{\gamma}{2}D_{2,T}}]$. Thus, the two prices can be computed separately when there are no circuit breakers, which implies that stock returns are independent.

Next, we derive the equilibrium prices in closed form for the two stocks and examine the impact of the jump and the stochastic disagreement on the market equilibrium.

For Stock 1, the disagreement process is governed by the mean-reverting process (6). The formula of equilibrium price $\hat{S}_{1,t}$ can be derived analytically and is presented in the following proposition.

PROPOSITION 1. *When there are no circuit breakers, the equilibrium price of Stock 1 is:*

$$\hat{S}_{1,t} = D_{1,t} + \mu_1^A(T-t) - 2\left(\frac{dA(t;\gamma)}{d\gamma} + \frac{dC(t;\gamma)}{d\gamma}\delta_{1,t}\right), \tag{14}$$

where $A(t;\gamma)$ and $C(t;\gamma)$ are given in Appendix A.

Proposition 1 shows that, in addition to the dividend payment, disagreement also affects the price of Stock 1. As a result, the instantaneous volatility of the stock price $\hat{S}_{1,t}$ is different from that of the dividend process.[9]

To show the importance of disagreement being stochastic, we next show what would happen if the disagreement were constant, that is, $\delta_{1,t} = \delta_{1,0}$ for all $t \in [0,T]$. In this case, the equilibrium price would simplify to

$$\hat{S}_{1,t} = D_{1,t} + \frac{\mu_1^A + \mu_1^B}{2}(T-t) - \frac{\gamma}{2}\sigma^2(T-t).$$

Thus, the equilibrium price of Stock 1 would be determined by the average beliefs of Type A and B investors on the expected growth rate of the dividend and the volatility of the stock price would be the same as the volatility of its dividend. Moreover, by applying Ito's lemma to the wealth process $W_t^A = \frac{\mathbb{E}_t^A[\pi_T^A W_T^A]}{\mathbb{E}_t^A[\pi_T^A]}$, we can find that the equilibrium

[9]It can be shown that the instantaneous volatility of the equilibrium price $\hat{S}_{1,t}$ is greater than the volatility of the dividend process $D_{1,t}$ when $T-t$ is small.

number of shares of Stock 1 held by Type A investors would be equal to

$$\hat{\theta}^A_{1,t} = \frac{1}{2} - \frac{1}{2\gamma}\frac{\delta_{1,0}}{\sigma^2}, \tag{15}$$

which implies that the equilibrium number of shares of Stock 1 held by Type B investors would be equal to

$$\hat{\theta}^B_{1,t} = \frac{1}{2} + \frac{1}{2\gamma}\frac{\delta_{1,0}}{\sigma^2}. \tag{16}$$

Because the number of shares held by investors in the equilibrium would be constant over time if the disagreement were constant, market closure would not have any impact on the equilibrium price in the case of constant disagreement. This result implies that stochastic disagreement is necessary for circuit breakers to have any impact through the market closure channel.

For Stock 2, an analytical expression of the equilibrium price in the case of stochastic disagreement can unlikely be obtained. However, it can be shown that if the disagreement $\delta_{2,t} = \delta_2$ is a constant, then

$$\mathbb{E}^A_t[\pi^A_{2,T} D_{2,T}] = \mathbb{E}^A_t[\eta^{1/2}_{2,T} e^{-\gamma D_{2,T}/2}] \cdot \left(D_{2,t} + (\mu_2 - \kappa\mu_J)(T-t) + \kappa^A\sqrt{\delta_2}\mu_J(T-t)e^{-\frac{\gamma}{2}\mu_J}\right).$$

Then by Equation (13), we have the equilibrium price of Stock 2 as in the following proposition.

PROPOSITION 2. *When there are no circuit breakers and $\delta_{2,t} = \delta_2$ (i.e., constant disagreement on the jump intensity), the equilibrium price of Stock 2 is:*

$$\hat{S}_{2,t} = D_{2,t} + \mu_2(T-t) + \kappa^A\sqrt{\delta_2}\mu_J(T-t)e^{-\frac{\gamma}{2}\mu_J}. \tag{17}$$

Proposition 2 shows that the equilibrium price is affected by the heterogenous beliefs through the geometric average of beliefs of Type A and Type B investors on the jump intensity. In addition, the instantaneous volatility (square root of instantaneous variance) of the equilibrium price under $\mathbf{P}^A$ is the same as that of the dividend process because the rest of the terms in (17) are deterministic.

Let $\hat{\theta}^A_{j,t}$ be the optimal shares of Stock i held by Type A investors. Then $dW^A_t = \hat{\theta}^A_{1,t} d\hat{S}_{1,t} + \hat{\theta}^A_{2,t} d\hat{S}_{2,t}$. Applying Ito's formula to $W^A_t = \mathbb{E}^A_t[\pi^A_T W^A_T]/\pi^A_t$ and collecting the coefficients of stochastic terms, we obtain the optimal shares holding of Stock 2 for Type

A investors as follows.

$$\hat{\theta}_{2,t}^{A} = \frac{1}{2} - \frac{1}{2\gamma\mu_J} \log \delta_2. \tag{18}$$

This shows that in the absence of circuit breakers, if the disagreement were constant, then the equilibrium trading strategy in Stock 2 for all investors would be to buy and hold and thus market closure would not have any impact on Stock 2 price. Therefore, as for Stock 1, stochastic disagreement is also important for Stock 2 to capture the market closure effect.

4. Equilibrium with Circuit Breakers

In this section, we study equilibrium prices when the circuit breaker rule is imposed in the market. We first solve for the indirect utility functions at the circuit breaker trigger time τ by maximizing investors' expected utility at $\tau \leq T$:

$$\max_{\theta_{1,\tau}^{i}, \theta_{2,\tau}^{i}} \mathbb{E}_{\tau}^{i}[u(W_{\tau}^{i} + \theta_{1,\tau}^{i}(S_{1,T} - S_{1,\tau}) + \theta_{2,\tau}^{i}(S_{2,T} - S_{2,\tau}))], \ i \in \{A, B\}, \tag{19}$$

with the market clearing condition $\theta_{j,\tau}^{A} + \theta_{j,\tau}^{B} = 1$ and the terminal condition $S_{j,T} = D_{j,T}$, where $\theta_{j,\tau}^{i}$ is the optimal number of shares of Stock j held by Type i investors at time τ, for $i \in \{A, B\}$ and $j = 1, 2$.

If the circuit breaker is triggered by a continuous decline in Stock 1's dividend, then the after-closure prices of both stocks will reflect their respective fundamental values because both dividends are continuous at the trigger time and investors can trade continuously. If there is a jump in Stock 2's dividend, then the index level corresponding to the after-jump dividend levels may fall strictly below the circuit breaker threshold h. To resolve this technical issue, as what is done in practice, we assume that investors can trade both stocks one more time to reflect the after-jump dividend levels after the circuit breaker is triggered by a jump in Stock 2's price caused by a jump in its dividend. [10] Therefore, at the market closure time, both stocks can reach their fundamental values regardless of which stock triggered the circuit breaker.

Exploiting the dynamics of $D_{j,t}$ and evaluating the expectation in the above optimiza-

[10] An alternative justification is that the jump can be viewed as an approximation of a deterministic steep decline (less than but very close to a 90-degree drop) and during the fast decline, Stock 1 or Stock 2 can trade freely and reach their fundamental values.

tion problems, we obtain a system of equations that determine $\theta^i_{j,\tau}$ for $i \in \{A, B\}, j = 1, 2$. Then the equilibrium prices are obtained through market clearing conditions. We summarize the result in the following proposition.

PROPOSITION 3. *Suppose that the market is halted at a stopping time $\tau < T$.*
(1) For Stock 1, the market clearing price at τ is given by

$$S^c_{1,\tau} = D_{1,\tau} + \mu^A_1(T-\tau) - \gamma\theta^A_{1,\tau}\sigma^2(T-\tau),$$

where the optimal share holding of Type A investors is

$$\theta^A_{1,\tau} = \frac{-\frac{1}{\tilde{k}_1}(1-e^{\tilde{k}_1(\tau-T)})\delta_{1,\tau} - \frac{k_1\bar{\delta}_1}{\tilde{k}_1}(T-\tau-\frac{1-e^{\tilde{k}_1(t-T)}}{\tilde{k}_1}) + I_\tau}{I_\tau + \gamma\sigma^2(T-\tau)}, \tag{20}$$

with $\tilde{k}_1 = k_1 - \frac{\nu}{\sigma}$ and

$$I_\tau = -\gamma\sigma^2(\tau-T) + \frac{2\nu\sigma\gamma}{\tilde{k}_1}(T-\tau-\frac{1-e^{\tilde{k}_1(\tau-T)}}{\tilde{k}_1}) + \frac{\nu^2\gamma}{\tilde{k}_1^2}(T-\tau-2\frac{1-e^{\tilde{k}_1(\tau-T)}}{\tilde{k}_1} + \frac{1-e^{2\tilde{k}_1(\tau-T)}}{2\tilde{k}_1}).$$

If $\tilde{k}_1 = 0$, the optimal share holding is simplified into:[11]

$$\theta^A_{1,\tau} = \frac{1}{\gamma}\left(\frac{\gamma\sigma^2 - \gamma\nu\sigma(\tau-T) + \frac{1}{2}k_1\bar{\delta}_1(\tau-T) + \frac{\nu^2\gamma}{3}(\tau-T)^2 - \delta_{1,\tau}}{-\nu\sigma(\tau-T) + \frac{\nu^3}{3}(\tau-T)^2 + 2\sigma^2}\right).$$

(2) For Stock 2, the market clearing price is given by

$$S^c_{2,\tau} := D_{2,\tau} + (\mu^A_2 - \kappa^A\mu_J)(T-\tau) + \kappa^A\mu_J(T-\tau)e^{-\gamma\theta^A_{2,\tau}\mu_J}.$$

The optimal share holding $\theta^A_{2,\tau}$ of Type A investors at τ is specified in Appendix B.2.

As in the case of no circuit breakers, because dividend processes are independent and investors have CARA preferences, the price of a stock only depends on its own dividend process at the circuit breaker trigger time.

[11]It can be verified that as $\tau \to T^-$, $\theta^A_{1,\tau} \to \frac{1}{2} - \frac{\delta_{1,T}}{2\gamma\sigma^2}$, which coincides with the optimal share holding of Stock 1 by Type A in the case of constant disagreement.

4.1 Circuit Breaker Trigger Time τ

The circuit breaker trigger time τ can be characterized using the dividend values. Because the market is closed when the sum of prices falls below (or reaches) the threshold h, we have

$$\begin{aligned} h &\geq S^c_{1,\tau} + S^c_{2,\tau} \\ &= D_{1,\tau} + D_{2,\tau} + \left(\mu_1^A - \gamma\sigma^2\theta^A_{1,\tau} + (\mu_2^A - \kappa^A\mu_J) + \kappa^A\mu_J e^{-\gamma\theta^A_{2,\tau}\mu_J}\right)(T-\tau). \end{aligned}$$

It follows that we may define the stopping time τ using the dividend processes as follows.

PROPOSITION 4. *Let h be the threshold. Define a stopping time*

$$\tau = \inf\{t \geq 0 : D_{1,t} + D_{2,t} \leq \underline{D}(t)\},$$

where

$$\underline{D}(t) = h - \left(\mu_1^A - \gamma\sigma^2\theta^A_{1,\tau} + (\mu_2^A - \kappa^A\mu_J) + \kappa^A\mu_J e^{-\gamma\theta^A_{2,\tau}\mu_J}\right)(T-\tau).$$

Then the circuit breaker is triggered at time τ when $\tau < T$.

Note that $D_{1,t} + D_{2,t}$ is a jump diffusion process; thus, the trigger time τ is the first time the jump-diffusion process hits or goes below $\underline{D}(t)$.

4.2 Equilibrium Prices before τ

After obtaining the market clearing prices and the optimal portfolios at τ, we now study the equilibrium stock prices for $t < \tau \wedge T$. For $i \in \{A, B\}$, let

$$G^i_\tau(\theta^{i,*}_{1,\tau}, \theta^{i,*}_{2,\tau}) = G^i_{1,\tau} + G^i_{2,\tau},$$

where $G^i_{1,\tau}$ and $G^i_{2,\tau}$ are given by (B.5),(B.6), and (B.16) in Appendix B. It can be shown that the indirect utility function of Type $i \in \{A, B\}$ at τ can be written as follows:

$$V^i(W^i_\tau, \tau) = \max_{\theta^i_{1,\tau}, \theta^i_{2,\tau}} \mathbb{E}^i_\tau[u(W^i_\tau + \theta^i_{1,\tau}(S_{1,T} - S_{1,\tau}) + \theta^i_{2,\tau}(S_{2,T} - S_{2,\tau}))] = -e^{-\gamma(W^i_\tau + G^i_\tau(\theta^{i,*}_{1,\tau}, \theta^{i,*}_{2,\tau}))}.$$

Then we are ready to solve the planner's problem at time $t < T \wedge \tau$:

$$\max_{W^A_{T\wedge\tau}, W^B_{T\wedge\tau}} \mathbb{E}^A_t[V^A(W^A_{T\wedge\tau}, T\wedge\tau) + \xi\eta_{T\wedge\tau}V^B(W^B_{T\wedge\tau}, T\wedge\tau)], \tag{21}$$

subject to the wealth constraint $W^A_{T\wedge\tau} + W^B_{T\wedge\tau} = S_{1,T\wedge\tau} + S_{2,T\wedge\tau}$.

Similar to the case without circuit breakers, it follows from the first order conditions and the wealth constraint that

$$W^A_{T\wedge\tau} = \frac{1}{2\gamma}\log(\frac{1}{\xi\eta_{T\wedge\tau}}) + \frac{1}{2}(S_{1,T\wedge\tau} + S_{2,T\wedge\tau}) + \frac{G^B_{T\wedge\tau} - G^A_{T\wedge\tau}}{2}, \tag{22}$$

$$W^B_{T\wedge\tau} = -\frac{1}{2\gamma}\log(\frac{1}{\xi\eta_{T\wedge\tau}}) + \frac{1}{2}(S_{1,T\wedge\tau} + S_{2,T\wedge\tau}) + \frac{G^A_{T\wedge\tau} - G^B_{T\wedge\tau}}{2}. \tag{23}$$

In addition, the state price density under Type A investors' beliefs is

$$\begin{aligned}\pi^A_t &= \mathbb{E}^A_t[\zeta(V^A(W^A_{T\wedge\tau}, T\wedge\tau))'] = \mathbb{E}^A_t[\gamma\zeta e^{-\gamma(W^A_{T\wedge\tau}+G^A_{T\wedge\tau})}] \\ &= \gamma\zeta\mathbb{E}^A_t[\eta^{1/2}_{T\wedge\tau}\cdot e^{-\frac{\gamma}{2}(S_{1,T\wedge\tau}+S_{2,T\wedge\tau}+G^B_{T\wedge\tau}+G^A_{T\wedge\tau})}],\end{aligned} \tag{24}$$

for some constant ζ, where $(V^A(W^A_{T\wedge\tau}, T\wedge\tau))'$ denotes the marginal utility of wealth. Thus, the stock price at $t < T\wedge\tau$ in equilibrium is given by

$$S_{j,t} = \frac{\mathbb{E}^A_t[\pi^A_{T\wedge\tau}S_{j,T\wedge\tau}]}{\mathbb{E}^A_t[\pi^A_{T\wedge\tau}]}, \quad j = 1, 2, \tag{25}$$

with

$$S_{j,T\wedge\tau} = \begin{cases} D_{j,T}, & \text{if } \tau \geq T, \\ S^c_{j,\tau}, & \text{if } \tau < T. \end{cases} \tag{26}$$

In Equation (25), because the stopping time τ depends on the circuit breaker threshold h, the equilibrium prices $S_{1,t}$ and $S_{2,t}$ also depend on h. On the other hand, in practice, h depends on the initial stock prices $S_{1,0}$ and $S_{2,0}$, because $h = (1-\alpha)(S_{1,0} + S_{2,0})$ (e.g., $\alpha = 0.07$ for Chinese markets). Therefore, to obtain the equilibrium prices $S_{1,t}$ and $S_{2,t}$, we need to solve the following fixed point problem in $S_{1,0}$ and $S_{2,0}$:

$$S_{j,0} = \frac{\mathbb{E}^A_0[\pi^A_{T\wedge\tau}S_{j,T\wedge\tau}]}{\mathbb{E}^A_0[\pi^A_{T\wedge\tau}]}, j = 1, 2, \tag{27}$$

where the right hand side is implicitly a function of the initial stock prices $S_{1,0}$ and $S_{2,0}$. The following proposition guarantees the existence and uniqueness of a solution to the above fixed point problem.

PROPOSITION 5. *If the initial equilibrium index value $\hat{S}_{1,0} + \hat{S}_{2,0}$ is positive in the absence of circuit breakers, there exists a unique solution to the fixed point problem (27) in the presence of circuit breakers.*

We can then compute the trading strategies as follows. The wealth process of Type A investors is

$$W_t^A = \frac{\mathbb{E}_t^A[\pi_{T\wedge\tau}^A W_{T\wedge\tau}^A]}{\mathbb{E}_t^A[\pi_{T\wedge\tau}^A]}, t < T \wedge \tau. \tag{28}$$

From the budget constraint we have

$$dW_t^A = \bar{\theta}_{1,t}^A dS_{1,t} + \bar{\theta}_{2,t}^A dS_{2,t},$$

where $\bar{\theta}_{1,t}^A$ and $\bar{\theta}_{2,t}^A$ are share holdings of Type A for Stock 1 and Stock 2, respectively. For $j = 1, 2$, we can recover the share holdings of Stock j at t by calculating quantities of $\mathbb{E}_t^A[dW_t^A \cdot dS_{j,t}]$, $\mathbb{E}_t^A[dS_{1,t} \cdot dS_{2,t}]$, and $\mathbb{E}_t^A[(dS_{j,t})^2]$ through simulations.

In the next section, we numerically compute the equilibrium prices and analyze the impact of circuit breakers.

5. Impact of Circuit Breakers

In this section, we examine the impact of circuit breakers on the dynamics of the market. The default parameter values for numerical analysis are set as follows, where daily growth rates and volatilities are used.[12] The algorithms used for the numerical analysis are presented in Appendix F.

$$\begin{array}{llll}
\mu_1 = 0.10/250, & \sigma = 0.4, & \nu = 0.5, & \\
k_1 = 0.1, & \delta_{1,0} = 0, & \bar{\delta}_1 = 0, & \\
\mu_2 = 0.10/250, & \mu_J = -0.25, & \kappa = 1, & \\
k_2 = 0.1, & \delta_{2,0} = 1, & \bar{\delta}_2 = 1, & \mu_\delta = 0.5, \\
\gamma = 1, & \alpha = 0.07, & T = 1. &
\end{array}$$

Because $\delta_{1,0} = 0$ and $\delta_{2,0} = 1$, Type B investors initially correctly estimate the expected growth rate of Dividend 1 and the jump intensity of Stock 2's dividend. Since our main goal is to examine the impact of circuit breakers in *bad* times when the market is volatile and the crash probability of some stocks is high (e.g., the U.S. market in the week of March 9, 2020 and the Chinese stock market in early January of 2016), we set the jump frequency high and the jump size large, along with a high volatility of Stock 1's dividend.

[12]We have analyzed the impact using a wide range of parameter valuee and have obtained the same qualitative results.

Because of the CARA preferences, the initial share endowment of the investors does not affect the equilibrium. The circuit breaker is triggered when the sum of the two prices (i.e., the index) first goes below the threshold $(1-\alpha)(S_{1,0}+S_{2,0})$, i.e., drops 7% from the initial value.

One alternative to the market-wide circuit breakers is to impose a circuit breaker separately on each stock (instead of on an index). With this separate circuit breaker on each stock, if a circuit breaker for a stock is triggered, only the trading in the corresponding stock is halted. For example, when the circuit breaker of Stock 1 is triggered, only the trading of Stock 1 is halted, but trading in Stock 2 is unaffected. Obviously, with separate circuit breakers, equilibrium prices remain independent, in sharp contrast to the case of market-wide circuit breakers. Let $S_{j,t}^{sep}, j = 1, 2$ denote the equilibrium prices of Stock j in this benchmark. We compare the impact of circuit breakers on the stock prices when they are on an index and when they are on individual stocks.

5.1 Equilibrium Prices

By Propositions 1 and 2, we obtain the initial equilibrium prices $\hat{S}_{1,0} = 0.8725, \hat{S}_{2,0} = 0.9703$ in the absence of circuit breakers. When there are separate circuit breakers on individual stocks, the equilibrium prices are $S_{1,0}^{sep} = 0.8719$ and $S_{2,0}^{sep} = 0.9577$, which are respectively lower than those without circuit breakers. In the presence of market-wide circuit breakers, we obtain the equilibrium prices $S_{1,0} = 0.8652$ and $S_{2,0} = 0.9418$. The prices of both stocks with separate circuit breakers are lower than those without circuit breakers, because market closure prevents risk sharing after the circuit breakers are triggered. In addition, with circuit breakers on an index, the prices are even lower. As we show later, this is because of the contagion effect of the circuit breakers on indices.

5.2 Crash Contagion

Because the circuit breaker based on a stock index is triggered when the index reaches a threshold, a crash in a group of stocks (e.g., from a downward jump in their dividends) may trigger the circuit breaker and cause the entire market to be closed down. As a result, the prices of otherwise independent stocks may also jump down because of the sudden market-wide closure. We refer this pattern of cross-stock serial crashes as *crash contagion.*

Let $\mathscr{C}$ denote the event that a crash in Stock 2's price triggers the circuit breakers at τ and Δt be a small time interval. In Panel A of Figure 1, we plot the distribution of

Stock 1's price change between $\tau - \Delta t$ and τ without circuit breakers, with or without a change in Stock 2's price (green line). In Figure 1, we plot the unconditional distribution of Stock 1's price change between τ and $\tau - \Delta t$ (blue line), the corresponding distribution conditional on the event $\mathscr{C}$ (red dashed line), and the conditional distribution of Stock 1's price change between τ and $\tau-$ (red line). The blue line in Figure 1 shows that without circuit breakers, the price change of Stock 1 between τ and $\tau - \Delta t$, with or without a crash in Stock 2's price, is normally distributed. This implies that without circuit breakers, there is no contagion across stocks. In contrast, as the red dashed line in Figure 1 shows, in the presence of circuit breakers, after a crash of Stock 2 that triggers a circuit breaker, the distribution of Stock 1's price change between τ and $\tau - \Delta t$ shifts leftward significantly compared to the unconditional distribution. This distribution shift indicates crash contagion from Stock 2 to Stock 1 in the presence of circuit breakers. Figure 1 indicates that on average a crash of Stock 2 's price causes Stock 1 price to drop, suggesting a positive correlation between the returns of the two stocks when the market crashes. Recall that in the absence of circuit breakers, Stock 1 price is continuous and thus Stock 1's price change between τ and $\tau-$ is zero. The red line of Figure 1 implies that in the presence of circuit breakers, not only there is contagion but also a crash in Stock 2's price can cause a crash (jumping down) in Stock 1's price, i.e., circuit breakers can result in a crash contagion. This discontinuity in Stock 1's price is due to the discontinuous change in the value of the stock due to the sudden market closure. Stock 1's price jumps down because the market closure reduces risk sharing and thus increases the riskiness of the stock.

Our findings are consistent with what happened in March 2020 in the U.S. stock market. The circuit breaker of the U.S. market was triggered four times in March 2020. The first one occurred at 9:34:13 am on March 9th, less than five minutes after the market opened. The second occurred at 9:35:43 on March 12th. Four days later, the market lasted only one second on March 16th before the circuit breaker was triggered. The fourth time occurred at 12:56:17 pm on March 18th. To illustrate that our results are consistent with what happened around the circuit breaker trigger time, we use high-frequency prices of the components of the S&P 500 index during the 10 minutes before the circuit breaker was triggered on March 18th, 2020 and sort components by their total dollar trading volumes. Simple regression of the return of the top 25%–50% of stocks inside the S&P 500 index on the lagged return of the top 25% of stocks suggests that, in the market crash of March 18th, 2020, the crash of the top 25%–50% of stocks followed that of the top 25% of stocks. Figure 2 depicts how returns of S&P 500 component stocks

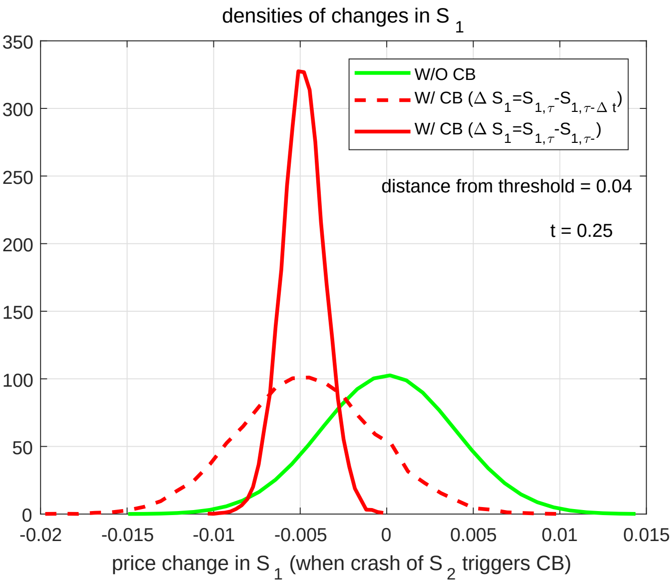

Figure 1. Distribution of changes in Stock 1's price when the circuit breaker is triggered by a jump in Stock 2's price. In the presence of a circuit breaker, the distribution is skewed negatively. Results for two methods of measuring the changes are presented. Meanwhile, in the absence of circuit breakers the price changes follows a normal distribution.

moved during the 10 minute period right before the market was halted.

Let Rt_t and Rb_t be the time t returns of the top 25% of stocks and the top 25%–50% of stocks, respectively. We obtain the regression result as follows.

$$\begin{aligned} Rb_t =& -0.01 + 0.5Rt_{t-\Delta t} + 0.1Rt_t + \epsilon_t, \\ t\text{-}stat:& \ (-9) \quad (10.8) \qquad (2.2) \end{aligned}$$

where Δt equals one second. The regression result indicates that those stocks with relatively low trading volumes followed the moves of those with high trading volumes. While this does not prove the causal relationship, it suggests a pattern that is consistent with the cross-stock contagion our model predicts.

A similar illustration is shown in Figure 3 for stocks in the China Securities Index (CSI) 300 index on January 4th, 2016 when the Chinese stock market crashed. Simple regression of the return of the top 25%-50% of stocks inside the CSI 300 index on the lagged return of the top 25% of stocks yields

$$\begin{aligned} Rb_t =& 10^{-4} + 0.78Rt_{t-\Delta t} + 0.25Rt_t + \epsilon_t, \\ t\text{-}stat:& \ (0.67) \quad (2.85) \qquad (0.92) \end{aligned}$$

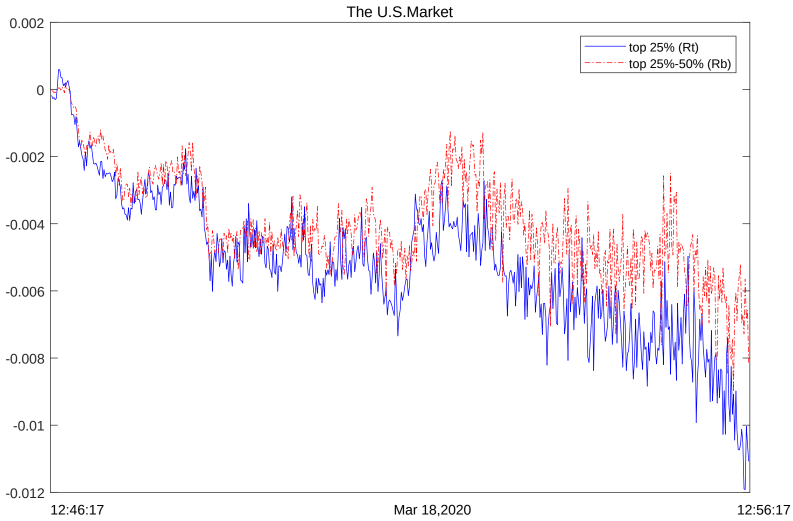

Figure 2. Evidence of contagion in real markets: the United States.

where Δt equals 3 seconds. This result suggests that, in the market crash of January 4th, 2016, the crash of the top 25%–50% of stocks followed that of the top 25% of stocks.

5.3 Increased Correlations

With circuit breakers based on indices, a discrete jump (crash) in a stock is not necessary to adversely affect other otherwise independent stocks. Intuitively, even after a small decline in the price of a stock, the index gets closer to the circuit breaker threshold and thus the market is more likely to be closed early, which may lower the prices of other otherwise independent stocks, which in turn makes the index even closer to the circuit breaker threshold, entering into a vicious circle. This contagion magnitude is typically smaller than that caused by a crash in a stock in normal times, but can become much more significant and create strong correlations when the circuit breaker is close to being triggered because of the magnified vicious circle effect. We next show that a gradual change in the price of a stock can indeed affect the price of another stock and can also cause high correlations among otherwise independent stocks when the index gets close to the circuit breaker threshold.

Consistent with our intuition, Figure 4 shows that the correlation between the two prices with circuit breakers increases significantly as the index gets close to the thresh-

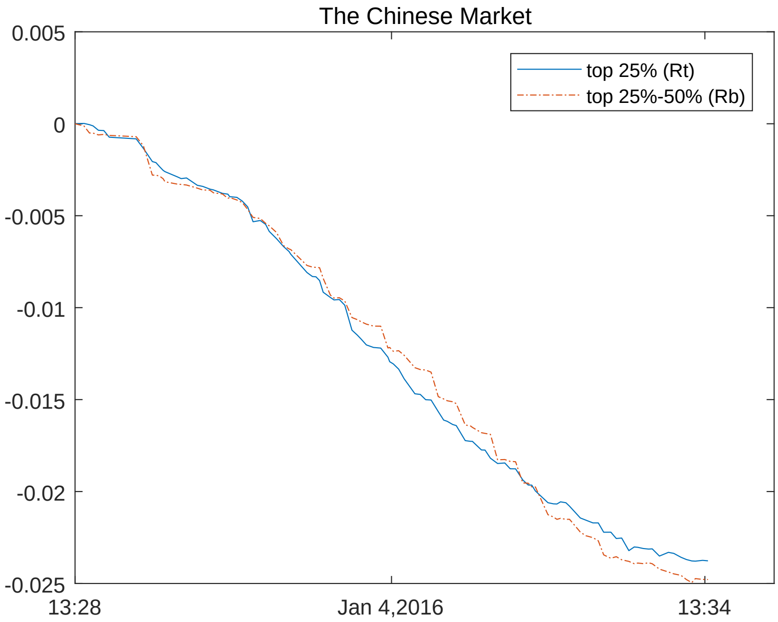

Figure 3. Evidence of contagion in real markets: China.

old.[13] When the index is far from the threshold and thus a market closure is unlikely, the correlation becomes close to zero, because the correlation without circuit breakers is zero. In addition, when the potential market closure duration is large ($T - t$ is large), the impact of the circuit breakers on the correlation is even greater, because the fear of a market closure is stronger when the potential market closure duration is longer. For example, conditional on the same distance of 0.02 from the threshold, if it is later in the day at $t = 0.75$, the correlation is 0.2, but the correlation increases to about 0.55 if it is early in the day at $t = 0$.[14]

[13]In the figure, "distance from threshold" is defined as the value of the index in exceed of the threshold. Because the equilibrium index level is determined jointly by the dividend levels of the two stocks, the way to vary the distance is not unique. In all the figures in this paper that plot against the distance to threshold we fix $D_{2,t}$ and vary $D_{1,t}$. We also used alternative ways such as fixing $D_{1,t}$ and varying $D_{2,t}$ and find similar results.

[14]So far the dividend processes are assumed to be uncorrelated and we show that a strong correlation of the stock prices can emerge due to circuit breakers. One concern may be that if the dividend processes are already correlated, then the additional correlation caused by the circuit breakers may be small and thus the effect of circuit breakers in increasing correlation may be small in practice. To address this concern, in an earlier version of the paper, we show that even when the dividends are correlated, the presence of circuit breakers can significantly increase the correlation of stock prices further. In addition, the presence of circuit breakers can even make negatively correlated stocks in the absence of circuit breakers become positively correlated. This reversal is because as the index gets close to the threshold, the common fear for market closure offsets the effect of the negatively correlated dividends and as a

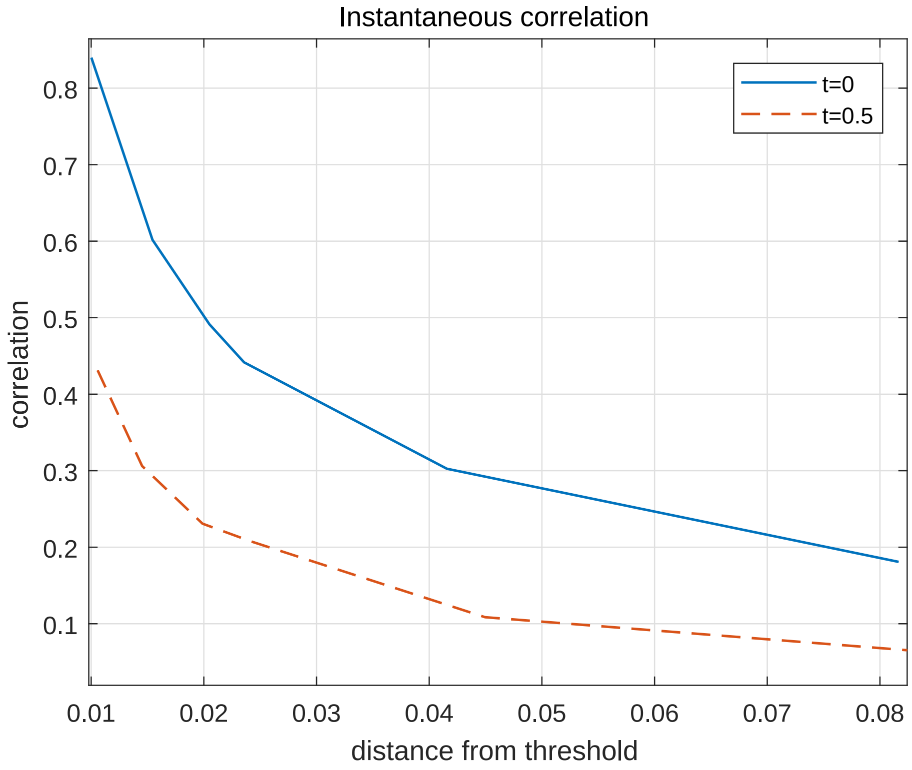

Figure 4. Instantaneous correlation.

5.4 Volatility Contagion and Volatility Amplification

Next, we show that in addition to crash contagion, circuit breakers can also cause volatility contagion among otherwise independent stocks, i.e., an increase in the volatility of one stock can cause an increase in that of another. Figure 5 plots the instantaneous volatility of Stock 2 against that of Stock 1 for $t = 0$ and $t = 0.25$ when the index level is 0.01 above the circuit breaker threshold as we change the volatility of Stock 1's dividend. Figure 5 indicates that, indeed, a higher volatility of Stock 1's dividend can cause a higher volatility of Stock 2. Intuitively, the stock price contagion causes the volatility contagion. As explained above, after some stocks fall in prices, the index gets closer to the circuit breaker threshold, other stock prices also fall due to the fear of the more likely market closure, which in turn drives the index even closer to the threshold, and so on. This vicious cycle implies that as the price change of one stock becomes more volatile, so does the price change of the other, resulting in volatility contagion, especially when the index is close to the circuit breaker threshold. In addition, when the time to horizon

result the correlation turns positive. These results are not presented in the current version to save space, but available from the authors.

is longer ($t = 0$), the effect of the circuit breakers is greater, and therefore the degree of contagion is larger as measured by the sensitivity of Stock 2's volatility change to Stock 1's volatility change (the red line slope).

One of the regulatory goals of the circuit breaker is to reduce market volatility in bad times. Because of the volatility contagion, we conjecture that contrary to regulators' intention, circuit breakers may increase the market volatility in bad times. Figure 6 plots the volatility of the index with circuit breakers against the index's distance from the circuit breaker threshold at two different time points $t = 0$ and $t = 0.5$. Figure 6 suggests that, indeed, contrary to the regulatory goal, circuit breakers can amplify the market volatility. This is because the vicious cycle effect described above can increase the sensitivity of stocks' prices to dividend shocks.

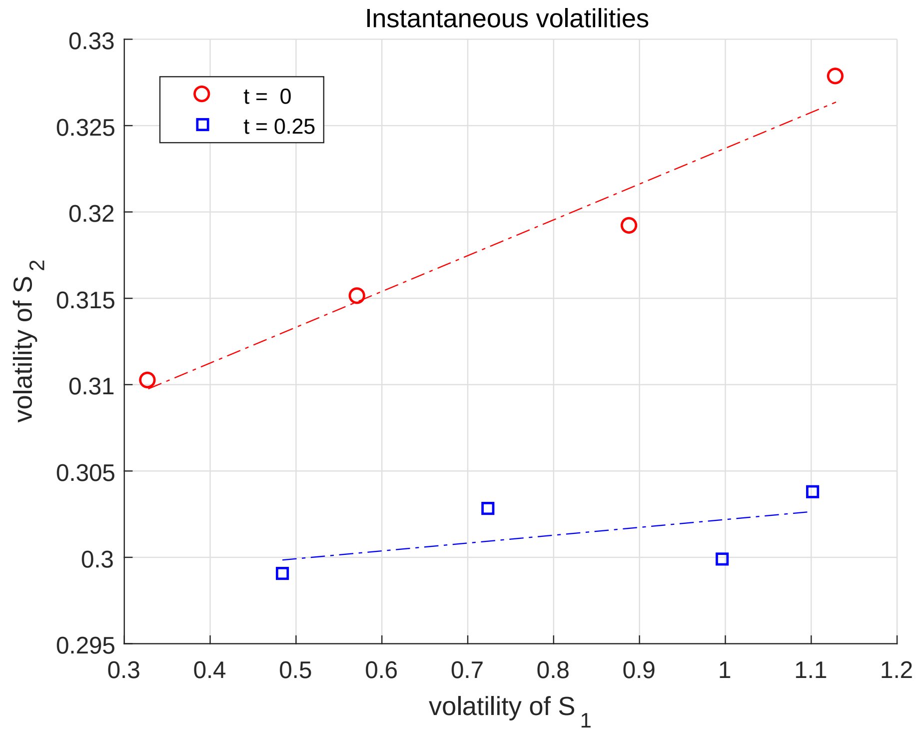

Figure 5. This figure shows that volatilities of Stocks 1 and 2 are correlated in the presence of circuit breakers.

5.5 Acceleration of Market Decline: The Magnet Effect

Circuit breakers are implemented to protect the market from a fast decline. Contrary to this intention, Chen, Petukhov, and Wang (2017) show in a single-stock setting that circuit breakers can accelerate a stock price decline compared to the case without circuit breakers. This acceleration is what is called the "magnet effect" by Chen, Petukhov,

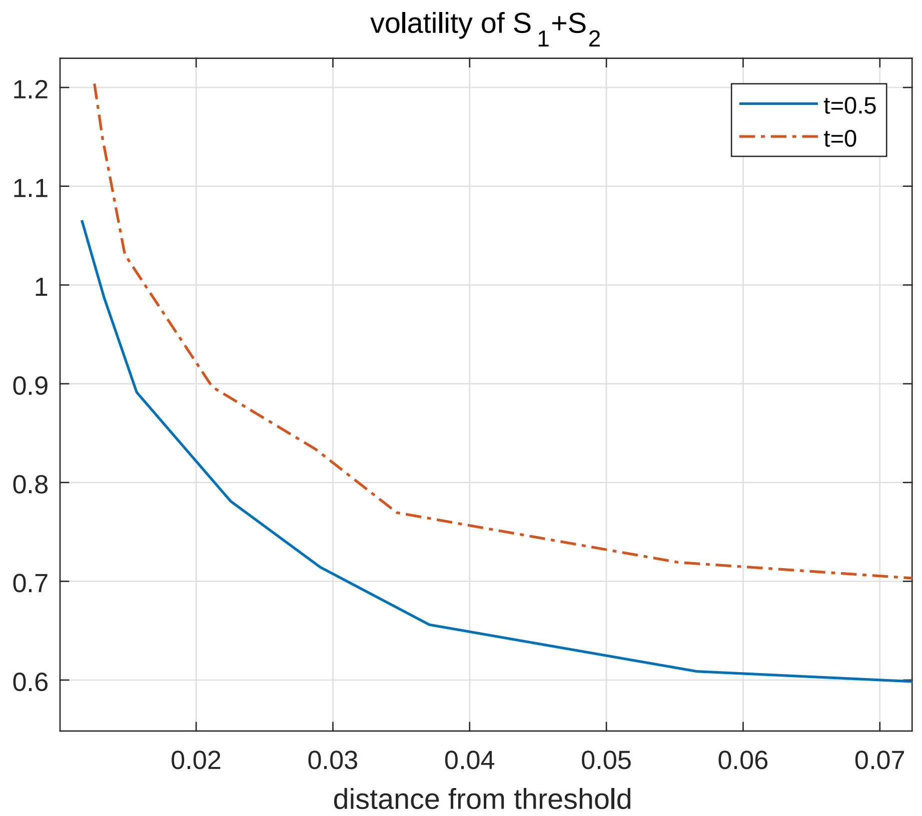

Figure 6. This figure plot the ratios of volatility with circuit breakers to that without circuit breakers against the distance from the circuit breaker threshold.

and Wang (2017). However, it is not clear how the presence of multiple stocks affects this magnet effect. Our following results suggest that, in the presence of circuit breakers on stock indices, the probability of falling to the index threshold compared to the case without circuit breakers is also increased, so the magnet effect found by Chen, Petukhov, and Wang (2017) is robust to a multiple-stock setting. In addition, the driving force of the magnet effect in our setting is different from that in Chen, Petukhov, and Wang (2017).

Figure 7 shows the probabilities of reaching the circuit breaker index threshold in a given time interval with circuit breakers on the index (red line), with separate circuit breakers on individual stocks (black dashed line), and without circuit breakers (blue line). It suggests that the probability of falling to the index threshold when there is a circuit breaker on the index is higher than that without any circuit breakers, which is in turn higher than that when circuit breakers are on individual stocks. This is because with circuit breakers on indices, when one stock goes down, the distance to the circuit breaker threshold is shorter and the likelihood of an early market closure is greater. As a result, other stock prices tend to go down, which in turn drags the index further downward, resulting in a downward accelerating vicious circle, contrary to regulators' intention. Because of the contagion effect across stocks, the magnet effect in a model with multiple stocks like ours can be stronger than that found in the single-stock setting of Chen, Petukhov, and Wang (2017), ceteris paribus. In addition, when the potential market closure duration is longer (e.g., at $t = 0$), this magnet effect is even stronger.

The main driving force for the magnet effect in Chen, Petukhov, and Wang (2017) is the fear that one has to liquidate a levered position at the market closure time because after market closure, leverage is prohibited by the solvency requirement. In contrast, in this paper there is no change in the leverage level allowed before and after a market closure. Figure 8 shows that when a separate circuit breaker is imposed on Stock 1, the probability of reaching the circuit breaker threshold is almost the same as that in the absence of a circuit breaker. Therefore, without the leverage effect and without the contagion effect of circuit breakers on the index, the magnet effect is almost zero. This suggests that different from Chen, Petukhov, and Wang (2017), the driving force behind the magnet effect in our setting is the contagion effect of circuit breakers, not the leverage constraint effect Chen, Petukhov, and Wang (2017).

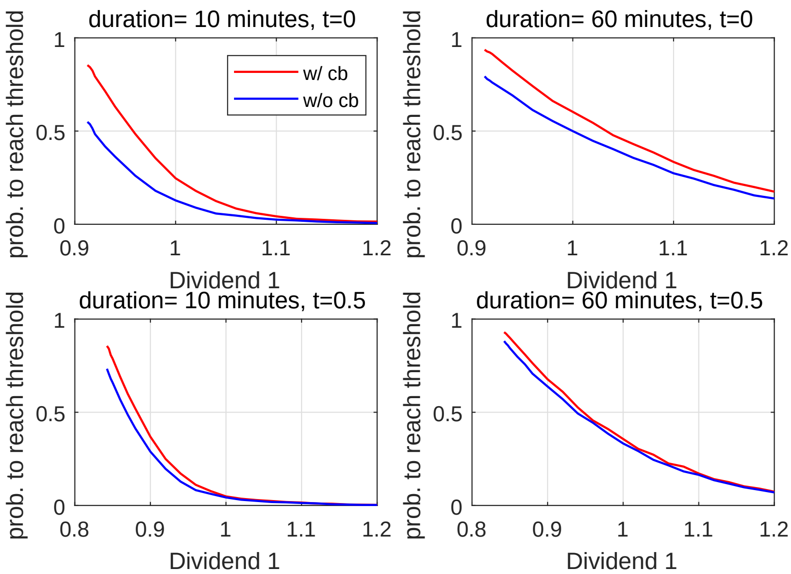

Figure 7. This figure shows the probability that prices will reach the threshold with or without a circuit breaker.

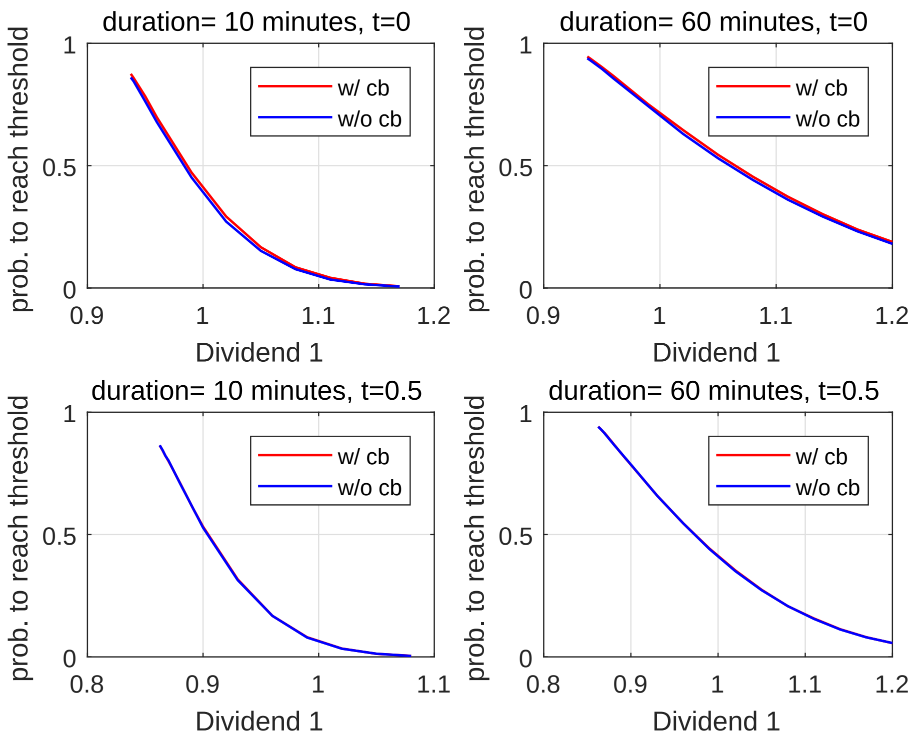

Figure 8. This figure shows the probability that prices will reach the threshold with or without a circuit breaker for Stock 1 with a separate circuit breaker.

6. Conclusion

Circuit breakers based on indices are commonly imposed in financial markets to prevent market crashes in bad times and reduce market volatility. We develop a continuous-time equilibrium model with multiple stocks to study how circuit breakers affect joint stock price dynamics and cross-stock contagion. Contrary to regulatory goals, we show that in bad times, circuit breakers can cause crash contagion, volatility contagion, greater volatilities, and high correlations among otherwise independent stocks. Our analysis suggests that international market plunges triggered by the COVID-19 pandemic may have been exacerbated by circuit breakers rules because of the contagion effect of these circuit breakers. Market-wide circuit breakers may be a source of financial contagion and a channel through which idiosyncratic risks become systemic risks, especially in bad times. An alternative circuit breaker approach based on individual stock returns instead of indices would alleviate such problems.

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Appendix

A Price of Stock 1: Without Circuit Breakers

We assume that the disagreement process $\delta_{1,t}$ is stochastic and follows Equation (6). When there are no circuit breakers, the equilibrium price of Stock 1 is independent of Stock 2 because of independent dividend processes. The price of Stock 1 can be obtained in closed-form as follows.

We first evaluate $\mathbb{E}_t^A[\pi_{1,T}^A]$. Ignoring constants, we need to calculate

$$\mathbb{E}_t^A[\eta_{1,T}^{1/2} e^{-\frac{\gamma}{2} D_{1,T}}] = \mathbb{E}_t^A[e^{Y_{1,T}}] \cdot f(t),$$

where $f(t)$ is a deterministic function and,

$$Y_{1,T} = \int_0^T (\frac{\delta_s}{2\sigma} - \frac{\gamma\sigma}{2}) dZ_s + \int_0^T (-\frac{\delta_s^2}{4\sigma^2}) ds.$$

To simplify notation, in the rest of Appendix A, we use δ_t, k, and $\bar{\delta}$ to denote $\delta_{1,t}$, k_1, and $\bar{\delta}_1$ respectively.

Conjecture $F(t, y, \delta, \delta^2) = e^{A(t)+B(t)y+C(t)\delta+\frac{H(t)}{2}\delta^2} = \mathbb{E}^A[e^{Y_T} | Y_t = y, \delta_t = \delta]$, with $A(T) = C(T) = H(T) = 0$ and $B(T) = 1$. Substituting the conjecture into the moment generating function of the process (Y_t, δ_t) and collecting the coefficients of y, δ, δ^2 and constants, we obtain four ordinary different equations:

$$\begin{aligned}
A'(t) + \frac{1}{8}\gamma^2\sigma^2 B(t)^2 + k\bar{\delta}C(t) + \frac{\nu^2}{2}(C(t)^2 + H(t)) - \frac{\gamma\sigma\nu}{2} B(t)C(t) &= 0, \\
B'(t) &= 0, \\
C'(t) - \frac{\gamma}{4} B(t)^2 + k\bar{\delta}H(t) - kC(t) + C(t)H(t)\nu^2 + \frac{\nu}{2\sigma} B(t)C(t) - \frac{\gamma\sigma\nu}{2} B(t)H(t) &= 0, \\
\frac{H'(t)}{2} - \frac{1}{4\sigma^2} B(t) + \frac{B(t)^2}{8\sigma^2} - kH(t) + \frac{\nu^2}{2} H(t)^2 + \frac{\nu B(t)H(t)}{2\sigma} &= 0.
\end{aligned}$$

The solution of the ODE system is obtained as follows.

$$
\begin{aligned}
B(t) &= 1, \\
H(t) &= \frac{e^{(D^+-D^-)v^2(t-T)}-1}{e^{(D^+-D^-)v^2(t-T)}D^- - D^+} D^+D^-, \\
C(t) &= \int_t^T e^{\int_t^s f(x)ds} g(s)ds = \frac{1}{\Delta(D^- - D^+e^{2\Delta(T-t)})} \\
&\cdot \Big(-\frac{\gamma}{4}((D^+ + D^-)e^{\Delta(T-t)} - D^+e^{2\Delta(T-t)} - D^-) - (k\bar{\delta} - \frac{\sigma\nu\gamma}{2})D^+D^-(e^{\Delta(T-t)}-1)^2 \Big), \\
A(t) &= \int_T^t (-\frac{1}{8}\gamma^2\sigma^2 - k\bar{\delta}C(s) - \frac{v^2}{2}(C(s)^2 + H(s)) + \frac{\gamma}{2}v\sigma C(s))ds,
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta =& \sqrt{k^2 + \frac{v^2}{2\sigma^2} - \frac{vk}{\sigma}}, \\
D^{\pm} =& \frac{k - \frac{v}{2\sigma} \pm \sqrt{k^2 + \frac{v^2}{2\sigma^2} - \frac{vk}{\sigma}}}{v^2}, \\
f(t) =& -k + v^2H(t) + \frac{v}{2\sigma}, \\
g(t) =& -\frac{\gamma}{4} + k\bar{\delta}H(t) - \frac{\gamma\sigma v}{2}H(t).
\end{aligned}
$$

Then

$$
\mathbb{E}_t^A[e^{Y_T}] = F(t, y, \delta, \delta^2; \gamma) = e^{A(t)+B(t)y+C(t)\delta+\frac{H(t)}{2}\delta^2}.
$$

Next, we consider the first derivative of F with respect to γ to obtain $\mathbb{E}_t^A[e^{Y_T}Z_T]$. We define

$$
A(t;\gamma) = A(t), C(t;\gamma) = C(t).
$$

Note that

$$
\begin{aligned}
\frac{dB(t)}{d\gamma} =& \frac{dH(t)}{d\gamma} = 0, \\
\frac{dC(t;\gamma)}{d\gamma} =& \int_t^T e^{\int_t^s f(x)dx}[-\frac{1}{4} - \frac{\sigma v}{2}H(s)]ds, \\
\frac{dA(t;\gamma)}{d\gamma} =& \int_T^t (\frac{-\sigma^2\gamma}{4} - k\bar{\delta}\frac{dC(s;\gamma)}{d\gamma} - v^2C(s;\gamma)\frac{dC(s;\gamma)}{d\gamma} + \frac{v\sigma}{2}C(s;\gamma) + \frac{\gamma v\sigma}{2}\frac{dC(s;\gamma)}{d\gamma})ds.
\end{aligned}
$$

Hence

$$
\mathbb{E}_t^A[e^{Y_T}Z_T] = -\frac{2}{\sigma}\frac{d}{d\gamma}\mathbb{E}_t^A[e^{Y_T}] = -\frac{2}{\sigma}\frac{d}{d\gamma}F(t, y, \delta, \delta^2; \gamma).
$$

Finally, the stock price in the equilibrium is given by

$$\begin{aligned}\hat{S}_{1,t} &= \frac{\mathbb{E}_t^A[\pi_{1,T}^A D_{1,T}]}{\mathbb{E}_t^A[\pi_{1,T}^A]} = \frac{\mathbb{E}_t^A[\pi_{1,T}^A D_{1,T}]}{F} = D_{1,0} + \mu_1^A T - 2\frac{\frac{dF}{d\gamma}}{F} \\ &= D_0 + \mu^A T - 2(\frac{dA(t;\gamma)}{d\gamma} + \frac{dy}{d\gamma} + \frac{dC(t;\gamma)}{d\gamma}\delta_t) \\ &= D_{1,0} + \mu_1^A T - 2(\frac{dA(t;\gamma)}{d\gamma} - \frac{\sigma}{2}Z_t + \frac{dC(t;\gamma)}{d\gamma}\delta_t).\end{aligned}$$

The last equality above holds because $Y_t = \int_0^t (\frac{\delta_s}{2\sigma} - \frac{\gamma\sigma}{2})dZ_s + \int_0^t (-\frac{\delta_s^2}{4\sigma^2})ds$ and $Y_t = y$ yield $dy/d\gamma = -1/2\sigma Z_t$. By $D_{1,t} = D_{1,0} + \mu_1^A t + \sigma Z_t$ (μ_1^A is constant), we obtain

$$\hat{S}_{1,t} = D_{1,t} + \mu_1^A (T-t) - 2(\frac{dA(t;\gamma)}{d\gamma} + \frac{dC(t;\gamma)}{d\gamma}\delta_t). \tag{A.1}$$

In case δ_t is constant, i.e., $v = k = 0$ and $\delta_t \equiv \delta_0$, we find that $dA(t)/d\gamma = -\sigma^2\gamma/4(t-T)$ and $dC(t;\gamma)/d\gamma = -1/4(T-t)$. Thus, $\hat{S}_{1,t} = D_{1,t} + \mu_1^A(T-t) + (\delta_0/2 - \sigma^2\gamma/2)(T-t)$. This is the equilibrium price of Stock 1 in the case of constant disagreement.

Since $H(t) \to 0$ as $t \to T$, we see that $dC(t;\gamma)/d\gamma$ is negative when $T-t$ is small. Thus, it follows (A.1) that the instantaneous volatility of the stock price $\sigma_{\hat{S}} = \sigma - 2\frac{dC(t)}{d\gamma}\nu$ is greater than the dividend volatility σ when $T-t$ is small, given ν is positive.

B Market Clearing Prices

B.1 Stock 1: Stochastic Disagreement

In the presence of circuit breakers, we cannot obtain the equilibrium price of Stock 1 directly. In this section, we derive the market clearing price of Stock 1 when a circuit breaker is triggered and the market is closed early. Because the two dividend processes are independent and we assume no leverage constraints when the market is halted, the market clearing prices for the two stocks are independent of each other.

The disagreement $\delta_{1,t}$ is stochastic following (6), therefore $\mu_{1,t}^B = \delta_{1,t} + \mu_1^A$ is stochastic as well. In the presence of a circuit breaker, we solve for the market clearing price when the market is halted. To do so, we solve the utility maximization problem

$$\max_{\theta_{1,\tau}^A} \mathbb{E}_\tau^A[-e^{-\gamma W_T^A}],$$

subject to $W_T^A = \theta_{1,\tau}^A(D_{1,T} - S_{1,\tau}) + W_\tau^A$, where W_t^A is the wealth of Type A investors at time t.

Using the dynamics $D_{1,T} = D_{1,\tau} + \mu_1^A(T-\tau) + \sigma(Z_T - Z_\tau)$, we obtain the optimal portfolio of agent A as follows.

$$\theta_{1,\tau}^A = \frac{D_{1,\tau} - S_{2,\tau} + \mu_1^A(T-\tau)}{\gamma\sigma^2(T-\tau)}. \tag{B.1}$$

Next, we study the utility maximization problem of agent B:

$$\max_{\theta_{1,\tau}^B} \mathbb{E}_\tau^B[-e^{-\gamma(W_\tau^B + \theta_{1,\tau}^B(D_{1,T} - S_{1,\tau}))}].$$

We first prove the following lemma.

Lemma B1. *Suppose θ is a constant, then*

$$\mathbb{E}_t^B[e^{-\gamma\theta D_{1,T}}] = e^{A(t,\theta)+B(t,\theta)D_{1,t}+C(t,\theta)\delta_{1,t}},$$

where

$$\begin{aligned}
A(t,\theta) &= \gamma\theta\mu_1^A(t-T) - \frac{\sigma^2}{2}\gamma^2\theta^2(t-T) + \frac{1}{\tilde{k}_1}(-\gamma\theta k_1\bar{\delta}_1 + \nu\sigma\gamma^2\theta^2)(T - t - \frac{1-e^{\tilde{k}_1(t-T)}}{\tilde{k}_1}) \\
&\quad + \frac{\nu^2\gamma^2\theta^2}{2\tilde{k}_1^2}(T - t - 2\frac{1-e^{\tilde{k}_1(t-T)}}{\tilde{k}_1} + \frac{1-e^{2\tilde{k}_1(t-T)}}{2\tilde{k}_1}), \\
B(t,\theta) &= -\gamma\theta, \\
C(t,\theta) &= \frac{-\gamma\theta}{\tilde{k}_1}(1-e^{\tilde{k}_1(t-T)}),
\end{aligned}$$

with $\tilde{k}_1 = k_1 - \nu/\sigma$. In particular, if $\tilde{k}_1 = 0$, then

$$\begin{aligned}
A(t,\theta) &= \gamma\theta\mu_1^A(t-T) - \frac{\sigma^2}{2}\gamma^2\theta^2(t-T) + \frac{1}{2}(-\gamma\theta k_1\bar{\delta}_1 + \gamma^2\theta^2\nu\sigma)(t-T)^2 - \frac{\nu^2\gamma^2\theta^2}{6}(t-T)^3, \\
B(t,\theta) &= -\gamma\theta, \\
C(t,\theta) &= \gamma\theta(t-T).
\end{aligned}$$

Lemma B1 can be proved by using the moment generating function of process $D_{1,t}$ and δ_t and solving an ODE system. Detailed deviations are omitted here.

By the lemma,

$$\mathbb{E}_\tau^B[-e^{-\gamma(W_\tau^B+\theta_{1,\tau}^B(D_{1,T}-S_{1,\tau}))}] = -e^{-\gamma W_\tau^B} e^{A(t,\theta_{1,\tau}^B)+C(t,\theta_{1,\tau}^B)\delta_{1,\tau}} e^{-\gamma\theta_{1,\tau}^B(D_{1,\tau}-S_{1,\tau})}.$$

Then the FOC with respect to $\theta_{1,\tau}^B$ yields that

$$\gamma S_{1,\tau} - \gamma D_{1,\tau} + \frac{\partial A(\tau,\theta_{1,\tau}^B)}{\partial\theta_{1,\tau}^B} + \frac{\partial C(\tau,\theta_{1,\tau}^B)}{\partial\theta_{1,\tau}^B}\delta_{1,\tau} = 0$$

or

$$S_{1,\tau} - D_{1,\tau} + \mu_1^A(\tau - T) - \frac{1}{\tilde{k}_1}(1 - e^{\tilde{k}_1(\tau-T)})\delta_{1,\tau} - \bar{\delta}_1\frac{k_1}{\tilde{k}_1}(T - \tau - \frac{1-e^{\tilde{k}_1(\tau-T)}}{\tilde{k}_1}) + \theta_{1,\tau}^B I(\tau) = 0, \tag{B.2}$$

where

$$I(t) = -\gamma\sigma^2(t-T) + \frac{2\nu\sigma\gamma}{\tilde{k}_1}(T-t-\frac{1-e^{\tilde{k}_1(t-T)}}{\tilde{k}_1}) + \frac{\nu^2\gamma}{\tilde{k}_1^2}(T-t-2\frac{1-e^{\tilde{k}_1(t-T)}}{\tilde{k}_1} + \frac{1-e^{2\tilde{k}_1(t-T)}}{2\tilde{k}_1}).$$

It follows (B.1) that

$$S_{1,\tau} = D_{1,\tau} + \mu_1^A(T-\tau) - \theta_{1,\tau}^A\gamma\sigma^2(T-\tau). \tag{B.3}$$

Together with (B.2) and the market clearing condition $\theta_{1,\tau}^A + \theta_{1,\tau}^B = 1$, we obtain the optimal share holding of Type A for Stock 1 at the time of market closure.

$$\theta_{1,\tau}^{A,*} = \frac{-\frac{1}{\tilde{k}_1}(1-e^{\tilde{k}_1(\tau-T)})\delta_{1,\tau} - \bar{\delta}_1\frac{k_1}{\tilde{k}_1}(T-t-\frac{1-e^{\tilde{k}_1(\tau-T)}}{\tilde{k}_1}) + I(\tau)}{I(\tau)+\gamma\sigma^2(T-t)}. \tag{B.4}$$

Therefore, we find the market clearing price $S_{1,\tau}$ by (B.3) where $\theta_{1,\tau}^A = \theta_{1,\tau}^{A,*}$ given by (B.4).

In particular, in the case $\tilde{k}_1 = 0$ (or $k_1 = \nu/\sigma$),

$$\theta_{1,\tau}^{A,*} = \frac{1}{\gamma}\left(\frac{\gamma\sigma^2 - \gamma\nu\sigma(\tau-T) + \frac{1}{2}k_1\bar{\delta}_1(\tau-T) + \frac{\nu^2\gamma}{3}(\tau-T)^2 - \delta_{1,\tau}}{-\nu\sigma(\tau-T) + \frac{\nu^2}{3}(\tau-T)^2 + 2\sigma^2}\right),$$

and substituting it into (B.1), it follows that

$$S_{1,\tau} = D_{1,\tau} + \mu_1^A(T-\tau) \\ + \frac{\gamma\sigma^2 - \gamma\nu\sigma(\tau-T) + \frac{1}{2}k\bar{\delta}(\tau-T) + \frac{\nu^2\gamma}{3}(\tau-T)^2 - \delta_{1,\tau}}{-\nu\sigma(\tau-T) + \frac{\nu^2}{3}(\tau-T)^2 + 2\sigma^2}\sigma^2(\tau-T).$$

Finally, it is worthy mentioning that $S_{1,\tau}$ may not be larger than $\hat{S}_{1,\tau}$ (the equilibrium price in the absence of circuit breakers at time τ). In fact, for a relative large positive δ_0 and small ν (say, less than half of the volatility σ), the coefficient of δ_t in (B.3) can always be less than the coefficient of δ_t in the formula of $\hat{S}_{1,\tau}$. Thus, along with a small γ, we can always have $S_{1,\tau} < \hat{S}_{1,\tau}$. Under these conditions, the market clearing price with circuit breakers can always be smaller than the price without circuit breakers at time τ.

Denote the market clearing price of Stock 1 by $S^c_{1,\tau}$. Then by (B.3),

$$S^c_{1,\tau} = D_{1,\tau} + \mu_1^A(T-\tau) - \gamma\theta^{A,*}_{1,\tau}\sigma^2(T-\tau).$$

In addition, we obtain the value function of Type B investors:

$$V_1^B(\tau, W_\tau^B) = \max_{\theta^B_{1,\tau}} \mathbb{E}^B_\tau[e^{-\gamma(W^B_\tau + \theta^B_{1,\tau}(D_{1,T}-S_{1,\tau}))}] = e^{-\gamma W^B_\tau} e^{-\gamma G^B_{1,\tau}},$$

where $-\gamma G^B_{1,\tau} = -\gamma\theta^B_{1,\tau}(D_{1,\tau} - S_{2,\tau}) + A(\tau, \theta^{B,*}_{1,\tau}) + C(\tau, \theta^{B,*}_{1,\tau})\delta_{1,\tau}$, or

$$G^B_{1,\tau} = \theta^{B,*}_{1,\tau}(D_{1,\tau} - S_{2,\tau}) - \frac{1}{\gamma}A(\tau, \theta^{B,*}_{1,\tau}) - \frac{1}{\gamma}C(\tau, \theta^{B,*}_{1,\tau})\delta_{1,\tau}, \tag{B.5}$$

and the value function of Type A investors:

$$V_1^A(\tau, W_\tau^A) = \max_{\theta^A_{1,\tau}} \mathbb{E}^A_\tau[e^{-\gamma(W^A_\tau + \theta^A_{1\tau}(D_{1,T}-S_{1,\tau}))}] = e^{-\gamma W^A_\tau} e^{-\gamma G^A_{1,\tau}},$$

where $-\gamma G^A_{1,\tau} = -\gamma\theta^{A,*}_{1,\tau}(D_{1,\tau} - S_{1,\tau}) - \gamma\theta^{A,*}_{1,\tau}\mu_1^A(T-\tau) + \frac{\gamma^2(\theta^{A,*}_{1,\tau})^2}{2}\sigma^2(T-\tau) = -\frac{\gamma^2(\theta^{A,*}_{1,\tau})^2}{2}\sigma^2(T-\tau)$, or

$$G^A_{1,\tau} = \theta^{A,*}_{1,\tau}(D_{1,\tau} - S_{1,\tau}) + \theta^{A,*}_{1,\tau}\mu_1^A(T-\tau) - \frac{\gamma(\theta^{A,*}_{1,\tau})^2}{2}\sigma^2(T-\tau) = \frac{\gamma(\theta^{A,*}_{1,\tau})^2}{2}\sigma^2(T-\tau). \tag{B.6}$$

B.2 Stock 2: Stochastic Disagreement on Jump Intensity

Note that for Stock 2 there is disagreement on the jump intensity of dividend process $D_{2,t}$, which follows

$$dD_{2,t} = (\mu_2^i - \lambda_t^i \mu_J)dt + \mu_J dN_{2,t}^i, \ i = A, B. \tag{B.7}$$

In this Appendix, we derive the market clearing price $S_{2,\tau}^c$ of Stock 2 when a circuit breaker is triggered.

Recall that $\delta_{2,t} = \kappa_t^B/\kappa^A$ satisfies a mean-reverting process as follows.

$$d\delta_{2,t} = -k_2(\delta_{2,t} - \bar{\delta}_2)dt + \mu_\delta dN_t. \tag{B.8}$$

Suppose that the circuit breaker is triggered at $\tau < T$. The individual optimization problem of Type $i \in \{A, B\}$ investors at τ is:

$$V_2^i(W_\tau^i, \tau) = \max_{\theta_{2,\tau}^i} \mathbb{E}_\tau^i[-\exp(-\gamma(W_\tau^i + \theta_{2,\tau}^i(D_{2,T} - S_{2,\tau})))], \tag{B.9}$$

subject to the market clearing condition $\theta_{2,\tau}^A + \theta_{2,\tau}^B = 1$, where W_τ^i is the wealth owned by Type i investors at time τ. Note that

$$\mathbb{E}_\tau^i[u(W_\tau^i + \theta_{j,\tau}^i(D_{2,T} - S_{2,\tau}))] = -e^{-\gamma W_\tau^i} e^{\gamma\theta_{2,\tau}^i S_{2,\tau}} \mathbb{E}_\tau^i[e^{-\gamma\theta_{2,\tau}^i D_{2,T}}].$$

For Type A agents we have

$$\mathbb{E}_\tau^i[e^{-\gamma\theta_{2,\tau}^i D_{2,T}}] = \exp\{-\gamma\theta_{2,\tau}^A D_{2,\tau} + (\tau - T)((\mu_2^A - \kappa^A\mu_J)\gamma\theta_{2,\tau}^A - \kappa^A(e^{-\gamma\theta_{2,\tau}^A\mu_J} - 1))\}; \tag{B.10}$$

and for Type B agents we have

$$\mathbb{E}_\tau^B[e^{-\gamma\theta_{2,\tau}^i D_{2,T}}] = \exp\{-\gamma\theta_{2,\tau}^B D_{2,\tau} + \vartheta_\tau g(\tau; -\gamma\theta_{2,\tau}^B) + \int_\tau^T (-\gamma\theta_{2,\tau}(\mu_2^A - \kappa^A\mu_J)) + k_2\bar{\delta}g(s; -\gamma\theta_{2,\tau}^B)ds\}$$
$$:= exp(-\gamma M_{2,\tau}^B), \tag{B.11}$$

where function $g(t; \alpha)$ satisfies

$$g'(t; \alpha) + \kappa^A(e^{\alpha\mu_J + \mu_\delta g(t)} - 1) - k_2 g(t; \alpha) = 0, \tag{B.12}$$

with $g(T) = 0$.
To solve the optimization problem (B.9), we find the first order conditions with respect to $\theta^i_{2,\tau}$ for $j \in \{1, 2\}, i \in \{A, B\}$ as follows.

$$D_{2,\tau} - S_{2,\tau} + (T - \tau)(\mu^A_2 - \kappa^A \mu_J + \kappa^A \mu_J e^{-\gamma\theta^A_{2,\tau}\mu_J}) = 0, \tag{B.13}$$

$$D_{2,\tau} - S_{2,\tau} + \delta_2 \frac{\partial g(\tau;\alpha)}{\partial \alpha}|_{\alpha=-\gamma\theta^B_{2,\tau}} + \int_\tau^T (\mu^A_2 - \kappa^A \mu_J) + k_2 \bar{\delta} \frac{\partial g(s;\alpha)}{\partial \alpha}|_{\alpha=-\gamma\theta^B_{2,\tau}})ds = 0, \tag{B.14}$$

where $g_\alpha(t) := \frac{\partial g(t;\alpha)}{\partial \alpha}$ satisfies an ODE as follows.

$$g'_\alpha(t) + \kappa^A e^{\alpha\mu_J + \mu_\delta g(t)}(\mu_J + \mu_\delta g_\alpha(t)) - k_2 g_\alpha(t) = 0, \tag{B.15}$$

with $g_\alpha(T) = 0$.

Along with the market clearing condition: $\theta^A_{2,\tau} + \theta^B_{2,\tau} = 1$, we can solve the optimal share holdings $\theta^A_{2,\tau} = \theta^{A,*}_{2,\tau}$, $\theta^B_{2,\tau} = \theta^{B,*}_{2,\tau}$ and the market clearing price $S_{2,\tau} = S^c_{2,\tau}$ from (B.13) and (B.14). No explict solutions like for Stock 1, we rely on numerical solutions in practice.

By (B.13), the market clearing price of Stock 2 can be expressed by $\theta^{A,*}_{2,\tau}$:

$$S^c_{2,\tau} = D_{2,\tau} + (\mu^A_2 - \kappa^A \mu_J)(T - \tau) + \kappa^A \mu_J (T - \tau) e^{-\gamma\theta^{A,*}_{2,\tau}\mu_J}.$$

Define

$$G^A_{2,\tau} = \theta^A_{2,\tau}\mu_2(T - \tau) + \theta^i_{2,\tau}(D_{2,\tau} - S^c_{2,\tau}) - \frac{\kappa^A}{\gamma}(T - \tau)(e^{-\gamma\theta^{A,*}_{2,\tau}\mu_J} - 1), \tag{B.16}$$

$$G^B_{2,\tau} = M^B_{2,\tau} - \theta^{B,*}_{2,\tau} S^c_{2,\tau},$$

where $M^B_{2,\tau}$ is defined in (B.11). Then by (B.10) and (B.11), the value function of Type i investors at τ can be expressed in terms of $W^i_{2,\tau}$ and $G^i_{2,\tau}$ as follows.

$$V^i_2(W^i_\tau, \tau) = -e^{-\gamma W^i_\tau} e^{-\gamma G^i_{2,\tau}}, i \in \{A, B\}.$$

By the expressions of $G^A_{2,\tau}$ and $G^B_{2,\tau}$, it is useful to notice that $S^c_{2,\tau} + G^A_{2,\tau} + G^B_{2,\tau}$ does not depend on $S^c_{2,\tau}$ directly. The quantity depends on the optimal share holdings at τ: $\theta^{A,*}_{2,\tau}$ and $\theta^{B,*}_{2,\tau}$.

C Learning and Heterogeneous Beliefs

Suppose

$$dD_t = \mu_t dt + \sigma d\bar{Z}_t.$$

The dividend D_t is observable but the growth rate μ_t is not. Agents A and B infer the value of μ_t through the information from the dividend. Assume that

$$d\mu_t = -k(\mu_t - \bar{\mu})dt + \sigma_\mu d\bar{Z}_t,$$

and $\mu_0 \sim N(a_0, b_0)$, a normal distribution with mean a_0 and standard deviation b_0. Agent $i \in \{A, B\}$ believes $k = k^i, \bar{\mu} = \bar{\mu}^i, \sigma_\mu = \sigma_\mu^i, a_0 = a_0^i, b_0 = b_0^i$. Both of them learn μ_t through $\{D_s\}_{s=0}^t$. Let $\mu_t^A = \mathbb{E}^A[\mu_t|\{D_s\}_{s=0}^t]$ and $\mu_t^B = \mathbb{E}^B[\mu_t|\{D_s\}_{s=0}^t]$. Then following the standard filtering results, we have (under the assumption: $\mu_t|\{D_s\}_{s=0}^t \sim N(\hat{\mu}, \nu)$)

$$\begin{aligned} d\mu_t^A &= -k^A(\mu_t^A - \bar{\mu}^A)dt + \nu^A dZ_t^A, \\ d\mu_t^B &= -k^B(\mu_t^B - \bar{\mu}^B)dt + \nu^B dZ_t^B, \end{aligned}$$

where $dZ_t^i = \frac{1}{\sigma}(dD_t - \mu_t^i dt), i = A, B$. Then

$$dD_t = \mu_t^A dt + \sigma dZ_t^A, \qquad dD_t = \mu_t^B dt + \sigma dZ_t^B.$$

Therefore, $Z_t^B + \frac{\delta_t}{\sigma_t} t$ is equal to Z_t^A almost surely, where $\delta_t = \mu_t^B - \mu_t^A$. In other words, $Z_t^B + \frac{\delta_t}{\sigma_t} t$ is a standard Brownian motion under agent A's probability measure $\mathbf{P}^A$.

Thus,

$$d\mu_t^B = -k^B(\mu_t^B - \bar{\mu}^B)dt - \frac{\nu^B}{\sigma}\delta_t dt + \nu^B dZ_t^A.$$

So we can obtain the general dynamics of the stochastic disagreement δ_t under learning. To validate the setting adopted in this paper, we let $\nu^A = 0$, $k^A = 0$, and $\mu_t^A = \mu^A$. That is, we assume that Type A investors take the long-time mean of the growth rate as the estimation and impose no learning. Then it follows that

$$\begin{aligned} d\delta_t = d(\mu_t^B - \mu^A) &= -(k^B + \frac{\nu^B}{\sigma})\delta_t dt - k^B(\mu^A - \bar{\mu}^B)dt + \nu^B dZ_t^A \\ &= -k^B\delta_t dt + k^B(\bar{\mu}^B - \mu^A)dt + \nu^B dZ_t^B. \end{aligned}$$

Further, let $k^B + \nu^B/\sigma = k$, $\nu^B = \nu$ and $(\bar{\mu}^B - \mu^A)/k = \bar{\delta}/\kappa^B$; we have reached the mean-reverting disagreement process assumed in the paper.

D The Fixed Point Problem

We prove the existence and uniqueness of a solution to the fixed point problem. First of all, based on the explicit expressions of the prices, we restrict the model parameters and the initial conditions (e.g., $D_{1,0}, D_{2,0}$) and assume that both $\hat{S}_{j,0}$ (the price without circuit breakers) and $S^c_{j,0}$ (the market clearing price) are positive for each $j = 1, 2$.

Recall that $S_{1,0}, S_{2,0}$ impact valuation of the expectations through the sum $S_{1,0} + S_{2,0}$ only. When the initial stock prices are $S_{1,0}$ and $S_{2,0}$, the threshold h is $(S_{1,0}+S_{2,0})(1-\alpha)$. So, we define

$$f_j(S_{1,0} + S_{2,0}) = \frac{\mathbb{E}^A_0[\pi^A_{T\wedge\tau} S_{j,T\wedge\tau}]}{\pi^A_0}, j = 1, 2.$$

and define a function $f : \mathcal{R} \to \mathcal{R}^2$ such that $f(S_{1,0} + S_{2,0}) = (f_1(S_{1,0} + S_{2,0}), f_2(S_{1,0} + S_{2,0}))^\top$, where $\top$ denotes the transpose of a vector. Then the fixed point problem is expressed as follows.

$$(S_{1,0}, S_{2,0})^\top = f(S_{1,0} + S_{2,0}).$$

Define $g(x) = f_1(x) + f_2(x) - x$, where $x \in \mathcal{R}$. When the threshold is zero, the circuit breaker is hardly triggered. Thus the equilibrium prices are close to the prices $\hat{S}_{0,1}$ and $\hat{S}_{2,0}$ respectively in the absence of circuit breakers. Given positive $\hat{S}_{0,1}$ and $\hat{S}_{2,0}$, we can obtain (specifically, for a sufficiently small volatility of $D_{1,t}$ and jump intensity of $D_{2,t}$): $g(0) = f_1(0) + f_2(0) > 0$. On the other hand, if the threshold is the sum of the market clearing prices $S^c_{1,0} + S^c_{2,0}$, the market is stopped immediately and the equilibrium prices must be the market clearing prices exactly. Thus, $g(\frac{S^c_{1,0}+S^c_{2,0}}{1-\alpha}) = f_1(\frac{S^c_{1,0}+S^c_{2,0}}{1-\alpha}) + f_2(\frac{S^c_{1,0}+S^c_{2,0}}{1-\alpha}) - \frac{S^c_{1,0}+S^c_{2,0}}{1-\alpha} = S^c_{1,0} + S^c_{2,0} - \frac{S^c_{1,0}+S^c_{2,0}}{1-\alpha} < 0$. It can be shown that $g(x)$ is a continuous function. Hence, there exists $x^* \in (0, \frac{S^c_{1,0}+S^c_{2,0}}{1-\alpha})$, such that $g(x^*) = 0$. Thus $f_1(x^*) + f_2(x^*) = x^*$.

Now define $(S^*_{1,0}, S^*_{2,0})^\top = f(x^*)$. Then $x^* = f_1(x^*) + f_2(x^*) = S^*_{1,0} + S^*_{2,0}$ and

$$(S^*_{1,0}, S^*_{2,0})^\top = f(x^*) = f(S^*_{1,0} + S^*_{2,0}).$$

Thus $(S^*_{1,0}, S^*_{2,0})^\top \in \mathcal{R}^2$ is a solution to the fixed problem. The existence is proved.

Next, we show that the solution is unique. To do so, it is sufficient to show that $g(x)$ is monotonic. For the sake of notional simplicity, we ignore super-script "A" of expectations and π^A_t below.

Let $D_0 = D_{1,0} + D_{2,0}$. Given an exogenous threshold h and initial dividend sum value D_0, let $S^{h,D_0}_t = S^c_{1,t} + S^c_{2,t}$, where $S^c_{1,t}$ and $S^c_{2,t}$ are the market clearing prices; let $\tau(h, D_0)$

denote the stopping time; and let π_t^{h,D_0} be the state price density, i.e.

$$\pi_t^{h,D_0} = (\eta_t)^{1/2} e^{-\frac{\gamma}{2} S_t^{h,D_0}} \cdot e^{\frac{G_t^A + G_t^B}{2}}.$$

We redefine

$$g(x) = g(x; D_0) = \frac{\mathbb{E}[\pi_{\tau(h,D_0)\wedge T} S^{h,D_0}_{\tau(h,D_0)\wedge T}]}{\mathbb{E}[\pi^A_{\tau(h,D_0)\wedge T}]} - x,$$

where $h = x(1-\alpha)$. Observe that $\tau(h, D_0) = \tau(0, D_0 - h)$ because the stopping time is determined by D_t and δ_t only. Then the market clearing (sum) price $S^{h,D_0}_{\tau(h,D_0)} = S^{0,D_0-h}_{\tau(0,D_0-h)} + h$ by the expressions of $S^c_{j,\tau}$, $j = 1, 2$. In addition, by the definition of G^i_τ, we see that $G^i_{\tau(h,D_0)} = G^i_{\tau(0,D_0-h)}$, $i = A, B$. Therefore

$$\pi^{h,D_0}_{\tau(h,D_0)} = e^{-\frac{\gamma}{2}h} \cdot \pi^{0,D_0-h}_{\tau(0,D_0-h)}. \tag{D.1}$$

Thus,

$$\begin{aligned} g(x; D_0) &= \frac{\mathbb{E}[\pi^{h,D_0}_{\tau(h,D_0)\wedge T} \cdot (S^{h,D_0}_{\tau(h,D_0)\wedge T} - x)]}{\mathbb{E}[\pi^{h,D_0}_{\tau(h,D_0)\wedge T}]} \\ &= \frac{\mathbb{E}[\pi^{0,D_0-h}_{\tau(0,D_0-h)\wedge T} \cdot (S^{0,D_0-h}_{\tau(0,D_0-h)\wedge T} - x + h)]}{\mathbb{E}[\pi^{0,D_0-h}_{\tau(0,D_0-h)\wedge T}]} \\ &= g(0; D_0 - h) - x + h = g(0; D_0 - h) - \alpha x. \end{aligned}$$

Given $h_1 < h_2$, we have $\tau(0, D_0 - h_1) \geq \tau(0, D_0 - h_2)$. Then,

$$\begin{aligned} &\mathbb{E}[\pi^{0,D_0-h_1}_{\tau(0,D_0-h_1)}] = \mathbb{E}[\mathbb{E}[\pi^{0,D_0-h_1}_{\tau(0,D_0-h_1)} | \tau(0, D_0 - h_2)]] = \mathbb{E}[\pi^{0,D_0-h_1}_{\tau(0,D_0-h_2)}] \\ &= \mathbb{E}[(\eta_{\tau(0,D_0-h_2)\wedge T})^{1/2} e^{-\frac{\gamma}{2} S^{0,D_0-h_1}_{\tau(0,D_0-h_2)\wedge T}} \cdot e^{\frac{G^A_{\tau(0,D_0-h_2)\wedge T} + G^B_{\tau(0,D_0-h_2)\wedge T}}{2}}] \\ &= \mathbb{E}[(\eta_{\tau(0,D_0-h_2)\wedge T})^{1/2} e^{-\frac{\gamma}{2} S^{0,D_0-h_2}_{\tau(0,D_0-h_2)\wedge T}} \cdot e^{\frac{G^A_{\tau(0,D_0-h_2)\wedge T} + G^B_{\tau(0,D_0-h_2)\wedge T}}{2}} \cdot e^{-\gamma/2(h_2-h_1)}] \\ &= \mathbb{E}[\pi^{0,D_0-h_2}_{0,D_0-h_2}] e^{-\gamma/2(h_2-h_1)}. \end{aligned}$$

Similarly,

$$
\begin{aligned}
\mathbb{E}[\pi^{0,D_0-h_1}_{\tau(0,D_0-h_1)} \cdot S^{0,D_0-h_1}_{\tau(0,D_0-h_1)}] &= \mathbb{E}[\mathbb{E}[\pi^{0,D_0-h_1}_{\tau(0,D_0-h_1)} \cdot S^{0,D_0-h_1}_{\tau(0,D_0-h_1)} | \tau(0, D_0 - h_2)]] \\
&= \mathbb{E}[\pi^{0,D_0-h_1}_{\tau(0,D_0-h_2)} \cdot S^{0,D_0-h_1}_{\tau(0,D_0-h_2)}] e^{-\gamma/2(h_2-h_1)} \\
&\geq \mathbb{E}[\pi^{0,D_0-h_2}_{\tau(0,D_0-h_2)} \cdot S^{0,D_0-h_2}_{\tau(0,D_0-h_2)}] e^{-\gamma/2(h_2-h_1)}.
\end{aligned}
$$

Finally, let $x_1 < x_2$ and $h_1 = x_1(1-\alpha), h_2 = x_2(1-\alpha)$. It follows that

$$
\begin{aligned}
g(x_1; D_0) = g(0; D_0 - x_1) - \alpha x_1 &= \frac{\mathbb{E}[\pi^{0,D_0-h_1}_{\tau(0,D_0-h_1)\wedge T} \cdot S^{0,D_0-h_1}_{\tau(0,D_0-h_1)\wedge T}]}{\mathbb{E}[\pi^{0,D_0-h_1}_{\tau(0,D_0-h_1)\wedge T}]} - \alpha x_1 \\
&\geq \frac{\mathbb{E}[\pi^{0,D_0-h_2}_{\tau(0,D_0-h_2)\wedge T} \cdot S^{0,D_0-h_2}_{\tau(0,D_0-h_2)\wedge T}]}{\mathbb{E}[\pi^{0,D_0-h_2}_{\tau(0,D_0-h_2)\wedge T}]} - \alpha x_1 \\
&= g(0; D_0 - h_2) = g(x_2; D_0) + \alpha x_2 - \alpha x_1 > g(x_2; D_0).
\end{aligned}
$$

Thus, $g(\cdot, D_0)$ is monotonic. This completes the proof of uniqueness.

E The Case of Correlated Dividend Processes

To impose a correlation between dividend processes, we assume that: under $\mathbf{P}^A$,

$$dD_{1,t} = \mu_1^A dt + \sigma_1 dZ_t, \tag{E.1}$$

$$dD_{2,t} = \mu_2 dt + \sigma_2 dZ_t + \mu_J dN_t, \tag{E.2}$$

and under $\mathbf{P}^B$:

$$dD_{1,t} = \mu_1^B dt + \sigma_1 dZ_t^B, \tag{E.3}$$

$$dD_{2,t} = \mu_2 dt + \frac{\sigma_2}{\sigma_1}\delta_t dt + \sigma_2 dZ_t^B + \mu_J dN_t, \tag{E.4}$$

where $\mu_1^B = \mu_1^A + \delta_t$ and

$$d\delta_t = -k(\delta_t - \bar{\delta})dt + \nu dZ_t,$$

or

$$d\delta_t = -k(\delta_t - \bar{\delta})dt + \frac{\nu}{\sigma_1}\delta_t dt + \nu dZ_t^B.$$

Then the two dividend processes are correlated with instantaneous correlation

$$\rho = \frac{\sigma_2}{\sqrt{\sigma_2^2 + \kappa\mu_J^2}}.$$

We assume no disagreement on jump intensity of the Poisson process N_t and study the equilibrium prices without or with circuit breakers.

E.1 The Equilibrium Prices without Circuit Breakers

The pricing formula has the same expression as that in the uncorrelated case.

$$\hat{S}_{j,t} = \mathbb{E}_t^A \left[\frac{\pi_T^A D_{j,T}}{\mathbb{E}_t^A[\pi_T^A]} \right], j = 1, 2,$$

where $\pi_T^A = \gamma\zeta\mathbb{E}_t^A[\eta_T^{1/2} \cdot e^{-\frac{\gamma}{2}(D_{1,T}+D_{2,T})}]$. However, the two prices cannot be evaluated separately anymore because the two dividend processes are correlated ($\sigma_2 \neq 0$).

E.2 The Equilibrium Prices with Circuit Breakers

We derive the market clearing prices when the market is closed early due to the circuit breaker.

Type A investors need to maximize the individual utility function

$$\max_{\theta_{1,\tau}^A, \theta_{2,\tau}^A} \mathbb{E}_t^A[-e^{-\gamma(\theta_{1,\tau}^A(D_{1,T}-S_{1,\tau})+\theta_{2,\tau}^A(D_{2,T}-S_{2,\tau}))}].$$

It results in first order conditions:

$$-\gamma(D_{1,\tau} - S_{1,\tau}) - \gamma\mu_1^A(T-\tau) + \gamma^2(\theta_{1,\tau}^A\sigma_1 + \theta_{2,\tau}^A\sigma_2)\sigma_1(T-\tau) = 0, \tag{E.5}$$

$$-\gamma(D_{2,\tau} - S_{2,\tau}) - \gamma\mu_2(T-\tau) + \gamma^2(\theta_{1,\tau}^A\sigma_1 + \theta_{2,\tau}^A\sigma_2)\sigma_2(T-\tau) - \gamma\mu_J\kappa^A e^{-\gamma\theta_2^A\mu_J} = 0. \tag{E.6}$$

For Type B investors, the optimization problem is

$$\max_{\theta_{1,\tau}^B, \theta_{2,\tau}^B} \mathbb{E}_t^B[-e^{-\gamma(\theta_{1,\tau}^B(D_{1,T}-S_{1,\tau})+\theta_{2,\tau}^B(D_{2,T}-S_{2,\tau}))}].$$

We first obtain an expression for the following expectation for any real numbers x and y:

$$\mathbb{E}_t^B[e^{x\int_t^T \delta_s ds + y(Z_T^B - Z_t^B)}] = e^{A(t;x,y)+C(t;x)\delta_t},$$

where

$$\begin{aligned} A(t;x,y) &= \frac{y^2}{2}(T-\tau) + k\bar{\delta}\int_t^T C(s;x)ds + \frac{\nu^2}{2}\int_t^T C(s;x)^2 ds + y\nu\int_t^T C(s;x)ds, \\ C(t;x) &= \frac{x}{k - \frac{\nu}{\sigma_1}}(1 - e^{(k-\frac{\nu}{\sigma_1})(\tau - T)}). \end{aligned}$$

Then let $y = -\gamma(\theta_{1,\tau}^B\sigma_1 + \theta_{2,\tau}^B\sigma_2)$ and $x = -\gamma(\theta_{1,\tau}^B + \theta_{2,\tau}^B\frac{\sigma_2}{\sigma_1})$; we obtain the first order conditions for the maximization problem of Type B:

$$-\gamma(D_{1,\tau} - S_{1,\tau}) - \gamma\mu_1^A(T-\tau) + \frac{dA(t;x,y)}{d\theta_{1,\tau}^B} + \frac{dC(t;x)}{d\theta_{1,\tau}^B}\delta_t = 0, \tag{E.7}$$

$$-\gamma(D_{2,\tau} - S_{2,\tau}) - \gamma\mu_2(T-\tau) - \gamma\kappa\mu_J(T-\tau)e^{-\gamma\theta_{2,\tau}^B\mu_J} + \frac{dA(t;x,y)}{d\theta_{2,\tau}^B} + \frac{dC(t;x)}{d\theta_{2,\tau}^B}\delta_t = 0. \tag{E.8}$$

Along with the market clearing condition $\theta_{j,\tau}^A + \theta_{j,\tau}^B = 1, j = 1, 2$, the four first order conditions determine the solution $S_{1,\tau}^*, S_{2,\tau}^*, (\theta_{1,\tau}^A)^*, (\theta_{2,\tau}^A)^*$, that is the market clearing prices and the share holdings at the market early closure time τ, respectively.

Next, as in the case of uncorrected dividend processes, we obtain the indirect utility functions for Type A and Type B investors and the state price density. The equilibrium stock prices at $t < \tau$ can be evaluated numerically by solving a fixed point problem similar to (27).

In the above, we deal with the case of no disagreement on the jump intensity. To incorporate a stochastic disagreement $\delta_{2,t}$ into the model, we can follow the procedure in Appendix B.2.

F Numerical Algorithms

P1: Solve for the Fixed Point Problem

Step 1. Initialize a solution and the threshold by letting $S_{1,0} = 1, S_{2,0} = 1$ and $h = (S_{1,0} + S_{2,0})(1 - \alpha)$.

Step 2. Generate M pairs of sample paths of $D_{1,t}$ and $D_{2,t}$ according to their dynamics

(1) and (2) and stochastic differential equations of (6), (7) of $\delta_{1,t}$ and $\delta_{2,t}$.

Step 3. Go through each sample ω: (1) Check whether the circuit breaker is triggered at some time τ before T. If it is triggered, let $S_{j,T\wedge\tau} = S^c_{j,\tau}$, which is the market clearing price at time τ. Otherwise, let $S_{j,T\wedge\tau} = D_{j,T}$. (2) Calculate $\pi^A_{T\wedge\tau} S_{j,T\wedge\tau}$ and $\pi^A_{T\wedge\tau}$ for each sample, where $\pi^A_{T\wedge\tau}$ is calculate by Eq. (24). (3) Find the averages of the two quantities over all samples. (4) Then let $\tilde{S}_{j,0}$ be the ratio of the two averages.

Step 4. If $\|\tilde{S}_0 - S_0\| < tol$, we find an approximated solution to the fixed point problem with accuracy tol. Otherwise, let $S_0 = \tilde{S}_0$ and go to Step 1.

P2: Find Stock Prices at any time $t' < \tau$

Step 1. Using S_0 obtained by P1 codes, let $h = (S_{1,0} + S_{2,0})(1 - \alpha)$ be the threshold of the circuit breakers.

Step 2. Generate M pairs of sample paths of $D_{1,t}$ and $D_{2,t}$ from t' to T.

Step 3. Go through each sample ω: (1) Check whether the circuit breaker is triggered at some time τ before T. If it is triggered, let $S_{j,T\wedge\tau} = S^c_{j,\tau}$, which is the market clearing price at time τ. Otherwise, let $S_{j,T\wedge\tau} = D_{j,T}$. (2) Calculate $\pi^A_{T\wedge\tau} S_{j,T\wedge\tau}$ and $\pi^A_{T\wedge\tau}$ for each sample, where $\pi^A_{T\wedge\tau}$ is calculate by Eq. (24). (3) Find the averages of the two quantities over all samples. (4) Then let $S_{j,t'}$ be the ratio of the two averages, that is the stock price at time t'.

P3: Calculate Correlation and Volatilities of $S_{1,t}$ and $S_{2,t}$

Step 1. Find equilibrium prices $S_{1,t}$ and $S_{2,t}$ at a give time $t < T$ by using P2 codes.

Step 2. Generate M sample pairs of $(D_{1,t+\Delta t},\ D_{2,t+\Delta t})$ given $(D_{1,t}, D_{2,t})$.

Step 3. For each $(D_{1,t+\Delta t},\ D_{2,t+\Delta t})$, calculate the equilibrium price $S_{1,t+\Delta t}$ and $S_{2,t+\Delta t}$ and find the price change $\Delta S_{j,t} = S_{j,t+\Delta t} - S_{j,t}$ for $j = 1, 2$.

Step 4. Calculate the correlation of the M pairs of $(\Delta S_{1,t}, \Delta S_{2,t})$, as well as the volatilities of $\Delta S_{1,t}$ and $\Delta S_{2,t}$.